

Comparison of output-only methods for condition monitoring of industrial systems

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Abstract: In the field of structural health monitoring or machine condition monitoring, the activation of nonlinear dynamic behavior complicates the procedure of damage or fault detection. Blind source separation (BSS) techniques are known as efficient methods for damage diagnosis. However, most of BSS techniques repose on the assumption of the linearity of the system and the need of many sensors. This article presents some possible extensions of those techniques that may improve the damage detection, e.g. Enhanced-Principal Component Analysis (EPCA), Kernel PCA (KPCA) and Blind Modal Identification (BMID). The advantages of EPCA rely on its rapidity of use and its reliability. The KPCA method, through the use of nonlinear kernel functions, allows to introduce nonlinear dependences between variables. BMID is adequate to identify and to detect damage for generally damped systems. In this paper, damage is firstly examined by Stochastic Subspace Identification (SSI); then the detection is achieved by comparing subspace features between the reference and a current state through statistics and the concept of subspace angle. Industrial data are used as illustration of the methods.

Keywords: KPCA; EPCA; NSA; BMID; SSI subspace; condition monitoring; statistics.

1. Introduction

Blind source separation (BSS) techniques allow to recover a set of underlying sources from observations without any knowledge of the mixing process or sources. BSS techniques are shown useful for modal identification [1], for damage detection and condition monitoring [2, 4] from output-only. Among the BSS family, one can cite for example Independent Component Analysis (ICA) [2], Second-Order Blind Identification (SOBI) [5] and an extension of SOBI, called Blind Modal Identification (BMID) [6] which can treat generally damped systems. Principal Component Analysis (PCA) [3] is a linear multivariable statistical method- known as an efficient method to compress large sets of random variables and to extract interesting features from a dynamical system. However, this method is based on the assumption of linearity. To some extent, many systems show a certain degree of nonlinearity and/or non-stationarity, and PCA may then overlook useful information on the nonlinear behavior of the system. As reported in [7], there are many types of damage that make an initially linear structural system respond in a nonlinear manner. Therefore, detection problem may necessitate methods which are able to study nonlinear systems.

Kernel Principal Component Analysis (KPCA) is a nonlinear extension of PCA built to authorize features such that nonlinear dependence between variables. The method is “flexible” in the sense that different kernel functions may be used to better fit the testing data. In the beginning, KPCA has interested many scientists in the domain of image processing [8, 9]. These researchers showed that KPCA may be more advantageous than other techniques such as PCA or Wavelet Transform etc. in encoding image structure. In the last five years, KPCA has been introduced in other fields of research (e.g. biological treatment process [10], machine monitoring [11, 12, 13]) and has shown its ability in the monitoring of nonlinear process.

A main drawback of BSS techniques cited above is the need of several sensors. If the number of sensors is too small, modal identification and/or damage detection may not be performed in good conditions. An alternative PCA-based method named Null Subspace Analysis (NSA), using block Hankel matrices was proposed to detect damages in bearings [14] and on an airplane mock-up [15]. Furthermore, the Hankel matrices were also exploited to enhance some other detection methods namely KPCA, BMID, called EKPCA and EBMID [16]. With the data generated by mean of block Hankel matrices, those methods have been proven to be efficient when the number of available sensors is small or even reduced to one sensor only [15-17].

The focus of this paper is the application of output-only health monitoring techniques to detect damaged mechanical components in industrial environment. First the PCA method is described briefly as it constitutes the background of the EPCA and KPCA methods. Then the definition of the block Hankel matrices is recalled to introduce the EPCA methods. KPCA and BMID are next presented as well as their enhanced versions. Two detection indicators are used which are based on the concept of subspace angle and on statistics. The methods are illustrated on two applications which consist in detecting damage in a rotating device and in controlling quality of welded joints in a steel processing plan. In both cases, only one sensor signal is exploited.

2. Principal Component Analysis (PCA)

Let us assume that a dynamical system is characterized by a set of vibration features collected in the matrix $\mathbf{X} \in \mathfrak{R}^{(m \times N)}$, where m is the number of sensors and N is the number of samples. In general, PCA involves a data normalization procedure, which leads to a set of variables with zero-mean and unitary standard deviation. This method, also known as Karhunen-Loève transform or Proper Orthogonal Decomposition (POD) [3], provides a linear mapping of data

from the original dimension m to a lower dimension p . The dimension p represents the physical order of the system or the number of principal components which affect the vibration features. In practice, PCA is often computed by a Singular Value Decomposition (SVD) of matrix \mathbf{X} , i.e.

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (1)$$

where \mathbf{U} and \mathbf{V} are orthonormal matrices, the columns of \mathbf{U} define the principal components (PCs). The order p of the system is determined by selecting the first p non-zero singular values in $\mathbf{\Sigma}$ which have a significant magnitude (“energy”) as described in [3]. A threshold in terms of cumulated energies is often fixed to select the effective number of PCs that is necessary for a good representation of the matrix \mathbf{X} . In practice, a cumulated energy of 75% to 95% is generally adequate for the selection of the active PCs.

