A shell fracture framework based on a full discontinuous Galerkin formulation combined with an extrinsic cohesive law

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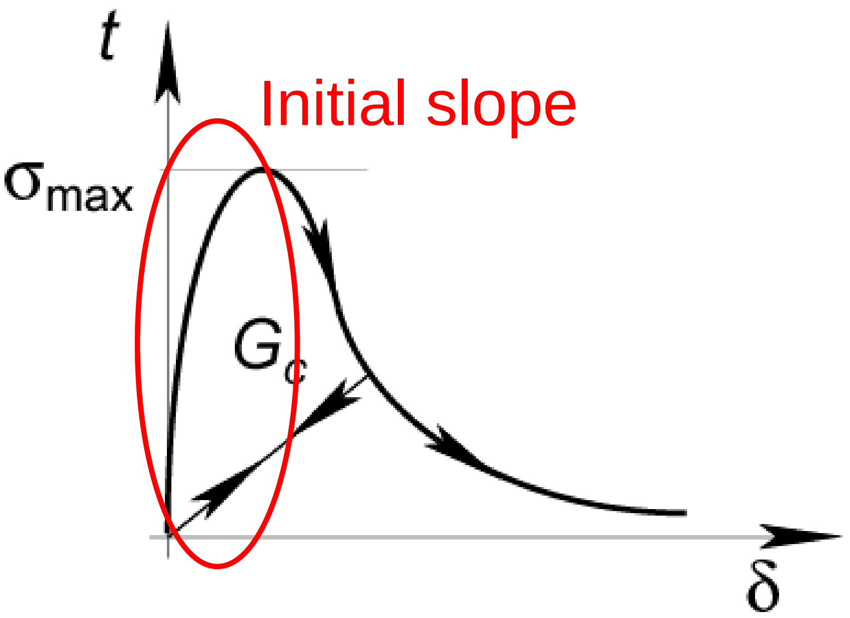
Context of cohesive approach

- Very appealing with finite element method

- Ideally
  - Consistent
  - Easy implementation (//)
- Two classical methods
  - Intrinsic
  - Extrinsic

Context of cohesive approach

- Intrinsic methods
  - Cohesive law

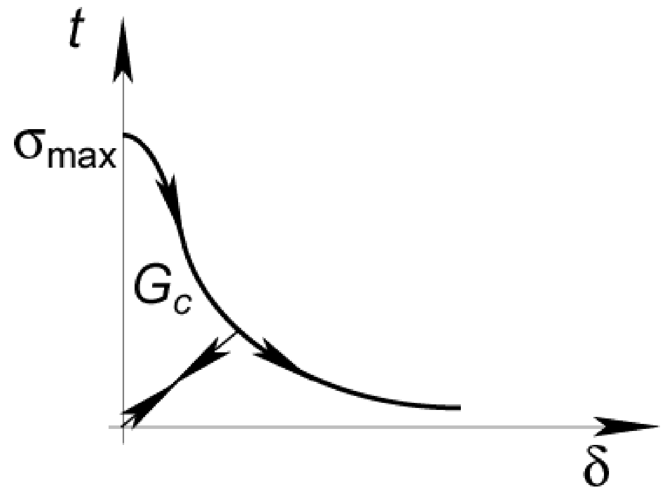


+ Easy implementation: cohesive element inserted at beginning

– Not consistent: initial slope modifies stiffness structure

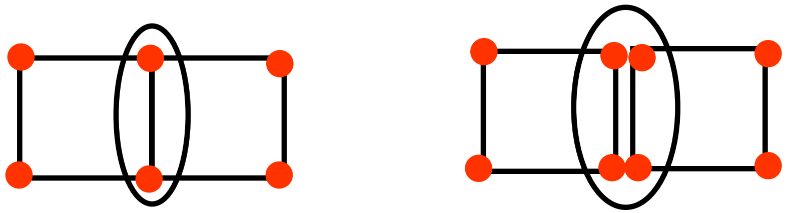
Context of cohesive approach

- Extrinsic methods
  - Cohesive law



+ Consistent

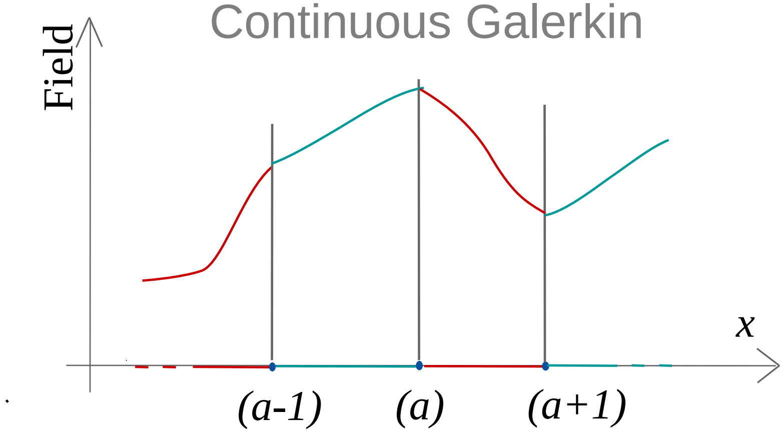
- Complex implementation: dynamic mesh modification



