MULTIBODY DYNAMICS 2011, ECCOMAS Thematic Conference
J.C. Samin, P. Fisette (eds.)
Brussels, Belgium, 4-7 July 2011

MODELING OF CONTACT BETWEEN STIFF BODIES IN AUTOMOTIVE TRANSMISSION SYSTEMS

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Keywords: Contact with friction, Impacts, Coefficient of restitution, TORSEN differential.

Abstract. *Many transmission components contain moving parts which can enter in contact. The TORSEN differentials are mainly composed of gear pairs and thrust washers. The friction involved by contacts between these two kind of parts is essential in the working principle of such differentials. In this paper, two different contact model are presented. The former uses an augmented Lagrangian technique and is defined between a rigid body and a flexible structure. The second contact formulation is a continuous impact modeling based on a restitution coefficient.*

1 INTRODUCTION

Multibody simulations of engine and vehicle dynamics are widely used in automotive design process. Current trend in industry addresses the development of reliable drivetrain models. In this way, there is a need to model completely the car from the motor to the vehicle dynamics. The driveline modeling would allow to improve the performance not only of the transmission devices, but also of the other subsystems of the vehicle. Indeed, the transmission components such as clutch, gear box or differential strongly interact with the engine, the car body and the suspensions. For instance, some noise and vibrations can be generated by the differentials and transmitted in the whole car structure with a direct impact on the comfort of the passengers.

In automotive as in other fields of mechanics, many transmission components include contacts between different parts. These contacts inhibit the relative motion in one or several directions but let free the motion in the other directions. The contact can be: bilateral or unilateral, rigid or flexible, frictional or frictionless. Several complex physical phenomena can be involved by contacts. For instance, when the relative velocity at the contact beginning is high for unilateral contact, the impact encountered can generate vibration waves in the body structure. Permanent plastic deformations can be induced. The friction can generate particular phenomena such as the well-known stick-slip due to the difference between the static and dynamic friction coefficients.

An accurate mathematical modeling of contact is needed among others to intend to improve the performance, reduce the losses and the weight of transmissions components. Nevertheless the modeling of such discontinuous and nonlinear behavior is not trivial and often leads to numerical problems.

In literature, three main contact modeling categories can be distinguished according to the behavior considered to model the bodies subjected to contact: rigid-rigid contact (Ref. [1]), flexible-rigid contact (Ref. [2]) or flexible-flexible contact (Ref. [3]). In the field of multibody systems dynamics, two different approaches are often used to formulate the contact condition: continuous contact modeling and instantaneous contact modeling. The continuous method does not need specific algorithmic tools to manage the impact phenomena. The contact forces are added in the equations of motion of the mechanism and a unique Newmark-type integration scheme can be used to solve the complete system. The positions and velocities of all bodies vary continuously and it is not necessary to stop the time integration at the moment of contact establishment (see Ref. [4] for more details). With instantaneous contact models, the multibody motion is divided into two periods, before and after the impact. While the displacements are continuous, a jump of the relative velocity is observed at the contact instant. These formulations are often related to nonsmooth dynamic methods (see References [5] and [6]). The discontinuities in the velocity field require the use of special integration methods (Ref. [7], [8] and [9]). For instance, event-driven approaches require the interruption of the time integration at each impact whereas time-stepping methods discretize in time the complete multibody system dynamics including the unilateral constraint(s) and the impact forces.

In this work, the application of a TORSEN differential is studied. This kind of limited slip differential is mainly composed of gear pairs and thrust washers. The axial force produced by the helical mesh leads to contact between the lateral circular faces of toothed wheels and the various thrust washers. The friction generated between these two bodies is at the origin of the locking effects, specific to the operation of TORSEN differentials. A unilateral frictional contact model is then essential to model accurately and reliably these differentials.

The objective of this paper is to study contact formulations appropriate for contacts between washers and gear flanks in dynamic simulations of flexible multibody systems, based on a nonlinear finite element approach. This method described in Ref. [10] is implemented in SAMCEF/MECANO and allows the modeling of complex mechanical systems composed of rigid and flexible bodies, kinematics joints and force elements. Based on absolute nodal coordinates, the description of flexible structural components naturally accounts for large rigid-body motions and elastic deflections. The numerical solution is based on a Newmark-type integration scheme with numerical dissipation, which is combined with a regularization of discontinuities and non-smooth phenomena in the system.

In the sequel on this paper, the working principle of TORSEN differential will be described in Section 2. The nonlinear finite element approach for flexible multibody systems is briefly presented in Section 3. Finally, two contact formulations are detailed in Section 4 and some simulation results are shown.

2 DESCRIPTION OF TORSEN DIFFERENTIALS

The two essential functions of a differential are to transmit the motor torque to the two output shafts and to allow a difference of rotation speed between these two outputs. In a vehicle, this mechanical device is particularly useful in turn when the outer wheels have to rotate quicker than the inner wheels to ensure a good handling.