3. Block Hankel matrices

The block Hankel matrices play an important role in subspace system identification [18]. Those matrices characterize the dynamics of the analyzed signals and have been used for modal identification and damage detection [15-17].

The covariance-driven block Hankel matrix is defined as:

$$\mathbf{H}_{r,c} = \begin{bmatrix} \Delta_1 & \Delta_2 & \dots & \Delta_c \\ \Delta_2 & \Delta_1 & & \Delta_{c+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_r & \Delta_{r+1} & \dots & \Delta_{r+c+1} \end{bmatrix} \quad (c \geq r) \quad (2)$$

where r, c are user-defined parameters ($r = c$ in this paper) and Δ_i represents the output covariance matrix defined by:

$$\Delta_i = \frac{1}{N-i} \cdot \sum_{k=1}^{N-i} \mathbf{x}_{k+i} \cdot \mathbf{x}_k^T \quad (0 \leq i \leq N-1) \quad (3)$$

The data-driven Hankel matrix is defined as:

$$\mathbf{H}_{1,2i} = \begin{bmatrix} x_1 & x_2 & \dots & x_j \\ x_2 & x_3 & & x_{j+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_j & x_{j+1} & \dots & x_{i+j+1} \\ \hline x_{j+1} & x_{j+2} & \dots & x_{i+j} \\ x_{j+2} & x_{j+3} & & x_{i+j+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{2i} & x_{2i+1} & \dots & x_{2i+j-1} \end{bmatrix} \equiv \begin{pmatrix} \mathbf{X}_p \\ \mathbf{X}_f \end{pmatrix} \equiv \begin{matrix} \text{"past"} \\ \text{"future"} \end{matrix} \quad (4)$$

where $2i$ is a user-defined number of row blocks, each block contains m rows (number of measurement sensors), j is the number of columns (practically $j = N - 2i + 1$, N is the number of sampling points).

4. Enhanced Principal Component Analysis (EPCA)

EPCA is basically a principal component analysis of the covariance-driven block Hankel matrix \mathbf{H} [15]. The smallest singular values of the matrix \mathbf{H} which correspond to components of low “energy” are actually associated to noise or to weak dynamics that may be neglected. Hence, the \mathbf{U} matrix is usually factorized in two subspaces, namely the active (\mathbf{U}_1) and the null subspace (\mathbf{U}_2) [15]. The SVD decomposition of the matrix \mathbf{H} becomes:

$$\mathbf{H} = [\mathbf{U}_1 \quad \mathbf{U}_2] \begin{bmatrix} \mathbf{S}_1 & 0 \\ 0 & \mathbf{S}_2 \end{bmatrix} [\mathbf{V}_1 \quad \mathbf{V}_2]^T \quad (5)$$

As stated in paragraph 2, the number of components in the active subspace \mathbf{U}_1 is user-defined. It should be chosen such that the dynamics of the signal is accurately modeled without accounting for the background noise. The size of the Hankel matrix is also a user-defined parameter.

5. Kernel Principal Component Analysis

The key idea of KPCA is first to define a nonlinear map $\mathbf{x}_k \mapsto \Phi(\mathbf{x}_k)$ with $\mathbf{x}_k \in \mathfrak{R}^m$, ($k = 1, \dots, N$) which defines a high dimensional feature space F , and then to apply PCA to the data in space F [8].

Let us define the kernel matrix \mathbf{K} of dimensions $N \times N$ such that:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \Phi(\mathbf{x}_i)^T \Phi(\mathbf{x}_j) \quad (6)$$

The following kernel functions may be used:

- polynomial kernel function,

$$K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i^T \mathbf{x}_j + 1)^d \quad (7)$$

where d is a positive integer

- radial basis function (RBF),

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\|\mathbf{x}_i - \mathbf{x}_j\|^2 / 2\sigma^2) \quad (8)$$

where $2\sigma^2 = w$ is the width of the Gaussian kernel

It is worth noting that in general, the above kernel functions give similar results if appropriate parameters are chosen [16]. The last function may present some advantages because of its flexibility in the choice of the associated parameter. For example, the width of the Gaussian kernel can be very small (<1) or quite large. Contrarily the polynomial function requires a positive integer for the exponent.

KPCA may be effectively enhanced by using the covariance-driven Hankel matrices which improves the sensitivity of the detection [16] and also the computation cost. The combined method will be called Enhanced KPCA (EKPCA) in the following.

6. Blind Mode Identification

BMID is a blind source separation technique (BSS) based on the Second-Order Blind Source Identification (SOBI). The principle of the method is to apply SOBI on an augmented and pre-treated dataset. Compared to SOBI, its main advantage for the identification problem, is to deal with generally damped system.

