STOCHASTIC BEHAVIOUR OF EUROPEAN STOCK MARKETS INDICES

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1. Introduction

This paper is concerned with modelling return generating processes in several European stock markets. Distributional properties of daily stock returns play a crucial role in valuation of contingent claims and mean-variance asset pricing models, as well as in their empirical tests. A common assumption underlying a considerable body of finance literature is that the logarithm of stock price relatives are independent and identically distributed according to a normal distribution with constant variance, while little attention is paid to the the empirical fit of the postulated process. For instance, the mean-variance asset pricing models of Sharpe (1964) and the option pricing model of Black and Scholes (1973) are based on the assumption of normally distributed returns. Moreover, the normality assumption and the parameter stability are necessary for most of statistical methods usually applied in empirical studies.

As early as 1963, Mandelbrot observed that returns series tend not to be independent over time, but characterised by succession of stable and volatile periods, that is, "large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes". He also observed that the distributions of returns are leptokurtic and proposed the family of the stable Paretian distributions as an alternative to the normal distribution. Such Paretian distributions with characteristic exponent of less than two indeed exhibit heavy tails and conform better to the distributions of returns series. Fama (1965) contributed further evidence supporting Mandelbrot's hypothesis.

While the approach of these studies is based on the empirical fit of observed stock return distributions, an alternative approach relies on describing the process that could generate distributions of returns having fatter tails than normal distributions. For instance, Paretz (1972), Blattberg and Gonedes (1974) showed that the scaled-t distribution, which can be derived as a continuous variance mixture of normal distributions, fits better daily stock returns than infinite variance stable Paretian distributions. Other models using different mixtures of normal to generate distributions that would account for the higher magnitude of kurtosis, are, among others, the Poisson mixtures of Press (1967) and the discrete mixtures of Kon (1984). Furthermore, Clark (1973), Merton (1982) and Tauchen and Pitts (1983) put forward models where the distribution of variance is a function of the arrival of the information rate, the trading activity and the trading volume. Such models are, however, too complex to be used in empirical applications.

There is yet no unanimity regarding the best stochastic return generating model. One of the most recent proposed class of return generating process in the literature that can capture the empirical characteristics of stock return series, i.e. changing variance and high level of kurtosis, is the class of autoregressive conditional heteroskedastic processes introduced by Engle (1982) and its generalized version by Bollerslev (1986). Empirical studies showed indeed that such processes are successful in modelling various time series. See, for example, French, Schwert and Stambaugh (1987), Baillie and Bollerslev (1989), Hsieh (1989) and Baillie and De Gennaro (1990). Interested readers can consult Taylor (1990) who provides many references of applications of this class of models in finance.

As far as stock markets are concerned, this class of models has been mainly applied to American stock markets with the exception of the Amsterdam (Corhay and Tourani Rad, 1990) and the London (Taylor and Poon, 1992) stock exchanges. In this paper we apply these models to stock price indices of several European countries. To that end, we have selected the following five countries: France, Germany, Italy, the Netherlands and the United Kingdom. The study of stock price behaviour in these markets is interesting in that it can provide further evidence in favour of or against the use of this type of models for describing stock price behaviour in smaller and thinner markets.

The rest of this paper is organised as follows. Section two presents the data and descriptive statistics of all the five countries. Section three determines which univariate autoregressive moving average model fits the data best for the countries of which returns series are serially dependent. In section four, the class of autoregressive conditional heteroskedastic models is presented and tests for the presence of heteroskedasticity in the returns series are carried out. The next section is then devoted to the estimation of the best model for each country. In the final section, comparisons between countries and implications are drawn.

2. Data and descriptive statistics

The indices of five European stock markets were collected from DATASTREAM for the period 1/1/1980 to 30/9/1990. They are indices for France (CAC General), Germany (Commerzbank), Italy (Milan Banca), the Netherlands (general CBS) and the U.K. (FT All-Shares). The daily returns of these market indices are continuously compounded returns. They are calculated as the difference in natural logarithm of the index value for two consecutive days, $R_t=\log(P_t)-\log(P_{t-1})$.

