Title

How to incorporate prior information in geophysical inverse problems: deterministic and geostatistical approaches.

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Abstract

Many geophysical inverse problems are ill-posed leading to non-uniqueness of the solution. It is thus important to reduce the amount of mathematical solutions to more geologically plausible models by regularizing the inverse problem and incorporating all available prior information in the inversion process. We compare three different ways to go beyond standard Occam's inversion for electrical resistivity tomography (ERT) using electromagnetic logging data in the context of salt water infiltration: a simple reference model, a structural constraint and a geostatistical constraint based on a vertical correlation length. Results with the traditional smoothness constraint yield small contrasts of resistivity, far from the reality revealed by borehole measurements. Incorporating prior information from boreholes clearly improves the misfit with logging data. If a good reference model can always be used, it can lead to misinterpretation if its weight is too strong. When the computation of the correlation length is possible, the geostatistical inversion gives satisfactory results everywhere in the section. In this specific case, the geostatistical approach seems to be a more robust way to incorporate prior information. The structural constraint seems to be more indicated when integrating information from other geophysical methods such as GPR or seismic.

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Introduction

Many geophysical inverse problems are ill-posed, due to the non-linearity of the model, noisy data sets and limited number of data versus number of parameters, leading to the non-uniqueness of the solution. Consequently, an infinite number of models are able to explain the data. A common way to manage such problems is to regularize them by additional constraints.

To regularize inverse problems in applied geophysics, the smoothness constrained regularization (Constable et al. 1987) has become standard. However, in a lot of cases, such constraint is not consistent with the prior geological knowledge of the study site. One thus wants to reduce the amount of mathematical solutions to more geologically plausible models by incorporating all available prior information in the inversion process (e.g. Blakely 1995, chapter 9).

In most cases, prior information results from borehole and logging data or from another geophysical method. In the latter case, joint interpretation or joint inversion are two possible ways to interpret the different data. Another possibility is to use one data set to constrain the inversion of the second data set (cooperative inversion). This technique is also useful to integrate borehole information in the inversion process.

In this paper, we compare three different ways to go beyond standard Occam's inversion for electrical resistivity tomography (ERT) using electromagnetic and geological logging data in the context of a salt water infiltration at the Belgian coast. The first method simply uses a reference model based on borehole data to constrain the solution. The second technique adds a structural constraint to delineate different facies (Kaipio et al. 1999). These two techniques can be qualified as deterministic. The third method uses geostatistical information to constrain vertically the inversion (Martin et al. 2010).

Methodology

We used the program CRTOMO (Kemna 2000) for ERT inversions. The three different methods used in this study can be differentiated by their objective functional. To incorporate a reference model to the smoothness constraint inversion, the regularization term in the objective function can be written as (Oldenburg and Li 1994):

$$\phi = \left\| W_d (d - f(m)) \right\|^2 + \lambda \left(\left\| W_m m + \alpha (m - m_0) \right\|^2 \right) \qquad (1)$$

where W_d is the data weighting matrix, f is the non-linear operator mapping the log conductivities of the model m to the log impedance data set d. W_m is the roughness matrix, λ is the regularization parameter, m_0 is the reference model and α weights the importance of the reference model. λ is optimized at each iteration to minimize the data misfit. At the last iteration, it is modified to fit exactly the assumed level of noise.

To add a structural constraint, we followed the methodology described by Kaipio et al. (1999). They modified W_m using Lie derivatives. At a point x close to a discontinuity, the smoothness constraint is modified such that the penalty to a rapid change of the model is smaller towards the discontinuity than along it.