SOBI considers the observed signals as a noisy instantaneous linear mixture of source signals. In many situations, multidimensional observations are represented as [5]:

$$\mathbf{x}(t) = \mathbf{y}(t) + \boldsymbol{\sigma}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\sigma}(t) \quad (9)$$

where

- $\mathbf{x}(t) = [x_1(t), \dots, x_m(t)]^T$ is an instantaneous mixture of source signals and of noise.
- $\mathbf{s}(t) = [s_1(t), \dots, s_p(t)]^T$ contains the signals issued from p narrow band sources, $p < m$.
- $\mathbf{y}(t) = [y_1(t), \dots, y_m(t)]^T$ contains the source assembly at a time t .

\mathbf{A} is the transfer matrix between the sources and the sensor, called the mixing matrix. Under certain conditions, the mixing matrix identifies to the modal matrix of the structure and the sources correspond to normal coordinates [1].

$\boldsymbol{\sigma}(t)$ is the noise vector, modeled as a stationary white, zero-mean random process. Furthermore it is assumed to be independent of the sources.

The BMID technique, proposed by McNeil and Zimmerman [6], consists in applying SOBI to a augmented and pre-treated dataset formed by the original data, denoted $\mathbf{x}_0(t)$, and their Hilbert transform pairs $\mathbf{x}_{90}(t)$. The mixing problem becomes double-sized:

$$\mathbf{x}^{(2m \times 1)} = \mathbf{A}^{(2m \times 2p)} \mathbf{s}^{(2p \times 1)} \quad (10)$$

$$\begin{bmatrix} \mathbf{x}_0^{(m \times 1)} \\ \mathbf{x}_{90}^{(m \times 1)} \end{bmatrix} = \mathbf{A}^{(2m \times 2p)} \begin{bmatrix} \mathbf{s}_0^{(m \times 1)} \\ \mathbf{s}_{90}^{(m \times 1)} \end{bmatrix} \quad (11)$$

BMID can also be enhanced by the Hankel matrices, particularly when only one sensor is used for the detection [16]. Like EPCA and EKPCA, the number of blocks of the Hankel matrix is a user-defined parameter.

7. Damage detection using the concept of angle between subspaces

The concept of subspace angle was introduced by Golub and Van Loan [19]. This concept can be used as a tool to quantify the spatial coherence between two data sets resulting from observation of a vibration system [3,16].

Given two subspaces (each with linear independent columns) $\mathbf{S} \in \mathbb{R}^{(m \times p)}$ and $\mathbf{D} \in \mathbb{R}^{(m \times q)}$ ($p \geq q$), the procedure is as follow. Carry out the QR factorizations

$$\begin{aligned} \mathbf{S} &= \mathbf{Q}_S \mathbf{R}_S, \quad \mathbf{Q}_S \in \mathbb{R}^{(m \times p)} \\ \mathbf{D} &= \mathbf{Q}_D \mathbf{R}_D, \quad \mathbf{Q}_D \in \mathbb{R}^{(m \times q)} \end{aligned} \quad (12)$$

The columns of \mathbf{Q}_S and \mathbf{Q}_D define the orthonormal bases for \mathbf{S} and \mathbf{D} respectively. The angles θ_i between the subspaces \mathbf{S} and \mathbf{D} are computed from singular values associated with the product $\mathbf{Q}_S^T \mathbf{Q}_D$:

$$\begin{aligned} \mathbf{Q}_S^T \mathbf{Q}_D &= \mathbf{U}_{SD} \mathbf{\Sigma}_{SD} \mathbf{V}_{SD}^T \\ \mathbf{\Sigma}_{SD} &= \text{diag}(\cos(\theta_i)), \quad i = 1, \dots, q \end{aligned} \quad (13)$$

The largest singular value is thus related with the largest angle characterizing the geometric difference between two subspaces.

The change in the system dynamics may be detected by monitoring the angular coherence between subspaces estimated from a reference observation set and from the observation set of a current state of the system. A state is considered as reference if the system operates in normal conditions (damage does not exist). Figure 1 shows a 2D example in which an active subspace built from two principal vectors which characterize the dynamic behavior of system.

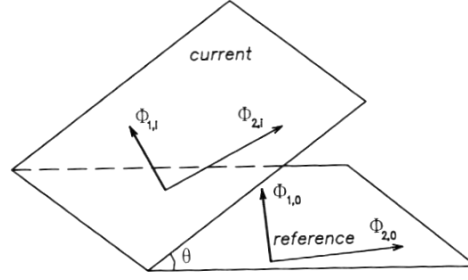


Figure 1 - Angle θ formed by active subspaces according to the reference and current states, due to a dynamic change [3]

In the case of EPCA, the considered subspaces are the active subspaces \mathbf{U}_1 . Based on KPCA, the subspaces are built by the kernel principal components. And finally, in BMID, the subspace is built by the first columns of the mixing matrix.