We first carry out a detailed analysis of the distributional and time-series properties of the stock market indices of the five countries. Descriptive statistics for each country

Table 1 — Sample Statistics on Daily Returns Series*

Statistics	France	Germany	Italy	Netherl.	U.K.
Mean ($x10^3$)	.4699	.2933	.6819	.6047	.5109
t(mean=0)	2.3998	1.3757	2.5259	2.9956	2.9908
Variance ($x10^3$)	.1074	.1274	.2043	.1142	.0817
Skewness	-1.5118	-1.1168	-.9916	-.5427	-1.6873
Kurtosis	16.941	15.035	11.738	13.4161	9.662
Range	0.1979	0.2117	0.2315	0.2183	0.1778
Median ($x10^3$)	0.0000	0.1105	0.0985	0.0000	0.8475
IQR (Q3-Q1)	0.0093	0.0111	0.0119	0.0109	0.0103
D-statistic	0.0971	0.0749	0.0999	0.0678	0.0567
Log-Likelihood	11406.	11167.	10506.	11321.	11789.

* Values of the tests statistically significant at the one per cent level are underlined.

are presented in table 1. They include the following distributional parameters: mean, variance, skewness, kurtosis, range, median and inter-quartile range (IQR). The value of the maximized likelihood function when a normal distribution is imposed on the data is also reported. It can be observed that there are differences across the countries regarding the mean and variance of the returns series. All means are statistically significant, except for Germany. Italy has the highest mean and variance of returns. All distributions are negatively highly skewed, indicating that they are non-symmetric. Furthermore, they all exhibit high level of kurtosis meaning distributions are more peaked and have fatter tails than normal distributions. The presence of negative skewness can be due to the inclusion of the crash of

October 1987 in the sample period. Corhay and Tourani Rad (1990) show that the skewness of the distribution of the Dutch index is negative but not statistically significant for the period before the 1987 crash. A direct test of normality has been carried out. Under the null hypothesis of identically, independently normal distribution of returns, the coefficients of skewness and excess kurtosis are both zero. Their sample estimates have standard deviations of $\sqrt{6/n}$ and $\sqrt{24/n}$, respectively, where n is the number of observations in the series. This test always rejects the null hypothesis at a very high level of significance. Moreover, the Kolmogorov-Smirnov D-Statistic for the null hypothesis of normality has been calculated, and it also rejects the normality assumption at a significant level of one per cent in all cases. The results confirm the well known fact that daily stock returns are not normally distributed, but are leptokurtic and skewed, whatever the country concerned. It also appears that the size of non-normality in stock returns of European markets is much more pronounced than that observed by Akgiray (1989) in the American market.

In order to test the hypothesis whether returns are strict white noise, i.e. random walk, the Box-Pierce test statistics up to lag 25 is calculated and presented in the table 2. This is a joint test that the first k autocorrelation coefficients are zero. Under the null hypothesis, that the sample autocorrelations are not asymptotically correlated, the Box-Pierce statistic, $Q=n\sum_{i=1}^{k}\rho(i)^2$, has chi-square distribution with k degrees of freedom, where $\rho(i)$ is the i-th autocorrelation. The values of Q are all significant at the

