Mathematical & Numerical Analysis of Coupled Mechanical and Hydraulic Effects Induced by a Blood Pressure Meter

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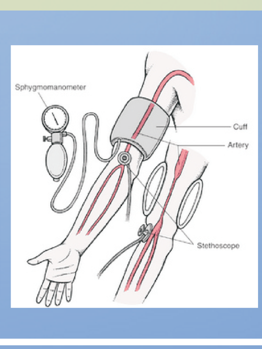
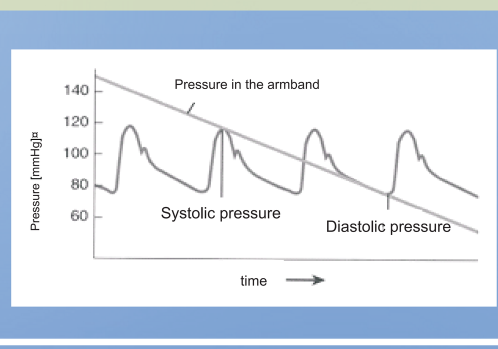
Analysis and Understanding of the Phenomenon's Physiology

Current method for blood pressure measurement

First, one have to induce a pressure through an armband, in order to stop the blood flow. Then, by letting this pressure decrease, the flow resumes and makes some noise called "Korotkoff's noise". The doctor then supplies two numbers. The first one correspond to the pressure when the flow begins (first noise) and the second one to the pressure when the flow stabilizes (no external effect anymore, stabilization of the noise). They are respectively called the systolic and diastolic pressure.

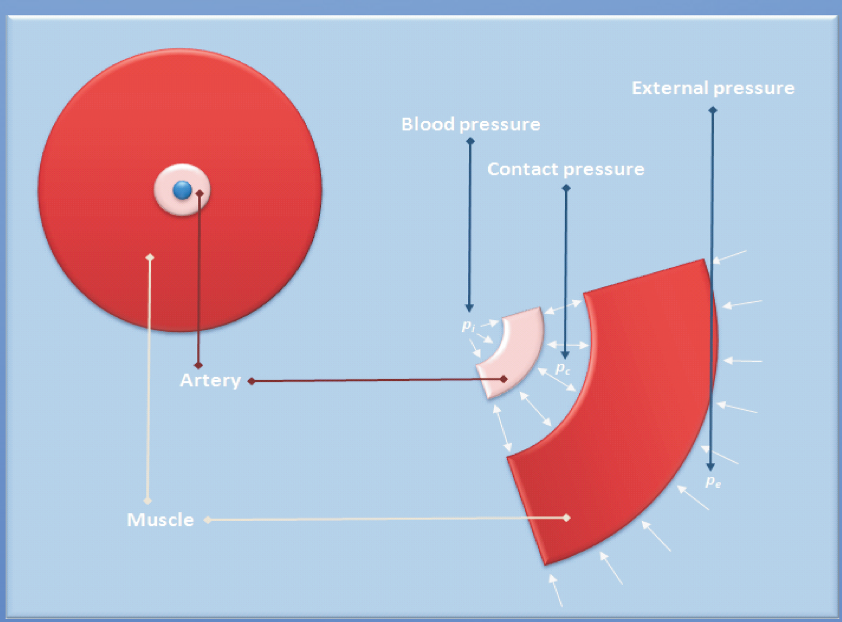




The results given by this method are affected by both the length and the circumference of the cuff, which are indeed not suitable for every patient, but actually more or less adjustable to the patient's anatomy.
The error caused by using a unfitted armband is for now unknown. This paper proposes to give the problem a new outline, by combining medical and engineering knowledge. From an engineering point of view, the problem can be parted in two related and coupled fields of the civil engineering, which are the "Solid Mechanics" and the "Fluid Mechanics", both studied at the MS²F Sector of ULg. In fact, the artery is modelled as a particular deformable pipe when the blood is a fluid with specific viscosity properties.

Biomechanics

The Artery as a Deformable Pipe...



$x,t \quad f \quad p_i \; x,t \; , p_e \; x,t$

Elastic materials
Cylindrical symmetry
Plane deformation state
Incompressible material
Perfect contact

Linear analysis (Hooke) — Non-linear analysis (Mooney-Rivlin)

Equations from Solid Mechanics

$x,t \quad f$ geometry, pressures x,t, materials

Variables

Anatomical parameters	Mechanical parameters	Physiological parameters
Radius Width	Hooke (E &) Mooney-Rivlin parameters	Pressure (given by the Hemodynamic)

Investigations

Despite a lot of researches and a wide range of existing informations contained in the literature, it is for now quite difficult to give the problem a thorough approach.
In fact, the behaviour of the materials and also the inherent parameters of the choosen material model remain limited.