8. Damage detection using statistics

The second type of indicator is based on the computation of errors in the reconstruction of the data. This method is known as “Novelty Analysis”. In the particular case of EPCA, the first columns \mathbf{U}_1 of \mathbf{U} may be used to project the measured features into the space characterized for a current state of the system. The loss of information in this projection can be assessed by re-mapping the projected data back to the original space:

$$\hat{\mathbf{H}} = \mathbf{U}_1 \mathbf{U}_1^T \mathbf{H} \quad (14)$$

The residual error matrix \mathbf{E} is estimated as

$$\mathbf{E} = \mathbf{H} - \hat{\mathbf{H}} \quad (15)$$

From the prediction error vector \mathbf{E}_k , the Novelty Index (NI) is defined using the Mahalanobis norm:

$$NI_k^M = \sqrt{\mathbf{E}_k \mathbf{R}^{-1} \mathbf{E}_k} \quad (16)$$

where $\mathbf{R} = \left(\frac{1}{N}\right) \mathbf{H} \mathbf{H}^T$ is the covariance of the features.

In the case of BMID, the projection is based on the active subspace defined by the first columns of the mixing matrix. For KPCA, the projection is realized on the first kernel principal components. For the last case, the statistic indicator is called squared prediction error (SPE), more details can be found in [10].

9. First application

9.1 Experimental setup

This industrial application concerns the case of electro-mechanical devices for which the overall quality at the end of the assembly line has to be assessed.

A set of nine rotating devices was instrumented with two accelerometers: one triaxial accelerometer was located on the flank of the component, and one monoaxial on the top as illustrated in Figure 2. Among this set of nine devices, five of them are known to be healthy (referenced OK-0 \rightarrow OK-4) and the other four are faulty (NOK-1 \rightarrow NOK-4).

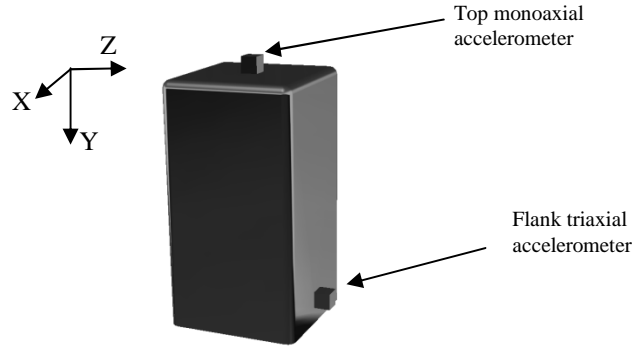


Figure 2 - Locations of the accelerometers on the analysed devices: one monoaxial accelerometer on the top and one tri-axial accelerometer on the flank of the device

A total of 4 915 200 points with a sampling frequency of 20 480 Hz were measured on each channel using a LMS Scadas III acquisition system. In the following chapters, only the data's in the Y direction collected by the flank accelerometer are used as it has been shown that the detection is the best in this direction [20].

9.2 Stochastic Subspace Identification results

The Stochastic Subspace Identification (SSI) technique is a well known output-only modal identification technique that treats directly recorded time signals [18]. The stabilization diagram is a useful tool to select the order of the system.

However, from only one sensor, the SSI method can only perform the diagnostic problem by detecting change of resonant frequencies. The following figures present the stabilization diagram for two healthy devices and two damaged devices. The frequencies pointed out by SSI do not seem to be consistent from one healthy device to another. The difference, in terms of detected frequencies between a healthy device and a damage device is also far from explicit.