Table 2 — Sample Autocorrelations of Daily Returns Series*

	France	Germany	Italy	Netherl.	U.K.
ρ1	.1461	.0284	.1367	-.0308	.1367
	(.0292)	(.0440)	(.0351)	(.0510)	(.0651)
ρ2	.0455	-.0545	-.0378	.0018	.0477
	(.0399)	(.0353)	(.0325)	(.0532)	(.0481)
ρ3	.0273	.0169	.0447	.0003	.0261
	(.0353)	(.0329)	(.0372)	(.0438)	(.0394)
ρ4	.0218	.0217	.0601	.0114	.0535
	(.0307)	(.0294)	(.0337)	(.0367)	(.0479)
ρ5	.0177	.0207	.0062	.0507	.0153
	(.0285)	(.0298)	(.0300)	(.0422)	(.0385)
ρ10	.0661	.0308	.0682	.0207	.0541
	(.0323)	(.0332)	(.0283)	(.0339)	(.0292)
ρ15	-.0035	-.0131	.0283	.0116	.0294
	(.0292)	(.0297)	(.0259)	(.0365)	(.0255)
ρ20	.0302	-.0027	.0413	.0011	.0099
	(.0277)	(.0271)	(.0267)	(.0218)	(.0269)
ρ25	-.0249	-.0172	.0005	-.0319	-.0305
	(.0268)	(.0199)	(.0258)	(.0226)	(.0209)
Q(25)	154.23	56.03	139.58	46.69	111.13
Q*(25)	65.66	22.52	52.64	17.32	26.30

* Values of the tests statistically significant at the one per cent level are underlined. Numbers in parentheses are heteroskedasticity-consistent standard errors.

one per cent level, which means that the null hypothesis of strict white noise is rejected, reflecting a rather long range of dependency in the returns series. However, it can be questioned whether this test accounts for the full probability distribution of the returns series since heteroskedasticity can lead to the underestimation of the standard error, $\sqrt{1/n}$, of each sample, and therefore to the overestimation of the t- and χ^2-statistics. Diebold (1987) provided a heteroskedasticity-consistent estimate of the standard error for the i-th sample autocorrelation coefficient:

$$S(i) = \sqrt{\frac{1}{n}\left(1 + \frac{\gamma_R{}^2(i)}{\sigma^4}\right)} \qquad (1)$$

where $\gamma_R{}^2(i)$ is the i-th sample autocovariance of the square data and σ is the sample standard deviation of the data. These adjusted standard errors are presented in the table 2 under their respective autocorrelation coefficient. It can be seen that only few autocorrelation coefficients are statistically different from zero. Using these adjusted standard errors, Diebold proposed an adjusted Box-Pierce statistic:

$$Q^* = \sum_{i=1}^{k}\left(\rho(i)/(S(i))^2\right) \qquad (2)$$

which is asymptotically chi-square distributed with k degrees of freedom, under the null hypothesis of no serial correlation in the data. The values of Q^* which are presented in table 2 are much lower than the non adjusted ones. They are significant at one per cent level for France and Italy only. So, even after adjusting for heteroskedasticity, there remains some significant autocorrelations in the series of returns for these two countries.

Looking at the autocorrelation coefficients, we can observe that the first order coefficients are 0.1461 for France, 0.0284 for Germany, 0.1367 for Italy, -0.0308 for the Netherlands and 0.1367 for the UK. These are significant at one per cent level for France and Italy, and at five percent for the UK, indicating that the daily index returns for these countries are first order serially correlated. This is in accordance with empirical findings for other stock markets. Stock indices usually show a rather large first-order autocorrelation, even if autocorrelation coefficients of individual stocks returns are very low. The presence of autocorrelation in the index returns is due to the existence of intertemporal cross covariance between stock returns which is caused by some friction in the trading process (Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980)). The rather low and insignificant coefficients of the first order serial correlation in the German and Dutch indices might be explained by the important weight of a few large, frequently and highly traded firms, of which returns are not autocorrelated, in the calculation of the indices.

A comparison between the values of Q and Q^* suggests that the rejection of serial independence using Q, which is based on the standard testing procedure, is due to the presence of heteroskedasticity in the returns series. The presence of significant values of Q^* in the French and Italian returns indices indicates, however, that these returns series are not white noise processes. Furthermore, the fact that the first lag autocorrelation is significant for these two countries at the one per cent level and for the UK at the five per cent level implies the rejection of white noise, i.e. uncorrelated process. Therefore, we have to eliminate the serial correlation in these three series before searching for appropriate models that could account for heteroskedasticity in the returns. One way to do this is to apply Autoregressive Moving Average (ARMA) models.