The main drawback of a conventional differential (open differential) is that the total amount of available torque is always split between the two output shafts with the same constant ratio. In particular, this is a source of problems when the driving wheels have various conditions of adherence. If the motor torque exceeds the maximum transferable torque limited by road friction on one driving wheel, this wheel starts spinning. Although they don't reach their limit of friction, the others driving wheels are not able to transfer more torque because the input torque is often equally splitted between the two output shafts.

The TORSEN differentials reduce significantly this undesirable side effect. This kind of limited slip differential allows a variable distribution of motor torque depending on the available friction of each driving wheel. For a vehicle with asymmetric road friction between the left and right wheels, for example, right wheels are on a slippery surface (snow, mud...) whereas left wheels have good grip conditions, it is possible to transfer an extra torque to the left lane. That allows the vehicle to move forward whereas it would be hardly possible with an open differential. However, the overall driving torque can't be applied on one output shaft while no load is exerted on the second shaft. When the difference between the 2 output torques becomes too large, the differential unlocks and lets different rotation speeds but keeps the same constant torque ratio.

When a TORSEN differential is used, the torque biasing is always a precondition before any difference of rotation speed between the two output shafts. Contrary to viscous coupling, TORSEN (a contraction of Torque-Sensing) is an instantaneous and pro-active process which acts before wheel slip.

The differential can be used either to divide the drive torque into equal parts acting on the traction wheels of the same axle, or to divide the output torque from the gearbox between the two axles of four-wheels drive vehicles. This second application is often called the transfer box differential or central differential. In a previous work, the central differential (type C TORSEN), which equips the Audi Quattro, has been modeled (see Ref. [11] for more details).

As depicted on Fig. 1, the TORSEN differential contains a housing in two parts as well as several gear pairs and trust washers. Due to the axial force produced by the helical mesh, several gear wheels can move axially and enter in contact with the various thrust washers fixed on the

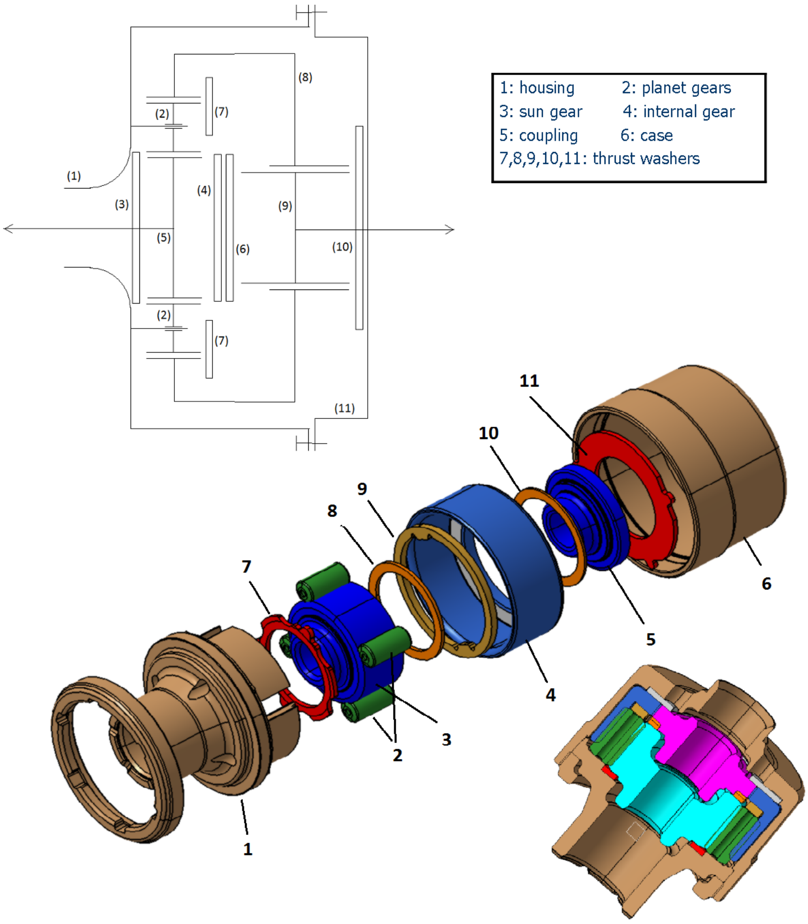

Figure 1: Kinematic diagram, exploded diagram and cut-away view of type C TORSEN differential

case or housing. The friction encountered by this relative sliding is at the origin of the locking effect of TORSEN differentials. The second important contribution to the limited slip behavior is due to the friction between the planet gears and the housing holes in which they are inserted. When one axle tries to speed up, all encountered frictions tend to slow down the relative rotation and involve a variable torque distribution between the output shafts. The biasing on the torque only results from the differential gearing mechanical friction.

This limited slip differential has four working modes which depend on the direction of torque biasing and on the drive or coast situation. According to the considered mode, the gear wheels rub against one or the other thrust washers which can have different friction coefficients and contact surfaces.