For the geostatistical regularization, borehole data were used to compute a vertical variogram. An anisotropy ratio was inferred from previous inversion results. Vertical and horizontal variograms were then used to calculate the covariance matrix C_m following the method of Chasseriau and Chouteau (2003). It leads to the objective functional described by Martin et al. (2010):

$$\phi(m) = \left\| W_d (d - f(m)) \right\|^2 + \lambda \left\| C_{\mathbf{m}}^{-0.5} (m - m_0) \right\|^2 \qquad (2)$$

Results and discussion

The example shown in figures 1 and 2 corresponds to a salt water infiltration at the Belgian coast. Two sea inlets were built crossing the fore dunes. Sea water has thereby access to two dune slacks. As a freshwater lens is present in the dune aquifer, exploited for the production of drinking water, it is important to know the spatial extent of the body of infiltrated salt water. The aquifer is mostly composed of sands, but a high clay content layer, 2 to 3 m thick, is present at an elevation between −4 and −7 m; it hinders the downward movement of salt water (for more details, see Vandenbohede et al. 2008).

ERT measurements were carried out in addition to borehole electromagnetic logging (EM39) to map the position of salt water. Boreholes contain important prior information to constrain ERT inversion. Indeed, standard smoothness constrained inversion yields poor results, not useful for a quantitative interpretation. The salt water body is spread over all the aquifer thickness (figure 1A), the contrast between salt and fresh water is small and far from the reality, as given by the EM39 logs (figure 2).

Several attempts were made to include borehole data; they are presented in figures 1B to 1F. First, a homogeneous reference model equal to 100 Ohm-m was used (1B), the value was chosen according to borehole measurements below −15 m, where the sensitivity is low and the model barely constrained by the data. It improves slightly the results but a more complex reference model composed of three layers (equal to 1000 Ohm-m from the surface to 4 m elevation, 5 Ohm-m from 4 m to an elevation between −4 and −7 m and 100 Ohm-m below) yields an improved correlation (figure 1C, figure 2).

However, the maximum conductivity value is still too low and the gradient of conductivity below the salt water body too large. An increased weighting factor α gives more importance to the reference model in the inversion (figures 1D and 1E). In the boreholes, the misfit is improved. The decrease in conductivity is almost perfectly resolved in P11, a little bit too sharp for P18 (figure 2).

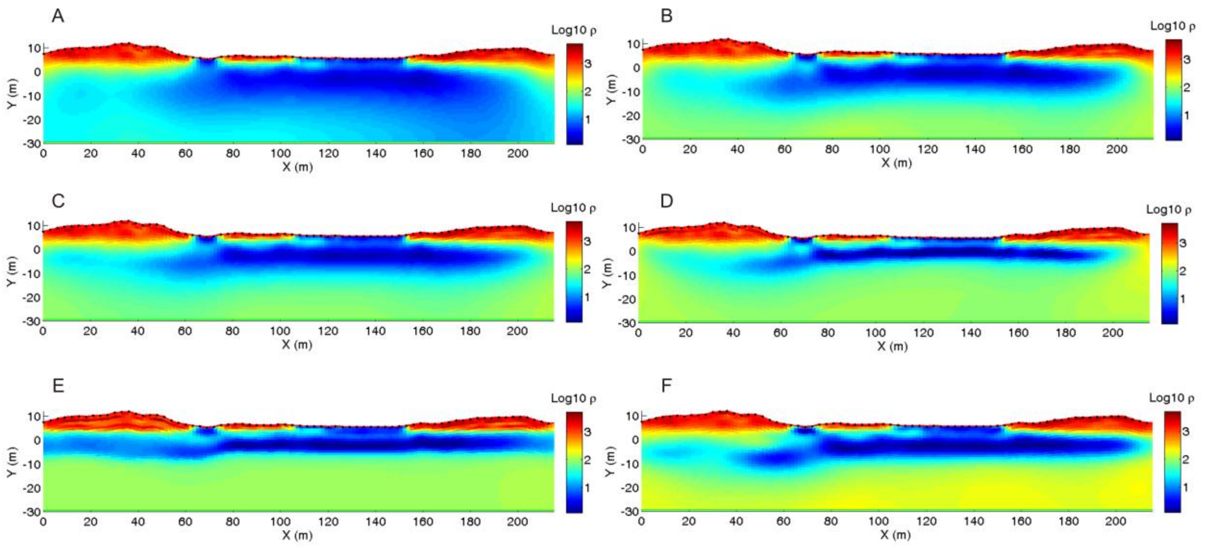

Figure 1 *Inversion results according to different methods. (A) Smoothness constrained solution without reference model. (B) Smoothness constrained solution with a reference model equal to 100 Ohm-m. (C) Smoothness constrained solution with a reference model with three layers (see the text for details). (D) Same as B but the weighting factor for the reference model is increased. (E) Same as C but the weighting factor for the reference model is increased. (F) Geostatistical inversion with a reference model equal to 100 Ohm-m.*