3. ARMA models

The class of univariate ARMA models might adequately represent the behaviour of the stock returns. Therefore, several ARMA models were applied to the returns of the three countries which exhibit rather significant serial dependence, namely, France, Italy and the UK. We found that an AR(1) fits all three returns series best.

$$R_t = \phi_0 + \phi_1 R_{t-1} + \varepsilon_t \tag{3}$$

The estimates of the above regression model for each country are presented in table 3. In order to observe whether the residuals ε_t obtained from equation (3) are uncorrelated, we applied the same tests for normality and serial correlation as for the returns series. As before, the values of the autocorrelation coefficients and their respective standard errors adjusted for heteroskedasticity are presented in table 4. The first order autocorrelation coefficient for all three countries is not significantly

Table 3 — The Autoregressive Model*

	France	Italy	U.K.
	Estimates of the model $R_t = \phi_0 + \phi_1 R_{t-1} + \varepsilon_t$		
ϕ_0	.0004	.0006	.0004
$t(\phi_0)$	2.0704	2.1861	2.6220
ϕ_1	.1461	.1367	.1367
$t(\phi_1)$	7.8140	7.3012	7.3075
F-statistic	61.0810	53.3160	53.4100
R^2	0.0213	.0187	.0187
Fuller's test	2743.18	2750.64	2750.59

* Values of the tests statistically significant at the one per cent level are underlined.

Table 4 — Sample Statistics on Daily Residuals Series*

	France	Italy	U.K.
Mean $(x10^3)$	.0000	.0000	.0000
t(mean=0)	.0000	.0000	.0000
Variance $(x10^3)$	.1052	.2005	.0802
Skewness	-1.4215	-.8295	-1.3075
Kurtosis	17.3579	12.1476	16.8439
Range	0.2005	0.2361	0.1813
Median $(x10^3)$	-0.0035	-0.3102	0.2390
IQR (Q3-Q1)	0.0089	0.0116	0.0102
D-statistic	0.0969	0.0997	0.0480

* Values of the tests statistically significant at the one per cent level are underlined.

different from zero. Furthermore, the results indicate that the AR(1) transformation of the returns provides an uncorrelated series of residuals. One can indeed observe that while the standard Box-Pierce statistics rejects the null hypothesis of no serial correlation at the one per cent level of significance for France and Italy, and at the five per cent level for the UK, the adjusted one does not reject the null hypothesis at the one percent level for all three countries.

The estimate of ϕ_1 is statistically significant at the one percent level and the Dickey-Fuller test for unit roots indicates that ϕ_1 is significantly less than one. The three series of returns appear to follow a stationary random walk. As far as the assumption of normality of the residuals is concerned, it can be rejected by the direct test of normality as well as by the Kolmogorov-Smirnov D-statistic at one percent significance level. The residual series appear to be leptokurtic and skewed. Moreover, again a comparison between values of Q and Q^* in table 5 indicates that the three residuals series still exhibit heteroskedasticity.

The presence of heteroskedasticity in stock prices and in the market model has been documented by, for example, Morgan (1976) and Giaccoto and Ali (1982). But while they focused on unconditional heteroskedasticity, in this paper we use Engle's Autoregressive Conditional Heteroskedastic (ARCH) model which focuses on conditional volatility movements. It is interesting to note that, according to Diebold et al. (1988), the presence of ARCH effect appears to be generally independent of unconditional heteroskedasticity. Excess kurtosis

Table 5 — Sample Autocorrelations of Daily Residuals Series*

	France	Italy	U.K.
$\rho 1$	-.0036	.0075	-.0045
	(.0323)	(.0393)	(.0660)
$\rho 2$	.0216	-.0646	.0275
	(.0419)	(.0321)	(.0537)
$\rho 3$	.0185	.0431	.0131
	(.0336)	(.0384)	(.0431)
$\rho 4$	.0208	.0554	.0495
	(.0300)	(.0332)	(.0444)
$\rho 5$	-.0227	.0049	.0056
	(.0301)	(.0301)	(.0394)
$\rho 10$	.0514	.0642	.0457
	(.0321)	(.0281)	(.0305)
$\rho 15$	-.0047	.0213	.0344
	(.0291)	(.0261)	(.0259)
$\rho 20$	.0280	.0368	.0084
	(.0272)	(.0267)	(.0263)
$\rho 25$	.0205	.0010	-.0271
	(.0267)	(.0252)	(.0208)
Q(25)	<u>76.32</u>	<u>92.09</u>	43.95
$Q^*(25)$	34.54	38.34	17.67