3 FINITE ELEMENT METHOD IN MULTIBODY SYSTEM DYNAMICS

For differentials, as for most automotive transmission components, it can be interesting to take flexibility in the system into account. For instance, the backlash between teeth in gear pairs and impact phenomena can generate some vibrations and noise which could be transmitted to the whole power train. In order to represent these physical phenomena, some bodies like the transmission shafts should be considered as flexible. The numerical model should also be able to manage the nonlinearities and high discontinuities involved by contact conditions.

In this work, the approach chosen to model the differentials is based on the nonlinear finite element method for flexible multibody systems developed by Géradin and Cardona [10]. This method allows the modeling of complex mechanical systems composed of rigid and flexible bodies, kinematics joints and force elements. Absolute nodal coordinates are used with respect to a unique inertial frame for each model node. Hence, there is no distinction between rigid and elastic coordinates which allows accounting in a natural way for many nonlinear flexible effects and large deformations. The cartesian rotation vector combined with an updated Lagrangian approach is used for the parametrization of rotations. This choice enables the representation of large rotations.

This approach to model flexible multibody systems is implemented in the software SAMCEF/MECANO commercialised by SAMTECH S.A. Discontinuities are managed with regularizations and the equations of motion for a dynamic system with holonomic constraints are stated in the form:

$$M(q)\,\ddot{q} + g^{gyr}(q,\dot{q}) + g^{int}(q,\dot{q}) + \phi_q^T(p\phi + k\lambda) = g^{ext}(t) \quad (1)$$

$$k\,\phi(q,t) = 0 \quad (2)$$

where q, $\dot{q}$ and $\ddot{q}$ are the generalized displacements, velocities and acceleration coordinates, $M(q)$ is the mass matrix, g^{gyr} is the gyroscopic and complementary inertia forces, $g^{int}(q,\dot{q})$ the internal forces, e.g. elastic and dissipations forces and $g^{ext}(t)$ the external forces. According to the augmented Lagrangian method, the constraint forces are formulated by $\phi_q^T(p\phi + k\lambda)$ where λ is the vector of Lagrange multiplier related to algebraic constraints $\phi = 0$; k and p are respectively a scaling and a penalty factor to improve the numerical conditioning.

Equations (1) and (2) form a system of nonlinear differential-algebraic equations. The solution is evaluated step by step using a second order accurate time integration scheme. For this study, the Chung-Hulbert scheme, which belongs to the family of the generalized α-method, has been used (see References [12], [13]). At each time step, a system of nonlinear algebraic equations has to be solved. In order to solve this system, a Newton-Raphson method is used.

As detailed in Ref. [11], the main kinematic constraints used in the TORSEN differential model are related with contact condition and gear pair. The contact modeling which yields inequality constraints in the equation of motion, will be discussed in the next section (4) and the formulation used to model gear element is developed in Ref. [14]. This formulation is available for describing flexible gear pairs in 3 dimensional analysis of flexible mechanism. Each gear pair is modeled between two physical nodes: one at the center of each gear wheel considered as a rigid body. Nevertheless the flexibility of the gear mesh is accounted for by a nonlinear spring and damper element inserted along the instantaneous normal pressure line. Several specific phenomena in gear pairs which influence significantly the dynamic response of gears are also included in the model: backlash, mesh stiffness fluctuation, misalignment, friction between teeth.

4 CONTACT MODELING

Several approaches can be adopted to model contacts for the application considered in this work. The main characteristics of contacts between lateral faces of wheels and thrust washers that can be deduced from the description of the operation of TORSEN differential (Section 2). Those contacts are:

unilateral Indeed, depending on the working mode, each gear is in contact with the thrust washers on the left or on the right but never with two washers at the same time.

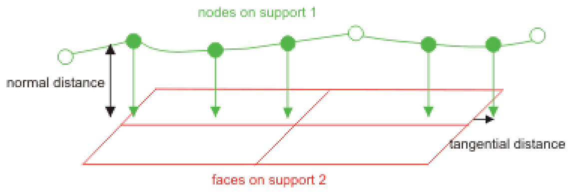

Figure 2: Contact condition - projection of slave node on master surface

frictional The friction being essential in the locking effect of TORSEN differential, it is mandatory to take friction into account.

enough robust to account for impact phenomena When working mode changes, the axial displacement of gears is fast and involve a impact phenomenon at the time of contact with the other washer.

Amongst possible contact models, a distinction can be done between the continuous classical methods which can be used with a standard Newmark-type integrator and the nonsmooth dynamic methods which need specific integration tools. Two different continuous contact models will be analyzed over in Sections 4.1 and 4.2.

4.1 Modeling of contact between a rigid and a flexible body

For implicit nonlinear analysis, the software SAMCEF/MECANO enables to define contact conditions between a rigid structure and a flexible part (flexible/rigid contact) or between two flexible parts (flexible/flexible contact). Contact relations are created between a set of nodes on the first support that will be connected to a facet (in case of flexible/flexible contact) or a surface (in case of flexible/rigid contact) on the second support. The contact is treated as a nonlinearity and the coupled iterations method is used. The contact formulation is based on a augmented Lagrangian approach, with kinematic constraints which are active in case of contact and inactive when the two bodies are not in contact.