On the left part of figure 1E, we see that the salt water is stretched to the origin of the profile. This is not visible for other inversions. It is due to the fact that the reference has a high weight, thus, the inversion tends to produce a resistivity value close to 5 Ohm-m in this zone, even if this is not expected. It illustrates the limitation of such a deterministic constraint. An incorrect reference model does not matter if its weight is small; when it is increased, it can lead to misinterpretation because the reference model is reproduced in the solution.

Figure 1F illustrates the results obtained with a geostatistical constraint and a homogeneous reference model. The comparison with borehole measurements shows that they represent well the thickness of the salt water body, even if the maximum conductivity value is different. The advantage of this inversion is that the thickness of the salt body is mainly constrained by the correlation length of the vertical variogram, it is not necessary to determine the best parameter or different reference models to find the optimum solution.

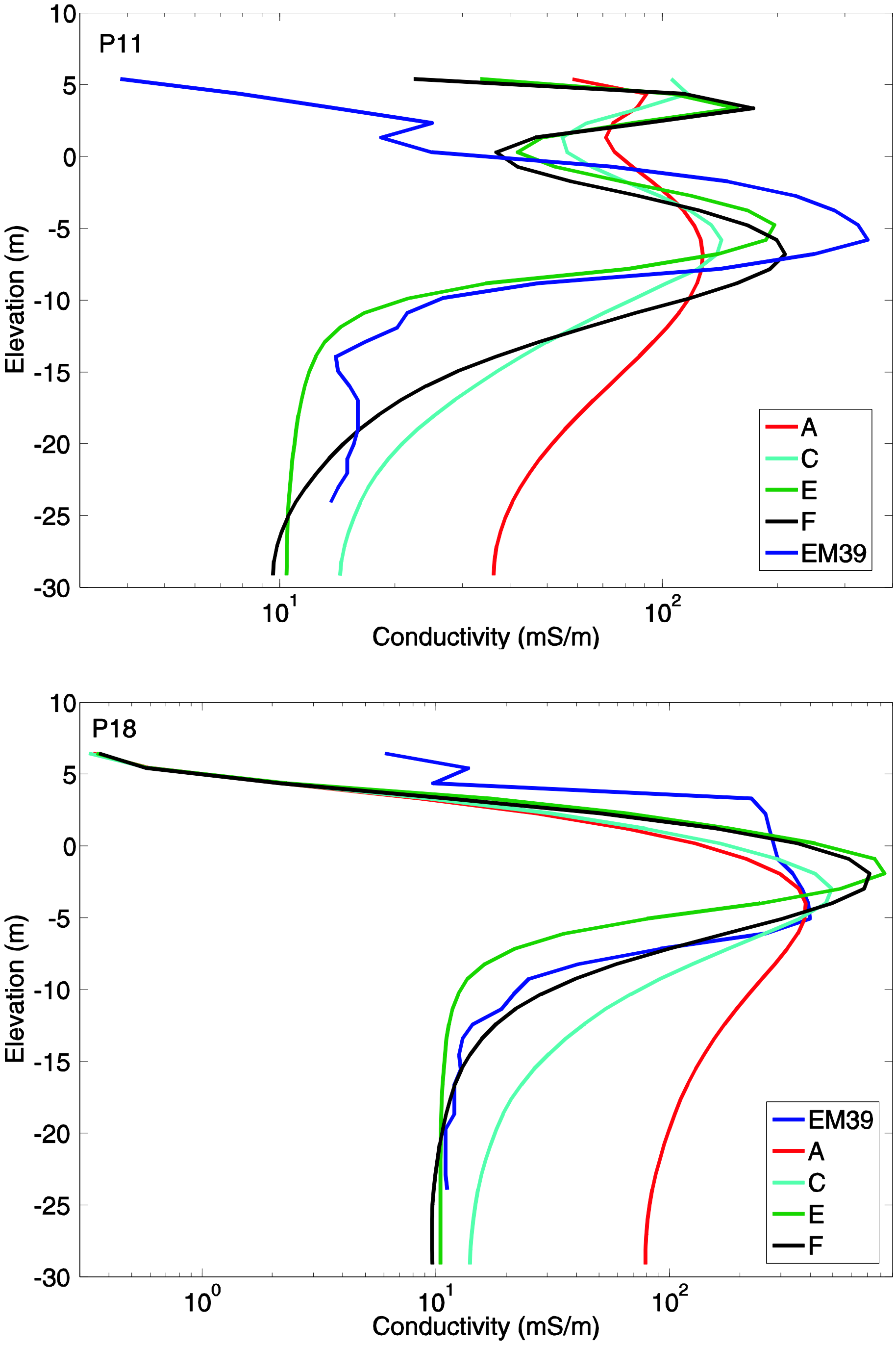

Figure 2 *Comparison of inversion results with EM39 measurements in well P11, located at abscissa 65 m on the profile of figure 1 (top), and P18, located at abscissa 167 m on the profile of figure 1. Letters refer to the inversion results of figure 1.*

Adding a structural constraint in figure 1B or 1C did not improve the results in this case, certainly because low resistivity values are due to salt water infiltration and clay lenses. The structural limit is thus unclear and difficult to determine precisely. However, this type of constraint worked well on two other sites where the water level and a bottom clay layer were imaged by GPR or borehole logs (not shown here). Constraining the ERT inversion with these structural limits enabled to improve highly the resolution within the saturated part of the subsurface.

Conclusions

The results show that the choice of the constraint to apply is highly dependent on the type of information available. If a reference model can always be used, the weight given in the inversion process is crucial to avoid a too strong constraint. Several attempts are necessary to deduce the best parameter to fit borehole measurements and there is no control on other parts of the model.