*statistical tests significant at the one per cent level are underlined. Numbers in parentheses are heteroskedasticity-consistent standard errors.

observed in both returns and residuals series can be related to conditional heteroskedasticity, that is, its presence can be due to a time varying pattern of the volatility. ARCH models and its extensions have been successfully applied, for instance, in foreign exchange markets by Baillie and Bollerslev (1989) and Hsieh (1989), and in stock markets by Akgiray (1989) and Baillie and De Gennaro (1990).

4. Conditional Heteroskedastic Models

a) ARCH and GARCH Models

The ARCH process imposes an autoregressive structure on the conditional variance which permits volatility shocks to persist over time. It can therefore allow for volatility clustering, that is, large changes are followed by large changes, and small by small, which has long been recognized as an important feature of stock returns behaviour. In this process, the conditional error distribution is normal, with a conditional variance that is a linear function of past squared innovations. The model, denoted by ARCH(p), is the following:

$$\varepsilon_t | \psi_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \qquad (4)$$

with $p>0$; $\alpha_i>0$, $i=0,\ldots,p$, and where ψ_t is the information set of all information through time t, and the ε_t are obtained from a linear regression model.

An important extension of the ARCH model is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process of Bollerslev (1986), denoted by GARCH(p,q). In this model, the linear function of the conditional variance includes lagged conditional variances as well. The equation (4) in the case of a GARCH model becomes:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j h_{t-j} \quad (5)$$

where also $q \geq 0$ and $\beta_j \geq 0$, $j=1,\dots,q$. The GARCH(p,q) model reduces to an ARCH(p) for $q=0$.

Before estimating (G)ARCH models, it is useful to test for the presence of ARCH properties on the returns series. This is the object of the next subsection.

b) Testing for ARCH presence

In an ARCH process, the variance of a time series depends on past squared residuals of the process. Therefore, the appropriateness of an ARCH model can be tested by means of a LM test, i.e. by regressing the squared residuals against a constant and lagged squared residuals (Engle, 1982).

$$\varepsilon_t^2 = \gamma_0 + \sum_{i=1}^{n} \beta_i \varepsilon_{t-i}^2 \quad (6)$$

Under the null hypothesis of no ARCH process, the coefficient of determination R^2 can be used to obtain the test statistic TR^2 which is distributed as a chi-square with i degrees of freedom. This LM test has been applied to our series up to lag 10 for all the five returns series. The values we obtained for the TR^2 are reported in table 6. They are all statistically significant at the one per cent level, which strongly indicates the presence of an ARCH process in the series.

Table 6 — LM test statistic*

	France	Germany	Italy	Netherl.	U.K.
ARCH(1)	164.2	356.3	330.8	643.0	1089.5
ARCH(2)	264.7	370.6	347.8	859.6	1090.7
ARCH(3)	280.9	381.5	431.0	859.8	1091.1
ARCH(5)	285.4	389.1	442.7	922.1	1141.1
ARCH(10)	311.1	423.8	492.5	942.3	1160.2

* All LM test statistics for ARCH(p) are significant at the one per cent level.

c) *Estimating (G)ARCH models*

The parameters of a (G)ARCH model are obtained through a maximum likelihood estimation. Given the return series and initial values of ε_l and h_l, for $l=0,\ldots,r$ and with $r=\max(p,q)$, the log-likelihood function we have to maximise for a normal distribution is the following:

$$L(\phi \mid p,q) = -\frac{1}{2}\,T\,\ln(2\pi) + \sum_{t=r}^{T} \ln\left(\frac{1}{\sqrt{h_t}}\right) \exp\left(\frac{-\varepsilon_t^2}{2h_t}\right) \qquad (7)$$

where T is the number of observations, h_t, the conditional variance, is defined by equations (4) and (5) for the ARCH and GARCH models respectively, ε_t^2 are the residuals obtained from the appropriate linear regression model according to the country in consideration.