Hemodynamics

The Blood as a Particular Fluid... Equations & Assumptions

Equations

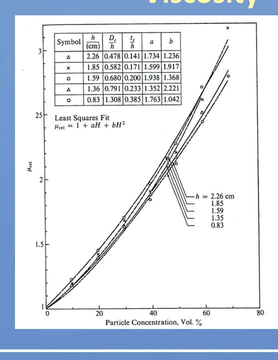
$$\frac{}{t} \qquad 0$$

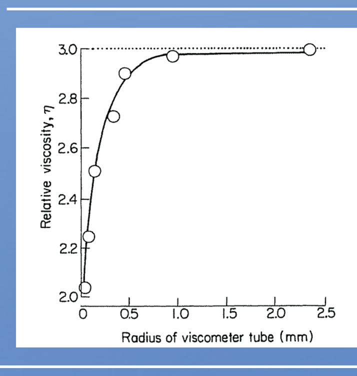
$$\frac{U}{t} \quad U \quad U \quad \frac{1}{} p \quad {}^2 U$$

Variables: & U

Viscosity

The viscosity must be modelled considering blood properties. The blood viscosity is related to the Hematocrit, and is obviously not Newtonian but rather binghamian. It is thus often expressed as a relative viscosity, being the ratio of the blood viscosity to the plasma's one, which is Newtonian.





In these first models, one shall use a constant viscosity, which is, within the range of velocities recorded during the phenomenon, quite acceptable.

First Modelling with a Specific Original Model

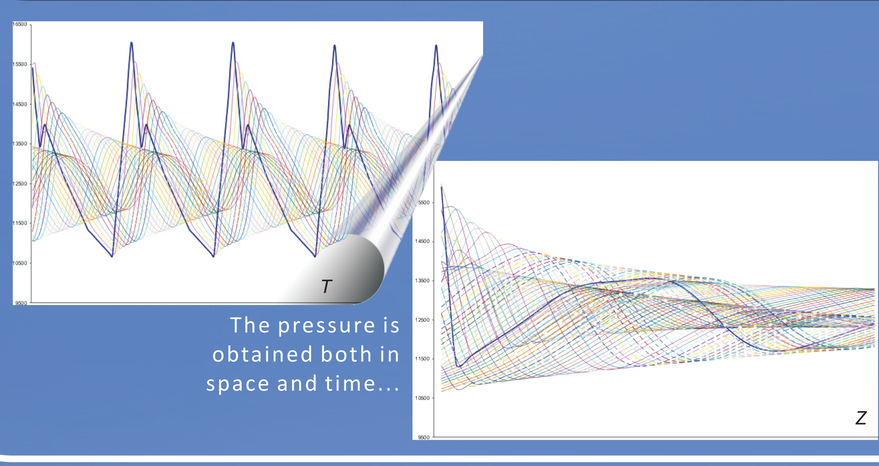
Remark

Before the utilization of WOLF 1D, the problem has been modelled using a specific original model developped in VisualBasic.
It is of the utmost importance to consider the results obtained with care. In fact, this first software consists of an initial approach, and needs this way a lot of hypothesis.
Limitations are caused by the relative knowledge of the phenomenon, and by the complexity of what was known as well.
The most restrictive hypothesis have been made concerning the geometry, resulting in a relative uncertainty about the boundary conditions especially.

Hypothesis

One of the main hypothesis concerns the boundary conditions, because of the complex geometry of the problem. In this first specific original model, these are supposed to be known, considering for this that both flow and pressure are measurable in space and time, and, moreover, that numerical values are known and therefore usable for our boundary conditions.

Results given by the First Model



The pressure is obtained both in space and time...

The pressure is obtained both in time and space. Althought these results are important, they do not procure any direct information about the error made by the use of an unadequate armband.
This can be resolved by comparing the results between two different cuffs, this at specific times, the representative moments being the ones corresponding to the beginning and to the stabilization of the flow (so be it the respective moments of systolic and diastolic pressures) at the stethoscope position.
The armband's dimensions, both the circumference and the length, are parameters in the developped model, it is thus rather easy to change them and also to compare the obtained results.