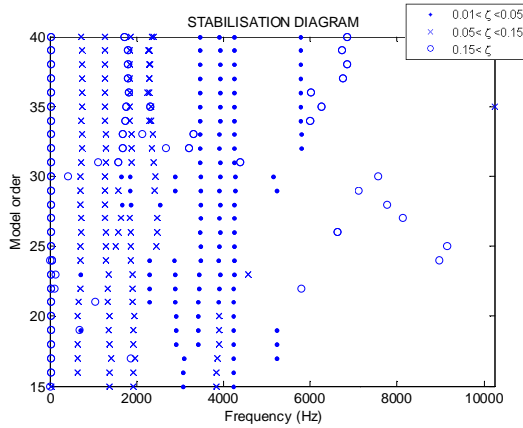


Figure 3 - Stabilization diagram for device OK-1

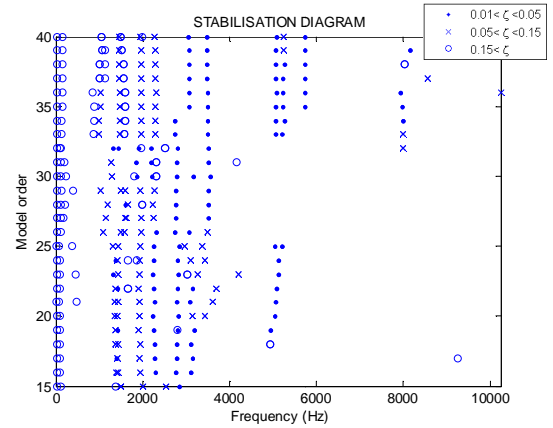


Figure 4 - Stabilization diagram for device OK-3

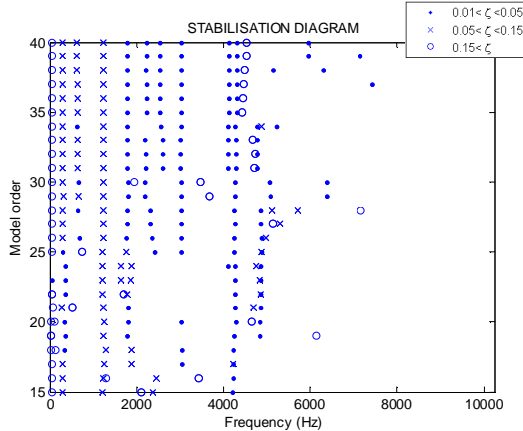


Figure 5 - Stabilization diagram for device NOK-4

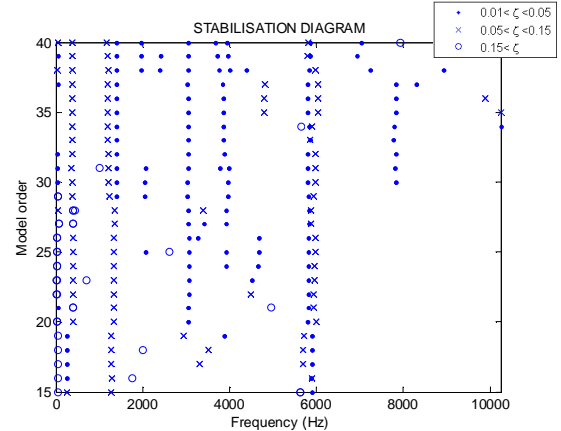


Figure 6 - Stabilization diagram for device NOK-1

9.3 Fault detection with EPCA, EKPCA and EBMID

The parameters of the different methods are set as follow:

Parameter	EPCA	EKPCA	EBMID
Number of Hankel blocks	Statistics: 28 ; Angle:6	35	8
Number of active principal components	Statistics: 26; Angle:2	3	2

Table 1 – Parameters of the diagnosis methods for the rotating devices.

The detection results of the different methods are presented in the following figures, using the statistics and the concept of angle between subspaces. When using statistics, the Mahalanobis norm is used for EPCA and EBMID; the SPE monitoring for EKPCA. The EKPCA method is applied with the width of the Gaussian kernel $w=3$. The device called OK-0 is used as the reference healthy device.