As the values of p and q have to be prespecified in the model, we tested several combinations of p and q. The values of the maximised likelihood functions for all pairs of p and q are presented in table 7. We also calculated the generalized likelihood ratio $LR=-2\{L(\phi_n)-L(\phi_a)\}$ of the maximised likelihood functions under the null hypothesis, i.e., the normal distribution, and the various alternate hypothesis. Under the null hypothesis LR is chi-square distributed with degrees of freedom equal to the difference in the number of parameters under the two hypotheses. The third column of table 7 gives the values of the LR test for each model. It can be observed that the value of the LR test for all (G)ARCH models is statistically significant at the one percent level, which means that all of these models fit the data more likely than does the normal distribution. In order to distinguish between an improvement in the likelihood function due to a better fit and an improvement due to an increase in the number of parameters, we also calculated Schwarz's order selection criterion, $SIC=-2L(\phi)+(\ln T)K$, where K is the number of parameters in the model. According to this criterion, the model with the lowest SIC value fits the data best. The SIC values are reported in the fifth column of table 7. The

Table 7 — Maximum log likelihoods for (G)ARCH models

Model p,q	Log likelihood	LR test	Schwarz criterion
	France		
Normal -	11406.50		
ARCH (1,0)	11560.95	308.90	-23113.96
ARCH (2,0)	11664.35	515.70	-23312.82
ARCH (3,0)	11677.31	541.62	-23330.80
	Germany		
Normal -	11167.36		
ARCH (1,0)	11335.34	335.96	-22662.74
ARCH (2,0)	11396.47	458.22	-22777.06
ARCH (3,0)	11490.67	646.62	-22957.52
GARCH (1,1)	11576.18	817.64	-23136.48
GARCH (2,1)	11578.72	822.72	-23133.62
GARCH (1,2)	11569.26	803.80	-23114.70
	Italy		
Normal -	10506.12		
ARCH (1,0)	10653.91	295.58	-21299.88
ARCH (2,0)	10757.64	503.04	-21499.40
ARCH (3,0)	10886.73	761.22	-21749.64
GARCH (1,1)	10992.68	973.12	-21969.48
GARCH (2,1)	...		
GARCH (1,2)	...		
GARCH (2,2)	10998.15	984.06	-21964.55
	The Netherlands		
Normal -	11321.07		
ARCH (1,0)	11483.11	324.08	-22958.28
ARCH (2,0)	11582.53	522.92	-23149.18
ARCH (3,0)	11597.34	552.54	-23170.86
GARCH (1,1)	11657.18	672.22	-23298.48
GARCH (2,1)	11650.63	659.12	-23277.44
GARCH (1,2)	11650.68	659.22	-23277.54
	The UK		
Normal -	11789.03		
ARCH (1,0)	11991.12	404.18	-23974.30
ARCH (2,0)	12046.90	515.74	-24077.92
ARCH (3,0)	12062.77	547.48	-24101.72
GARCH (1,1)	12097.46	616.86	-24179.04
GARCH (2,1)	...		
GARCH (1,2)	12091.50	604.94	-24159.18
GARCH (2,2)	...		

... indicates where the optimization routine failed.

GARCH(1,1) model has the lowest SIC values for all countries except France. For the latter the ARCH(3) supersedes the other models.