Integration of the whole Process into WOLF 1D

General Framework of Integration

- Viscosity
- Transient flow
- Boundary conditions

Discharge of the **Blood** through the **Artery**

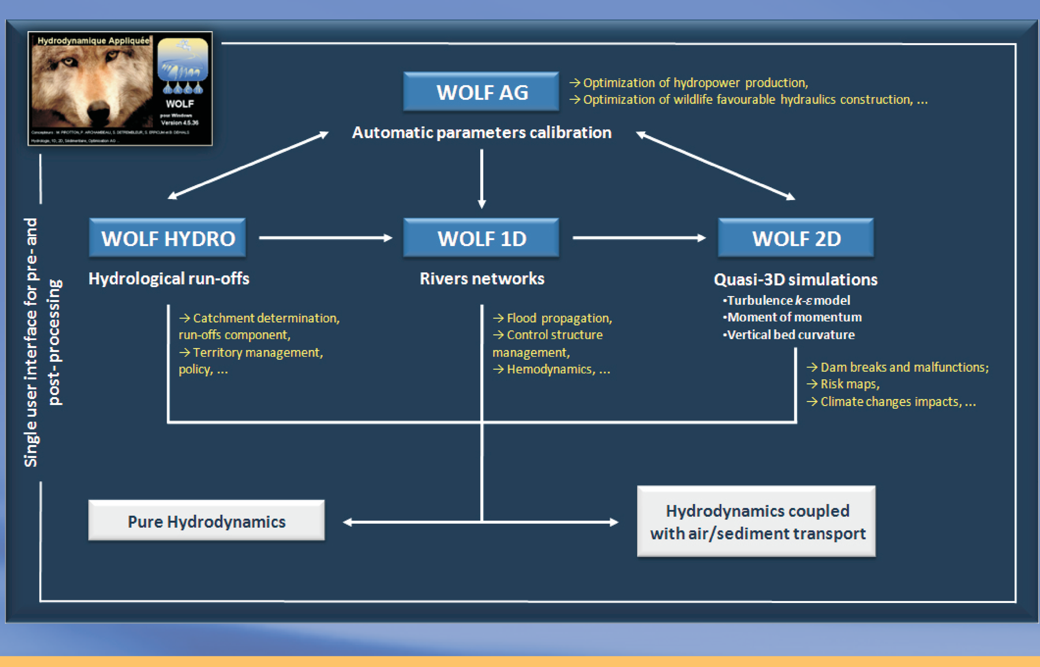
Blood	Artery
Specific fluid	Deformable pipe

3 equations (Navier-Stokes & Pipe)
3 unknows (Q, p & Ω)

Integration of the whole process into WOLF 1D

- Section law
- Specific material parameters

General Framework of Development

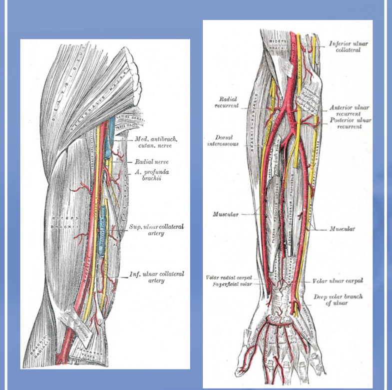


WOLF : an integrated package of free surface and pressurized flow models

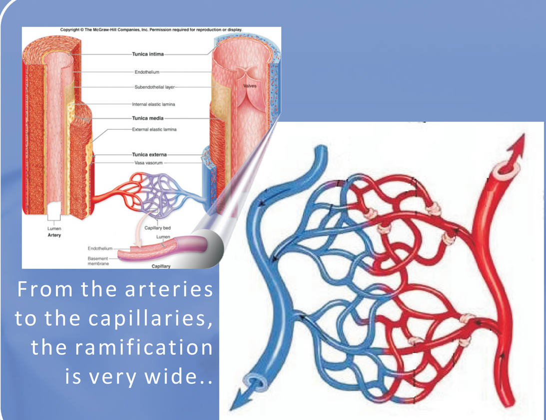
The modelling system WOLF has been developed for almost ten years at the University of Liege (HACH). WOLF includes a complete set of numerical models for simulating free surface and pressurized flows
A user-friendly GIS interface makes the pre- and post-processing operations very convenient. Import and export operations are easily feasible from and to various classical GIS tools. Different layers of maps can be handled to analyse any physical or morphological informations.
The validation of the models has been performed continuously for many years and is still running. Further researches are also currently undertaken.