In the TORSEN differential model, the gear wheels are considered as rigid bodies. Then, the rigid/flexible contact model has been used in order to model contacts between washers and gears. Only this option will be described in the following. The contact algorithm consists of two steps. The first one searches for the projection of each slave node of the flexible body in the master surface of the rigid body (Fig. 2), computes the distance (d) between the node and the surface and measures the displacement variation during the time step in the tangent directions to the surface (Δu_1,Δu_2).

The variations δd, $\delta\Delta u_1$, $\delta\Delta u_2$ can be expressed according to variations of nodal unknowns X_D (positions and rotations of the master node and position of the slave node in potential contact)

$$\delta d = \underline{n}^T\, \delta \underline{X}_D \quad (3)$$

$$\delta\Delta u_1 = \underline{t}_1^T\, \delta \underline{X}_D \quad (4)$$

$$\delta\Delta u_2 = \underline{t}_2^T\, \delta \underline{X}_D \quad (5)$$

with $\underline{n}$ the normal and $\underline{t}_1, \underline{t}_2$ the tangents to the rigid surface.

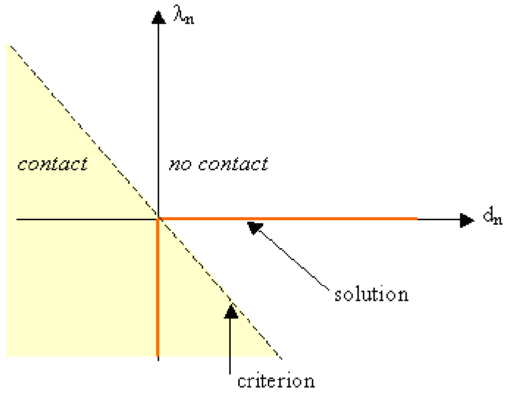

Figure 3: Contact criterion and solution

The second step sets the contact condition. Three Lagrange multipliers λ_i are introduced: one for the contact and two for the friction.

$$\sigma_n = k\lambda_n + pd \tag{6}$$

$$\sigma_{t_1} = k\lambda_1 + p\Delta u_1 \tag{7}$$

$$\sigma_{t_2} = k\lambda_2 + p\Delta u_2 \tag{8}$$

k is a scaling factor and p is a regularization parameter.

In order to know if the contact is active or not, a test based on σ_n, is carried out (Fig. 3).

Inactive contact

If σ_n is positive, there aren't any contact force nor friction force applied on the nodes. At the element level, the forces are computed from the virtual work:

$$\delta \underline{q}^T \underline{F} = -(\delta\lambda_n k\lambda_n + \delta\lambda_{t_1} k\lambda_{t_1} + \delta\lambda_{t_2} k\lambda_{t_2}) \tag{9}$$

At equilibrium the Lagrange multipliers are equal to zero and the iteration matrix is computed easily.

Active contact

If σ_n is negative, the contact est effective and the forces at the element level can be computed from the virtual work:

$$\delta \underline{q}^T \underline{F} = \delta d(pd + k\lambda_n) + \delta\lambda_n kd \tag{10}$$

The iteration matrix related to the contact is symmetric and the Lagrange multiplier λ_n at equilibrium represents the contact force (scaled by k).

In summary, a contact element is defined between each node candidate on contact of the flexible body and the surface related to the master node of the rigid body. Three Lagrange multipliers are introduced for each individual contact element.

With this Lagrange multipliers method, the contact condition is considered as infinitely rigid. This can lead to convergence problems, in particular when the normal relative velocity of bodies is not negligible at the time of contact establishment. Nevertheless, it is possible to relax

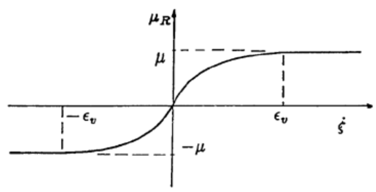

Figure 4: Regularized friction coefficient

slightly the contact condition using a penalty method, which is obtained by setting the Lagrange multiplier to zero in Eq. 10. In this case, a small penetration of the rigid body inside the flexible body is allowed. The penalty function can be linear or nonlinear and can be seen physically as a spring that is active in compression but not in traction. This method enables to reduce the discontinuity involved when the contact condition becomes active.

In order to have a smoother response, it can also be useful to account for damping in the contact model. This damping force is equal to the velocity of the clearance multiplied by a damping coefficient which can be a function of the normal distance.

Friction torques have a key role in the working principle of TORSEN differentials. Therefore, friction effects have to be taken into account in all contact conditions used in the differential model. The friction force F_{fr} is directly proportional to the normal reaction between the point and the surface by means of a friction coefficient.

$$F_{fr} = \mu \, |F_{norm}| \tag{11}$$

A regularized friction coefficient μ_R is often used instead of the constant coefficient μ to avoid a discontinuity in F_{fr} when the sign of the relative sliding velocity changes (see Fig. 4). The regularization function is defined as

$$\mu_R(\dot{\xi}) = \begin{cases} \mu(2 - \frac{|\dot{\xi}|}{\epsilon_v})\frac{\dot{\xi}}{\epsilon_v} & |\dot{\xi}| < \epsilon_v \\ \mu\frac{\dot{\xi}}{|\dot{\xi}|} & |\dot{\xi}| \geq \epsilon_v \end{cases} \tag{12}$$

where $\dot{\xi}$ is the relative sliding velocity, μ is the friction coefficient and ϵ_v is the regularization tolerance. When friction is modeled in the contact condition, the iteration matrix becomes unsymmetric. So, using a non-symmetric resolution algorithm can improve the convergence properties.