The sum of $\sum_{i=1}^{p}\alpha_i+\sum_{j=1}^{q}\beta_j$ in the conditional variance equations measures the persistence of the volatility. Engle and Bollerslev (1986) have shown that if this sum is equal to one, the GARCH process becomes an integrated GARCH or IGARCH process. Such integrated model implies the persistence of a forecast of the conditional variance over all future horizons and also an infinite variance of the unconditional distribution of ε_t. We calculated the sum of the parameters $\sum_{i=1}^{p}\alpha_i+\sum_{j=1}^{q}\beta_j$ for the appropriate GARCH models. They are respectively 0.9923, 1.0005, 0.9761, 0.9520 and 0.4329 for France, Germany, Italy, the Netherlands and the UK. It can be noticed that it is less than unity for four countries, though rather close to one, which indicates a long persistence of shocks in volatility. This means that this model is second order stationary and that the second moment exists for these four countries. The unconditional variances of residuals, shown in table 8, are respectively $\sigma_\varepsilon^2=\alpha_0/(1-\alpha_1-\beta_1)$ for Italy, the Netherlands and the UK and $\sigma_\varepsilon^2=\alpha_0/(1-\alpha_1-\alpha_2-\alpha_3)$ for France, and, for returns, it is $\sigma_R^2=\sigma_\varepsilon^2/(1-\phi_1^2)$.

As for Germany, the sum $\alpha_1+\beta_1$ is greater than unity, indicating that the series is not stationary and that an integrated model is more appropriate, i.e. the conditional variance follows an integrated process. The GARCH(1,1) model has therefore been reestimated with the restriction

Table 8 — Model Estimates*

	France	Germany	Italy	Netherl.	U.K.
ϕ_0 (thousands)	.6881	.6246	.8061	.9255	.6775
$t(\phi_0)$	4.1215	4.0899	3.8546	5.6614	4.6100
ϕ_1	.2031	-	.1909	-	.1519
$t(\phi_1)$	9.5706	-	7.7043	-	7.5064
α_0 (thousands)	.0581	.0021	.0064	.0053	.0053
$t(\alpha_0)$	24.7654	3.7088	6.2055	4.9619	5.1517
α_1	.1739	.1406	.1394	.1113	.1131
$t(\alpha_1)$	6.6687	8.5353	9.0820	7.6555	7.5200
α_2	.1612	-	-	-	-
$t(\alpha_2)$	6.4882	-	-	-	-
α_3	.0978	-	-	-	-
$t(\alpha_3)$	4.3288	-	-	-	-
β_1	-	.8594	.8367	.8407	.8153
$t(\beta_1)$	-	55.5779	52.0732	40.7251	35.6342
$\sum\alpha_i+\sum\beta_j$	.4329	1.0000	.9761	.9520	.9284
σ_ε^2 $(x10^3)$	.1025	-	.2678	-	.0740
σ_R^2 $(x10^3)$	.1069	-	.2779	.1104	.0758

* t statistics significant at the one percent level are underlined.

that $\alpha_1+\beta_1=1$. Table 8 contains the results of fitting GARCH(1,1) process to the returns series of Italy, the Netherlands and the UK, ARCH(3) to that of France, and finally IGARCH(1,1) for Germany. All estimated coefficients, except that of ϕ_0 and α_0 for Germany, are statistically significant at the one percent level. Interestingly, the estimates of α_0 are much smaller than the sample variances of returns or residuals reported in tables 1 and 4, indicating that conditional variances are changing over time.

5. Conclusions

This paper provides empirical support that the class of autoregressive conditional heteroskedasticity models is generally consistent with the stochastic behaviour of daily stock returns in five European countries. The results show that stock market indices exhibit a significant level of non linear dependence which cannot be accounted for by the random walk model. Descriptive statistics and normality tests reveal that the distribution of returns is not normal, whatever the country concerned, and that three out the five country indices exhibit significant first order autocorrelation. It has further been shown that the residuals obtained after applying an AR(1) model, which accounts for the presence of autocorrelation in the returns, exhibit non linear dependence and non normality. Then we observed the presence of ARCH in the returns series and tested various models belonging to the class of autoregressive conditional heteroskedasticity models. Our results reveal that this class of models supersedes the random walk model. And among the different models the GARCH(1,1) fits the data best for Italy, the Netherlands and the UK, the ARCH(3) for France and IGARCH(1,1) for Germany.

6. References

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