When the physical parameter can be measured in several boreholes at different depths, the computation of a variogram is possible and a geostatistical constraint seems well suited. However, in most cases, there will remain some uncertainty on the horizontal correlation length. The main advantage of this technique is to use borehole measurements only indirectly (to calculate the variogram); the misfit observed in the borehole is thus expected to be the same elsewhere in the section (if the variogram is representative of the whole site).

However, when only lithological or when other geophysical data sets are available, a structural constraint can be satisfactory. It enables to disconnect different lithological facies and create sharp contrast whereas standard Occam inversion would lead to smooth transitions.

References

Blakely, R.J. [1995] *Potential theory in gravity and magnetic applications*. Cambridge University Press, Cambridge.

Chasseriau, P. and Chouteau, M. [2003] 3D gravity inversion using a model of parameter covariance. *Journal of applied geophysics*, **52**, 59-74.

Constable, S.C., Parker, R.L. and Constable, C.G. [1987] Occam's inversion : a practical algorithm for generating smooth models from electromagnetic sounding data. *Geophysics*, **52**(3), 289-300.

Kaipio, J.P., Kolehmainen, V., Vauhkonen, M. and Somersalo, E. [1999] Inverse problems with structural prior information. *Inverse problems*, **15**, 713-729.

Kemna, A. [2000]. *Tomographic inversion of complex resistivity – theory and application*. PhD thesis, Ruhr-University of Bochum.

Martin, R., Kemna, A., Hermans, T., Nguyen, F., Vandenbohede, A. and Lebbe, L. [2010] Using geostatistical constraints in electrical imaging for improved reservoir characterization. *AGU Fall Meeting 2010*, San Francisco, California, 13-17 December, Abstract H23L-06.

Oldenburg, D.W. and Li, Y. [1994] Subspace linear inverse method. *Inverse problems*, **10**, 1-21.

Vandenbohede, A., Lebbe, L., Gysens, S., Delecluyse, K. and DeWolf, P. [2008] Salt water infiltration in two artificial sea inlets in the Belgian dune area. *Journal of hydrology*, **360**, 77-86.