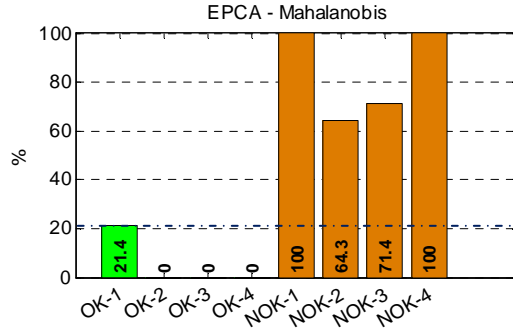


Figure 7 – EPCA detection using statistics

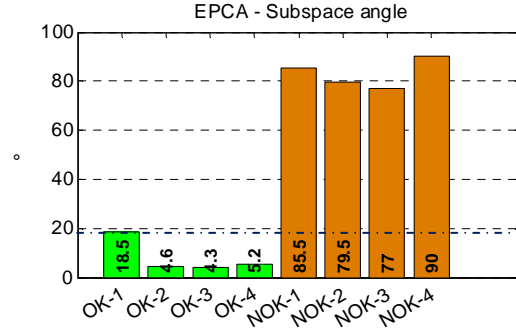


Figure 8 – EPCA detection using the concept of subspace angle

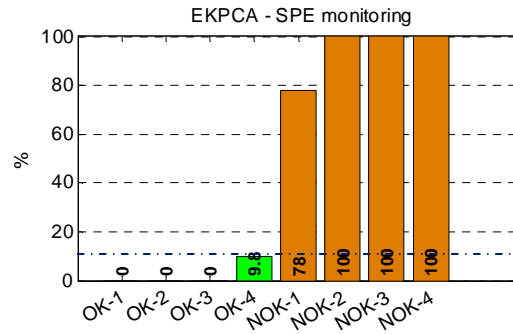


Figure 9 – EKPCA detection using statistics

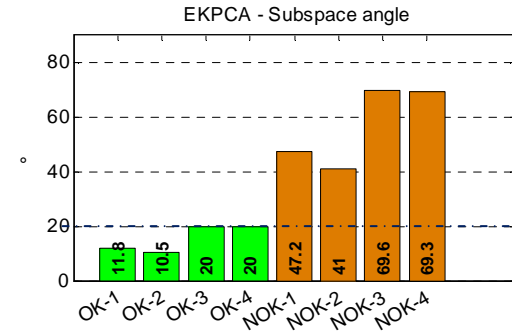


Figure 10 – EKPCA detection using the concept of subspace angle

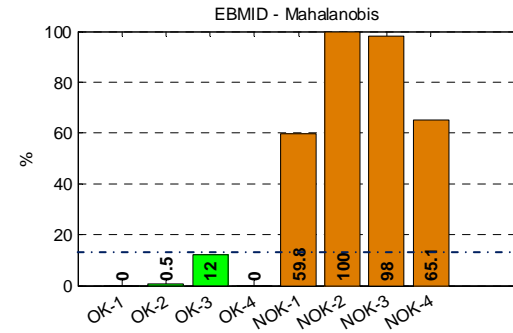


Figure 11 – EBMID detection using the statistics

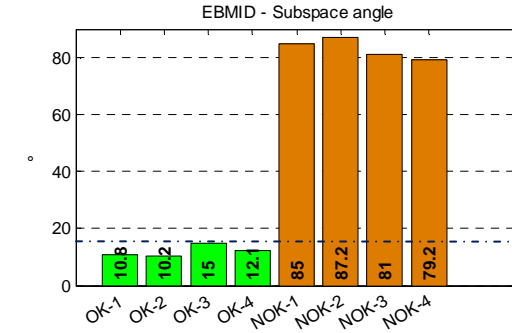


Figure 12 – EBMID detection using the concept of subspace angle

All the methods were able to successfully diagnose the faulty devices. Only the EKPCA detection using the subspace angle appears to less clear than the other results. Notice also the consistent results of the EBMID method.

10. Second application

10.1 Experimental setup

The second example involves an industrial welding machine from a steel processing plan. The machine was instrumented with a monoaxial accelerometer on the upper forging wheel (as illustrated in figure 13). The purpose of this wheel is to flatten the welded joint during the welding process.

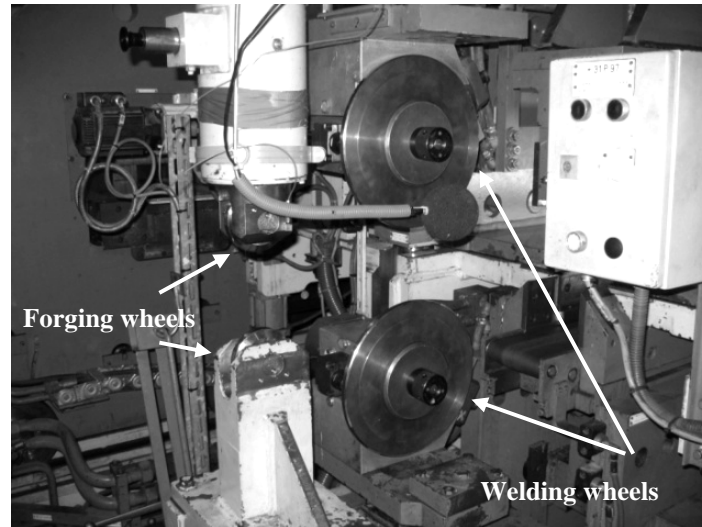


Figure 13 - Location of the accelerometer on the forging wheel of the welding machine

Vibration measurements were acquired during the whole welding process (about 4 seconds) at a sampling frequency of 25600 Hz using a National Instrument data acquisition device.

The quality of the welded joints depends on several parameters (welding current, pressure, relative position of the metal strips,...). In this example, four distinct parameters were altered and multiple alteration levels were considered, leading to a batch of 27 welded joints with out-of-range parameters. Six welded joints were realized using nominal parameters for false-positive testing (see Table 2).

<i>Name</i>	<i>Modified parameter</i>	<i>Number of samples</i>	<i>Weld quality</i>
Welding OK	All parameters at nominal level	6	Good
Welding A	-33% covering	3	Acceptable
Welding B	-66% covering	3	Bad
Welding C	-33% compensation	3	Good
Welding D	-66% compensation	3	Acceptable
Welding E	-10% current	3	Acceptable
Welding F	-20% current	3	Bad
Welding G	-10% forging pressure	3	Good
Welding H	+5% forging pressure	3	Acceptable
Welding I	-66% covering and compensation	3	Bad

Table 2 – Realized welded joints. The parameters “Covering” and “Compensation” determine the relative position of the metal strips.

A modification of a welding parameter does not always lead to a bad welding. A microscopic quality control of each welded joint was realized after the measurement campaign to assess their actual quality (Figure 14). Welded joints realized with nominal parameters as well as welded joints from the C and G series were diagnosed good, welded joints named A, D, E, H were diagnosed acceptable, and welded joints B, F, I were diagnosed bad (Table 2).

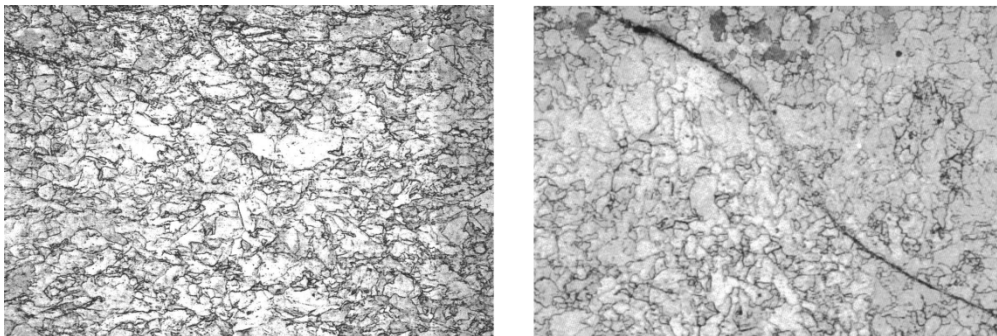


Figure 14 - Microscopic control of the welded joints. The figure on the left is an example of a good welded joints and the one on the right represents a faulty welded joint.