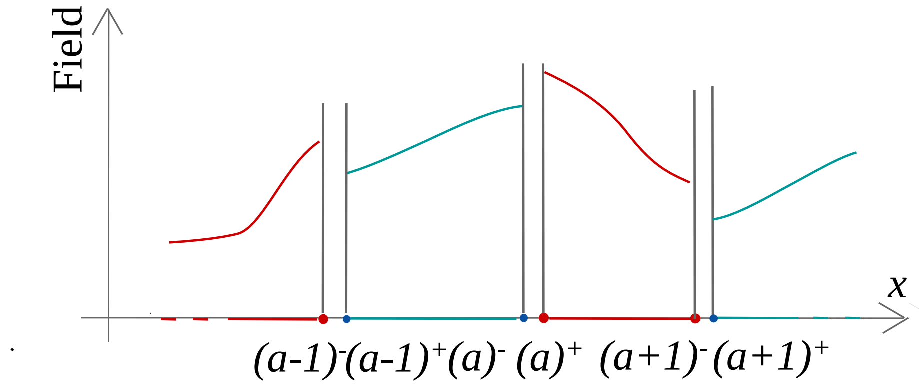
Consistent without mesh modification ?

- Discontinuous Galerkin (DG) formulation
  - Discontinuous tests functions at interfaces