A complete model of the type C TORSEN differential has been achieved in a previous work (see Ref. [11]). This model contains five rigid-flexible contact conditions. Owing to the high axial velocity of the gear wheels when the differential switches from one working mode to another one, a linear penalty function and a damping force have been used to allow the convergence of the integration algorithm. The penalty function enables small interferences between the gear wheel and the thrust washer whereas the damping tends to slow down the impact velocity. The damping function is a slope function with more and more damping as the penetration increases. The damping force is actually active before the effective contact to anticipate the contact and reduce the shock phenomenon. This anticipating damping has been used to facilitate the convergence but is also present in real operation due to the film of lubricating oil between the contacting surfaces. However, the damping coefficient has been chosen to obtain acceptable numerical results without identification of oil properties. The contact stiffness value or penalty coefficient has also been set only to ensure the convergence and not based on physical data.

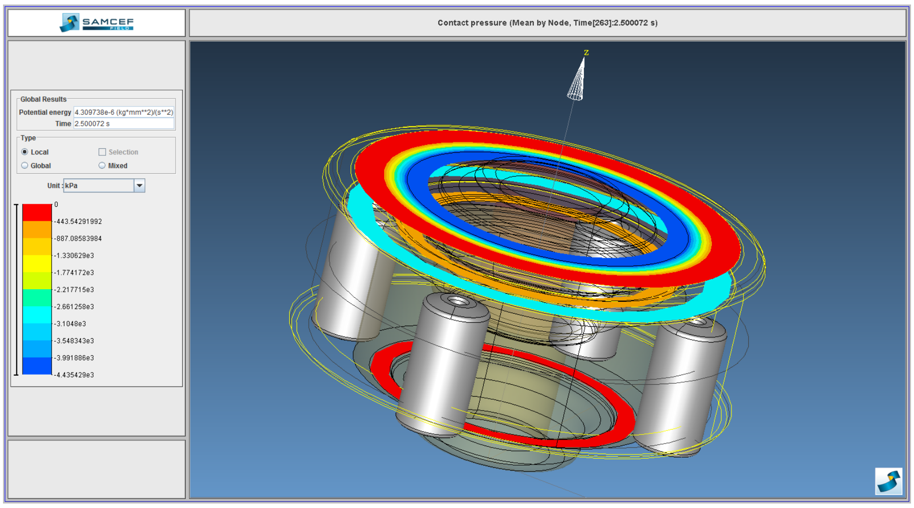

Figure 5: Contact Pressure on thrust washers

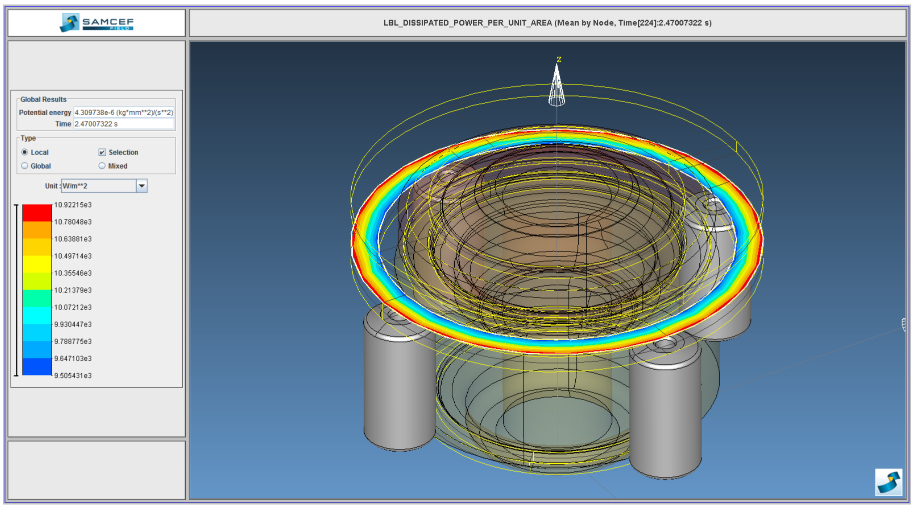

Figure 6: Power dissipated by friction on the washer housing-internal gear

If this value is to big, the discontinuity is not relaxed, leading to numerical difficulties, and if the value is too small the penetration of the two contacting bodies is too high, which is not physically acceptable.

Figure 5 illustrates the contact pressures for all the contact elements introduced in the model for the drive to rear mode and Figure 6 shows the spatial distribution of power dissipated by the friction for the washer between the housing and the internal gear. The maximum dissipated power is located near the outer radius of the ring because the sliding is more and more important when the distance from the rotation axis increases.