10.2 Stochastic Subspace identification results

The stabilization diagrams shows interesting results. In the case of a good welded joint, one stabilized frequencies is pointed out in the range [500-900] Hz, although the value of this frequency may change from one healthy device to another (Figure 15 and 16). In the case of a bad welded joint, no stabilized frequency can be found in the same frequency range, (Figure 17 and 18).

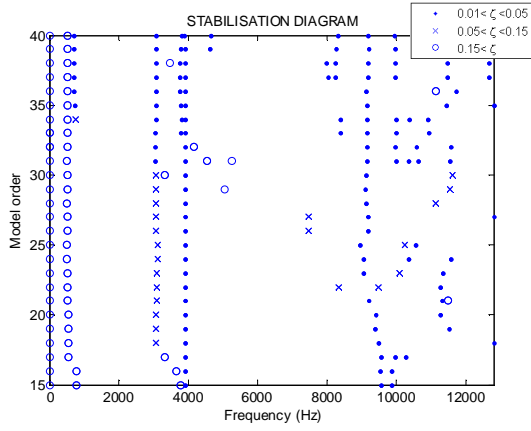


Figure 15 - Stabilization diagram for welding OK-1

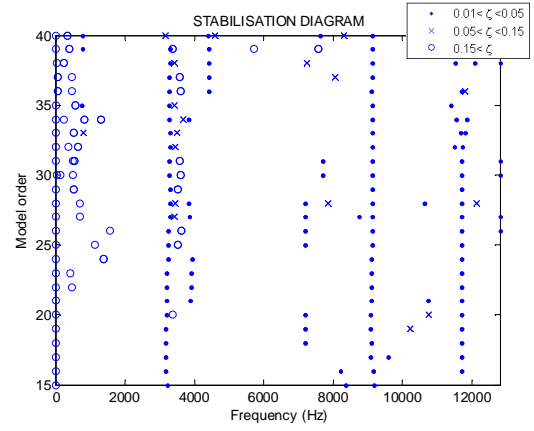


Figure 17 - Stabilization diagram for welding B-3

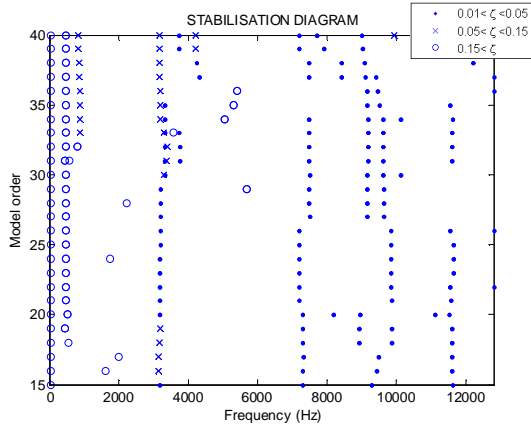


Figure 16 - Stabilization diagram for welding OK-5

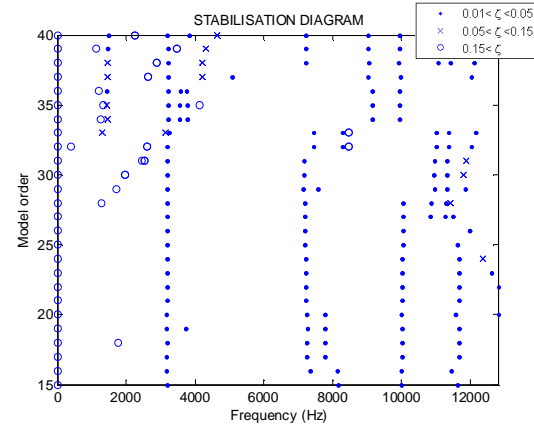


Figure 18 - Stabilization diagram for welding F-3

10.3 Fault detection results with EPCA, EKPCA and EBMID

The parameters of the different methods are presented in Table 3. The radial basis function was used for the EKPCA method with $w=150$.

Parameter	EPCA	EKPCA	EBMID
Number of Hankle Blocs	35	30	10
Number of active principal components	4	7	Statistics:10 ; Angle:1

Table 3 – Parameters of the diagnosis methods for the welded joints.