Additional Reflexions about the Boundary Conditions

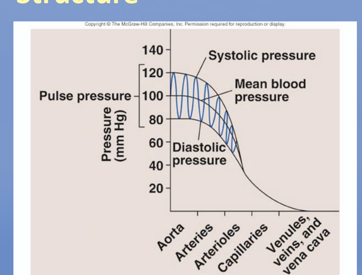
Architecture



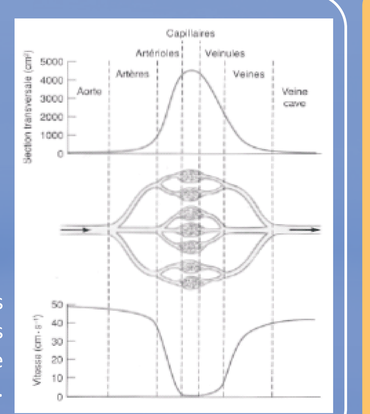
Structure



From the arteries to the capillaries, the ramification is very wide...

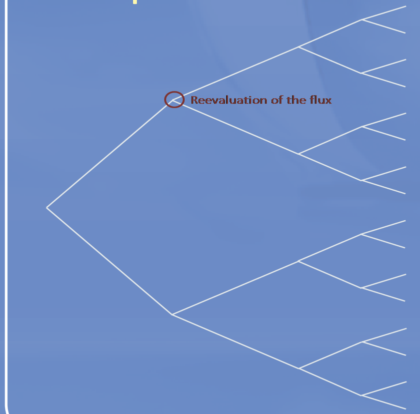


Both the pressures and the velocities are known in the capillaries...

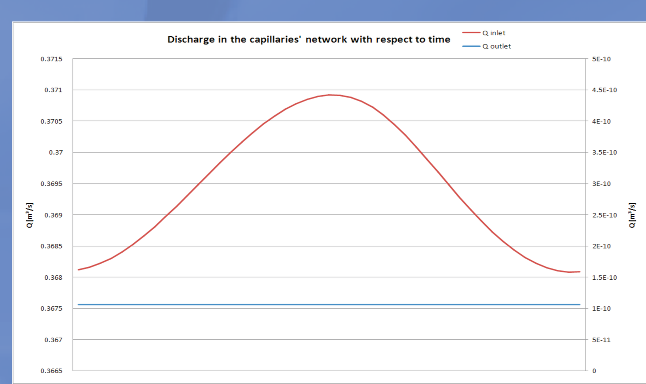


The knowledge of the boundary behaviour is limited. Besides, having access to numerical values would require probing or in vivo tests, what is out of our skills. At the MS²F, the fact that both velocities and pressures are known in the capillaries has been used. The boundary conditions are thus implemented as capillaries conditions, what could be done using branches reminding the architecture of vessels in the arm.

Idealized scheme of the capillaries' network



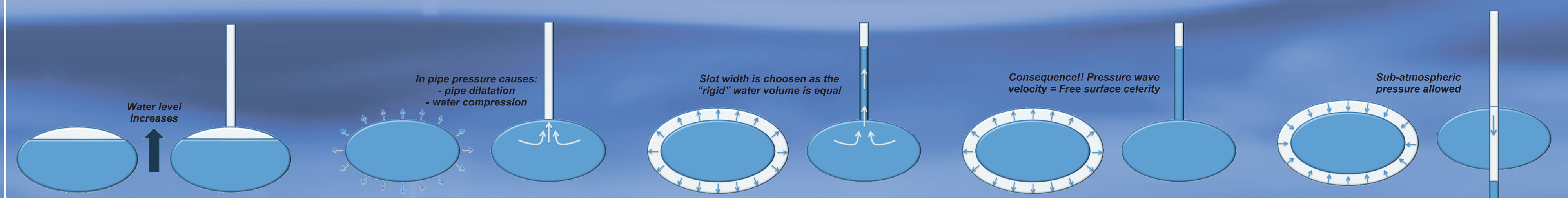
Boundary conditions modeling, first results



By considering only one way (and then reevaluating the flux), one can obtain the discharge through the capillaries' network.
Two main parameters influence the behaviour. These are the slot's width (it actually represents, among others, the material's deformability) and the friction coefficient.
If these are appropriately chosen, the wanted dampening and the known drop can be well represented.

Specifities of the Modeling System

Treatment of the section's dilatation



The classical Preismann hypothesis sets out that a pressurized flow can be equally simulated through the free surface flow set of equations by adding a narrow slot at the top of the pipe.

The modelling system WOLF gives results concerning the flow and the section, resolving actually the classical Navier-Stokes equations, integrated on the wetted surface, with implicit development. To allow the implementation of an external pressure, the code shall be extended to analytical establishment of sections, and also of the parameters depending on the sections, like *e.g.* the celerity and the Preisman slot.

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