A global validation of the TORSEN differential model has been carried out by comparison of the TDR values (Torque Distribution Ratio), for each working mode, computed by simulation with experimental results on a test bench (see Ref. [11] for more details). Dynamic analyses have been performed using the Chung-Hulbert generalized-α integration scheme (Ref. [12]).

However, several drawbacks in this contact model have been noticed during these simula-

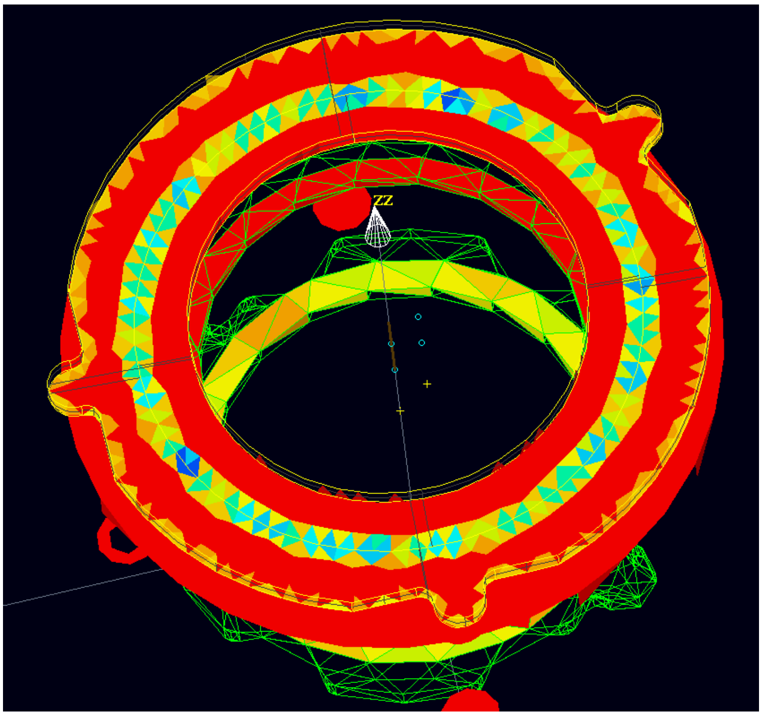

Figure 7: Contact pressure Irregularities due to washers meshing with tetrahedrons

tions. To manage the impact phenomena at the switching time between two working modes, damping and stiffness parameters have been introduced. The fitting of these parameters is not easy. Besides, the automatic time step method adopted decreases the time step to very small values ($\approx 10^{-9}s$) at the impact time, which leads to large simulation computation time.

A particular attention must be put on the meshing of the thrust washers. The type of finite element discretization can influence the convergence properties and the results accuracy. It was observed that modeling the thrust washers with a volume behavior is better than with shell finite element. Furthermore, it is better if the contact surface is composed of quadratic elements which are thereafter extruded to obtain hexahedron elements. The irregularities in the results due to a mesh with tetrahedrons are depicted on Figure 7.

The main disadvantage is the high computational time needed owing to the important number of configuration parameters required by this contact formulation. Indeed, this contact condition requires to model at least one of two contacting bodies with a flexible behavior. However, for the differential models, it is not needed to model all the thrust washers with a flexible behavior. A contact model defined between two rigid bodies seems to be more suitable and would allow to reduce the computational time because the number of nodal variables would be considerably reduced.

4.2 Continuous modeling of impacts between rigid bodies

In this section, another contact model for the gear wheels - thrust washers contacts is presented. This new contact formulation is based on a restitution coefficient and has been implemented in the user element environment of the software SAMCEF/MECANO. This framework allows the user to add its own elements (kinematic joint, force element,. . .) in the library already available in the software.

As explained in the previous section, the main drawback of the classical rigid/flexible contact model available in SAMCEF/MECANO is the quite high computational time needed for complex and large models containing several contact conditions. It is the reason while a impact model between two rigid bodies has been considered. In order to avoid to modify the solver, a continuous contact model has been chosen.

During impacts between rigid bodies, some kinetic energy is lost. Indeed, impacts can initiate wave propagation in the bodies which absorb parts of the kinetic energy until they vanishes owing to material damping. High stresses might also occur near the impact point and involve plastic deformation which also contributes to kinetic energy loss as well as visco-elastic material behavior. Macro-mechanically, these various sources of kinetic energy loss are often summarized and expressed by a coefficient of restitution. The loss of kinetic energy described by the coefficient of restitution depends on the shapes and material properties of the colliding bodies as well as on their relative velocities. However, the restitution coefficient cannot be computed within the multibody system simulation. It has to be roughly estimated from experience, measured by costly experiments or determined by numerical simulations on a fast time scale (Ref. [1]).