The detection results are shown in the following figures:

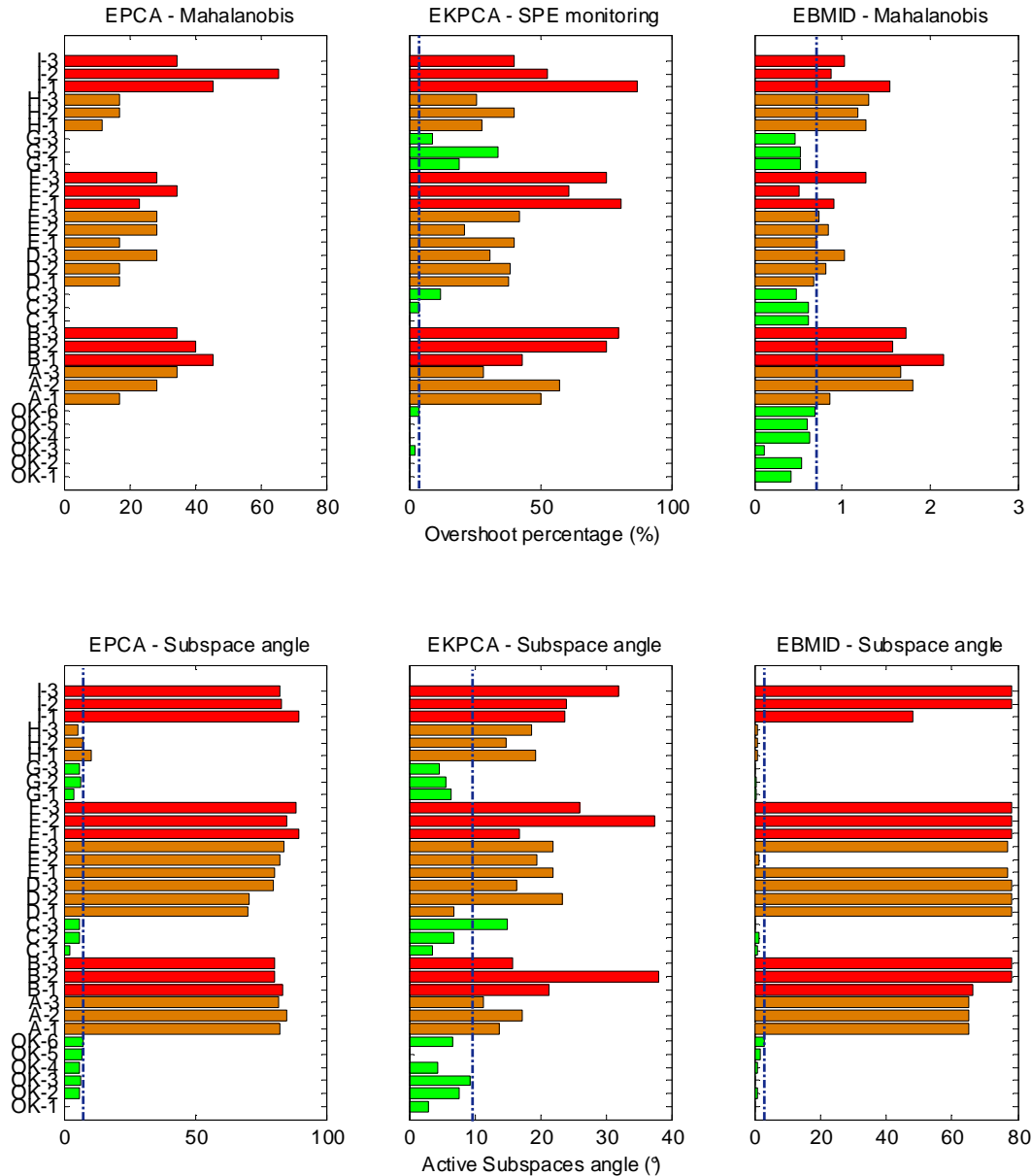


Figure 19 - Detection result for the welded joints

EPCA monitoring shows good results either with the statistics or with the subspace angle. In this last case however, the weldings H-2 and H-3 are not detected as faulty. However, the bad weldings are all correctly detected.

The EKPCA method also gives interesting results but seems to be a little bit too responsive to small changes in the dynamics of the signals. Indeed, with the statistical indicator, welding C-3 and the entire G-series give rather important indexes. With the subspace angle indicator, the G-series is no longer diagnosed faulty but the welding D-1 is not detected. Yet, all the bad weldings are again correctly detected.

EBMD presents some contrary results. With the statistics, the method fails to detect one welding which belongs to the bad welding group (F-2). Also, the values of the indicator are very small (only a few percent). However, with the subspace angle all the bad weldings are correctly diagnosed. Only welding E-2 is not noticed as faulty while the others weldings from this series are obviously detected. The entire G-series (belonging to the “Acceptable” group) is not detected as faulty.

11. Conclusion

Vibration monitoring is a useful tool to diagnose fault or to detect damage in a non intrusive way but the placement of multiple sensors can be time consuming and is not always achievable in industrial applications.

The EPCA, KPCA and BMID method can detect damage by comparison with a reference, healthy signal. Through the use the Hankel matrices, the “enhanced” version of KPCA and BMID and EPCA can be applied to damage detection problem where only one sensor is available, which is an appreciable advantage.

Two industrial applications have been presented. In both cases, the presented methods performed quite well in detecting faulty systems. However, it is worth noting that the detection results are highly dependent upon the parameters of the methods.

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