There exists different definitions for the coefficient of restitution: kinematic (e_N), kinetic (e_P) or energetic (e_E).

$$e_N = -\frac{\dot{g}_{n_e}}{\dot{g}_{n_s}} \tag{13}$$

$$e_P = \frac{\Delta P_r}{\Delta P_c} = \frac{\int_{t_c}^{t_e} F \, \mathrm{d}t}{\int_{t_s}^{t_c} F \, \mathrm{d}t} \tag{14}$$

$$e_E^2 = -\frac{T_r}{T_c} = -\frac{\int_{t_c}^{t_e} F \, \dot{g}_n \, \mathrm{d}t}{\int_{t_s}^{t_c} F \, \dot{g}_n \, \mathrm{d}t} = -\frac{\int_{h_c}^{h_e} F \, \mathrm{d}h}{\int_{h_s}^{h_c} F \, \mathrm{d}h} \tag{15}$$

where $\dot{g}_{n_s}$ and $\dot{g}_{n_e}$ are respectively the relative velocity between the two bodies in normal direction before and after impacts; the time intervals $[t_s, t_c]$ and $[t_c, t_e]$ correspond to the compression and restitution phases; ΔP_c and ΔP_r are the impulse during the compression and restitution phases; T_c and T_r are the deformation energies during the compression and restitution phases; F is the contact force and $h = -g_n$ is the penetration allowed between the two bodies.

An impact with $e = 1$ means no energy loss (complete elastic contact), whereas $e = 0$ corresponds to a total loss of energy (plastic or inelastic contact). In any case, the restitution coefficient verifies $0 \leq e_i \leq 1$.

These three definitions of restitution coefficient are equivalent unless the configuration is eccentric and the direction of slip varies during impact or if the bodies are rough. Some differences can also appear in case of frictional contact or if several impacts occur simultaneously (see Ref. [15] for more details).

A penalty approach is used for this continuous contact model whereby a small penetration h is allowed. The contact force is computed from this local penetration by a force law

$$F(h, \dot{h}) = k \, h^n + c \, h^n \, \dot{h} \tag{16}$$

where k is the contact stiffness and c is a damping parameter.

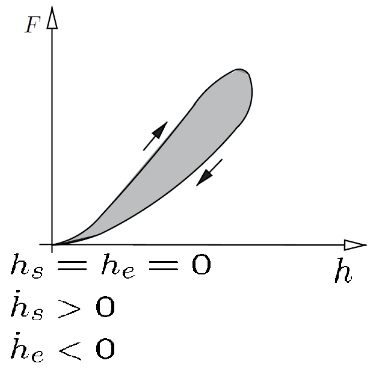

Figure 8: Force law for continuous impact modeling

In order to avoid a jump at the beginning of the impact and tension force at the end of the impact, the classical viscous damping term ($c\,\dot{h}$) has been multiplied by h^n.

As depicted in Fig. 8,this force law yields a hysteresis loop with $h_s = h_e = 0$ for the force-penetration curve. The enclosed areas represents the kinetic energy loss during impact.

The parameters k and d have to be chosen in order to have a realistic values for the impact duration, the local penetration and the kinetic energy loss. One way to set the damping parameter consists of computing this coefficient as a function of the restitution coefficient. According to the contact configuration, various expressions are available in the literature (see for example [4]). For the contact considered in this work between gear wheels and washers, the expression (17) seems relevant and yields a good approximation of the kinetic energy loss for large coefficients of restitution ($e > 0,8$).

$$c = \frac{3(1-e^2)}{4}\frac{k}{\dot{h}_s} \tag{17}$$

where $\dot{h}_s$ is relative normal velocity between bodies at the contact beginning.

This contact model has been implemented in the user element framework of SAMCEF/MECANO. The force (Eq. 16) applies on the two bodies while there are in contact as well as the contribution of this contact element to the global iteration matrix of the system have to be specified in the user subroutine. In order to compute the tangent stiffness matrix and the damping matrix included in the iteration matrix of the contact element, the incremental form of the virtual work principle can be used.

$$\delta \mathrm{d}W = \delta \mathrm{d}h\, F(h,\dot{h}) + \delta h\, \mathrm{d}F(h,\dot{h}) \tag{18}$$

$$\delta \mathrm{d}W = \delta \mathrm{d}h\, F(h,\dot{h}) + \delta h\, (\frac{\partial F}{\partial h}\mathrm{d}h + \frac{\partial F}{\partial \dot{h}}\mathrm{d}\dot{h}) \tag{19}$$

After some algebraic manipulations, this expression is restated as follows in order to identify the tangent stiffness matrix and damping matrix:

$$\delta \mathrm{d}W = \delta \underline{q}^T \underbrace{\frac{\partial \underline{F}_{int}}{\partial \underline{q}}}_{\underline{\underline{K_T}}} \mathrm{d}\underline{q} + \delta \underline{q}^T \underbrace{\frac{\partial \underline{F}_{int}}{\partial \underline{\dot{q}}}}_{\underline{\underline{C}}} \mathrm{d}\underline{\dot{q}} \tag{20}$$

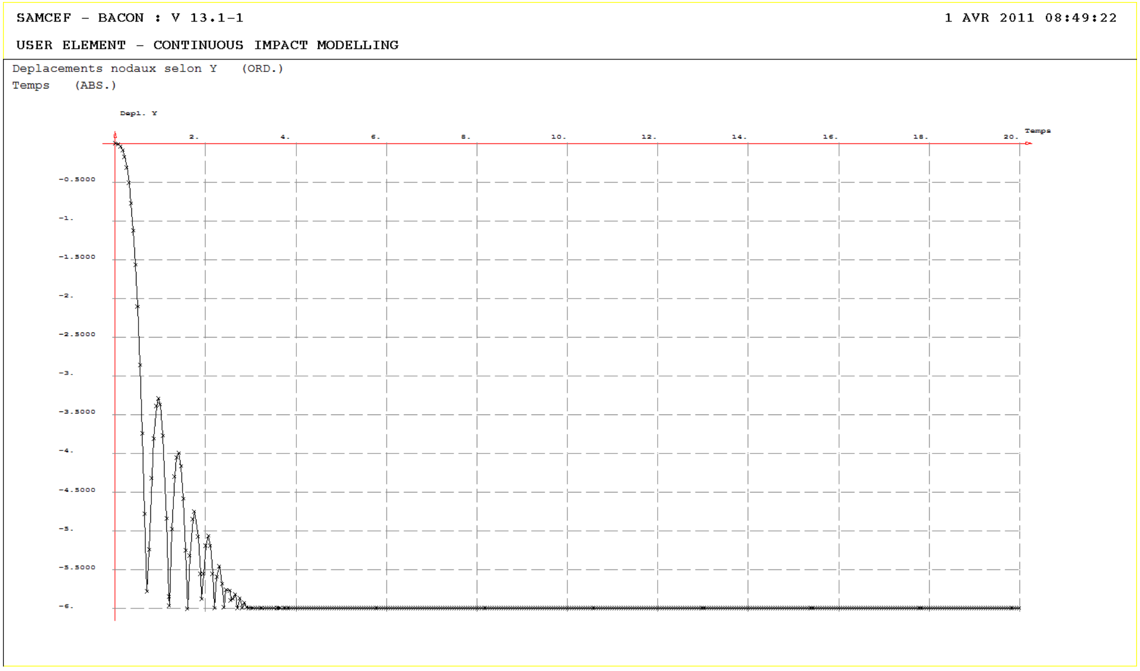

Figure 9: Axial vertical displacement of a rigid body in the gravity field with impact on the ground ($e = 0,8$)

where $\underline{q}$ is the vector of nodal degree of freedom used by the contact element. In the version implemented up to now, this vector contains the position parameters of the node located at the center of the contact surface of two bodies candidate to contact ($\underline{q}^T = (x_A \; y_A \; z_A \; x_B \; y_B \; z_B)$). The tangent stiffness matrix and damping matrix can be expressed as:

$$\underline{\underline{K_T}} = \left[\left(-\frac{F}{h} + \frac{\partial F}{\partial h} - \frac{\partial h}{\partial \dot{h}}\frac{\dot{h}}{h}\right)\underline{n}\,\underline{n}^T + \frac{F}{h}\underline{\underline{I}} + \frac{\partial F}{\partial \dot{h}}\underline{n}\frac{\dot{\underline{x}}_{AB}^T}{h}\underline{\underline{I}}\right] \tag{21}$$

$$\underline{\underline{C}} = \left[\frac{\partial F}{\partial \dot{h}}\underline{n}\,\underline{n}^T\right] \tag{22}$$

with $\underline{x}_{AB}^T = (x_B - x_A \;,\; y_B - y_A \;,\; z_B - z_A)$, the vector between nodes A and B; $\underline{n}$ the normal direction to the contact surface.

In order to make a first validation of this formulation, an academic example is studied. Figure 9 shows the axial displacement of a rigid body in the gravity field and entering in contact with the ground located $6\ mm$ below. A restitution coefficient of 0.8 has been used for this simulation.

For the moment, friction is not implemented in this contact model but could be added without great difficulties. The TORSEN differentials are often located into the gear box housing and are then lubricated with oil through holes in the differential case. The modeling of the thin film fluid could be considered in future developments. The formulation could also be extended to model the contact between plates in clutch or between synchronization devices in gear boxes.

5 CONCLUSIONS

Frictional contacts between gear wheels and thrust washers are essential for the locking effects of TORSEN differentials. In a first step, rigid/flexible contact models based on a augmented Lagrangian approach have been used to model the contact conditions in the differential.

Although a global validation of the model could be achieved, several drawbacks have been noticed in this contact formulation. More particularly the computational time is rather high and the response to impact phenomena leads to numerical difficulties.

A rigid/rigid continuous contact model based on a restitution coefficient has also been studied for differential models. This contact formulation has been implemented in SAMCEF/MECANO and tested for an academic benchmark. In the future, the friction will be added in the contact model and the element will be exploited in the complete differential model. The numerical results and the computational time could then be compared with the simulations using the initial rigid/flexible contact formulation.

6 ACKNOWLEDGEMENTS

The first author would like to acknowledge the Belgian National Fund for Scientific research (FRIA) for its financial support.

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