Continuous Galerkin

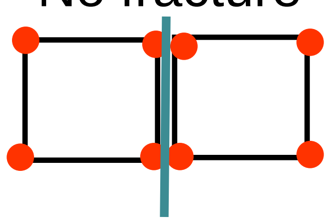


Discontinuous Galerkin



  - Consistency at interface

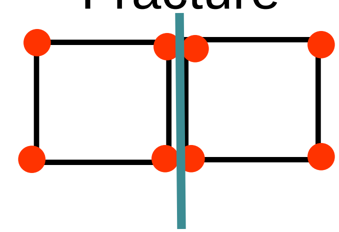
No fracture



DG terms ensure continuity

Switch when fracture crtiterion is reached

Fracture



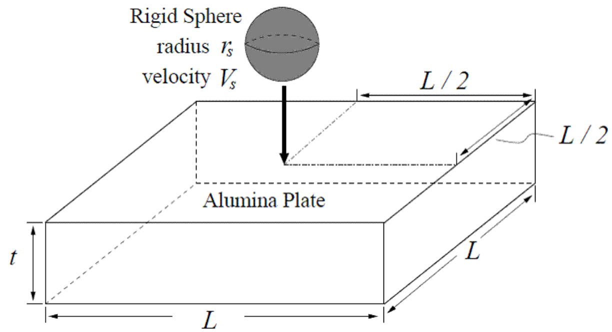
Extrinsic cohesive element

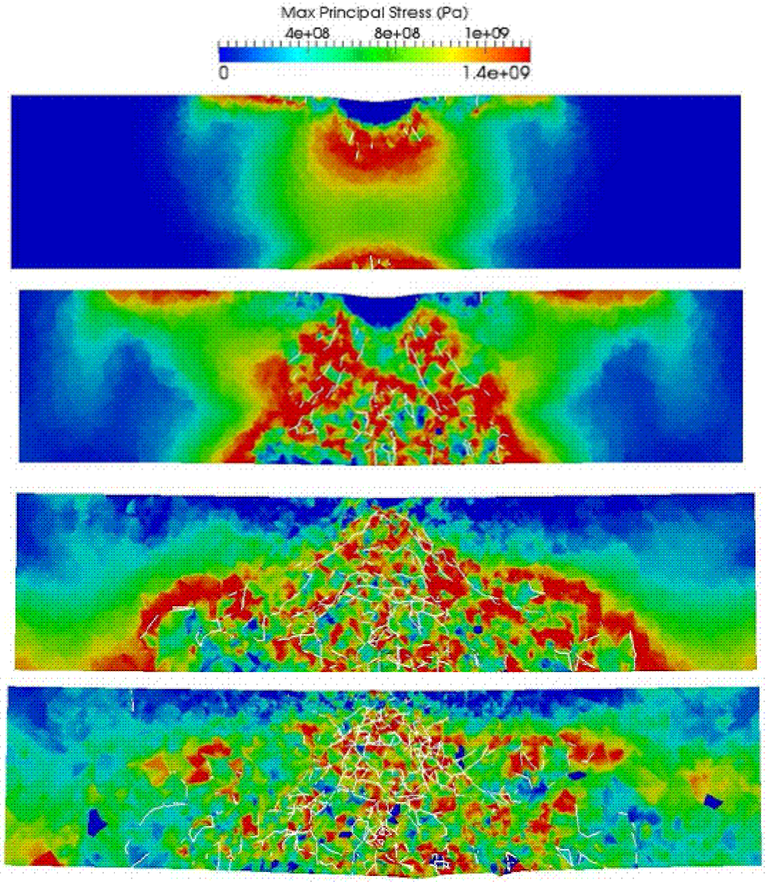
Easy implementation

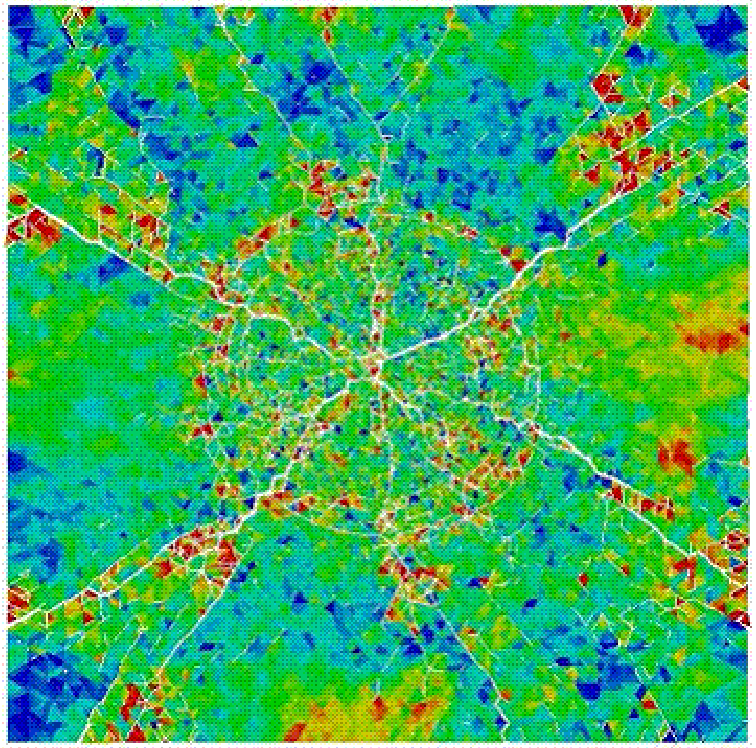
DG / Extrinsic cohesive law combination

- Application to 3D cases

  [Radovitzky, Seagraves, Tupek & Noels CMAME 2010]

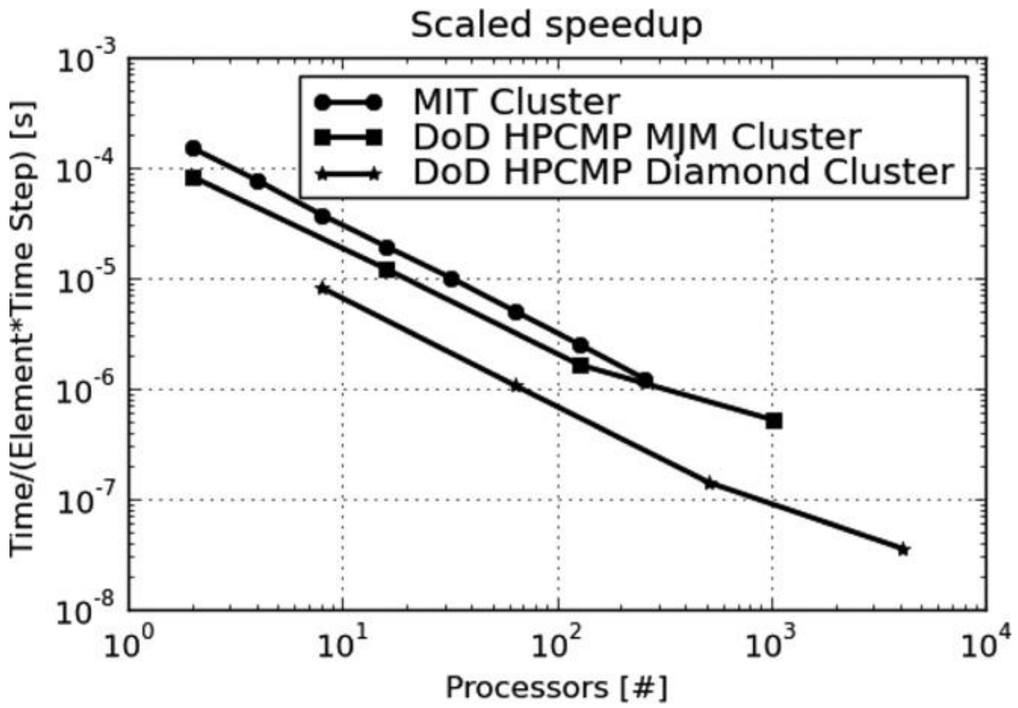






DG / Extrinsic cohesive law combination

- Application to 3D cases
  [Radovitzky, Seagraves, Tupek & Noels CMAME 2010]
  - Scalable approach



- Extension to thin bodies (beam, plate & shell) ?

Topic

- DG Kirchhoff-Love shell formulation
- Coupling with extrinsic cohesive law (ECL)
- DG/ECL applications
- Conclusions & perspectives

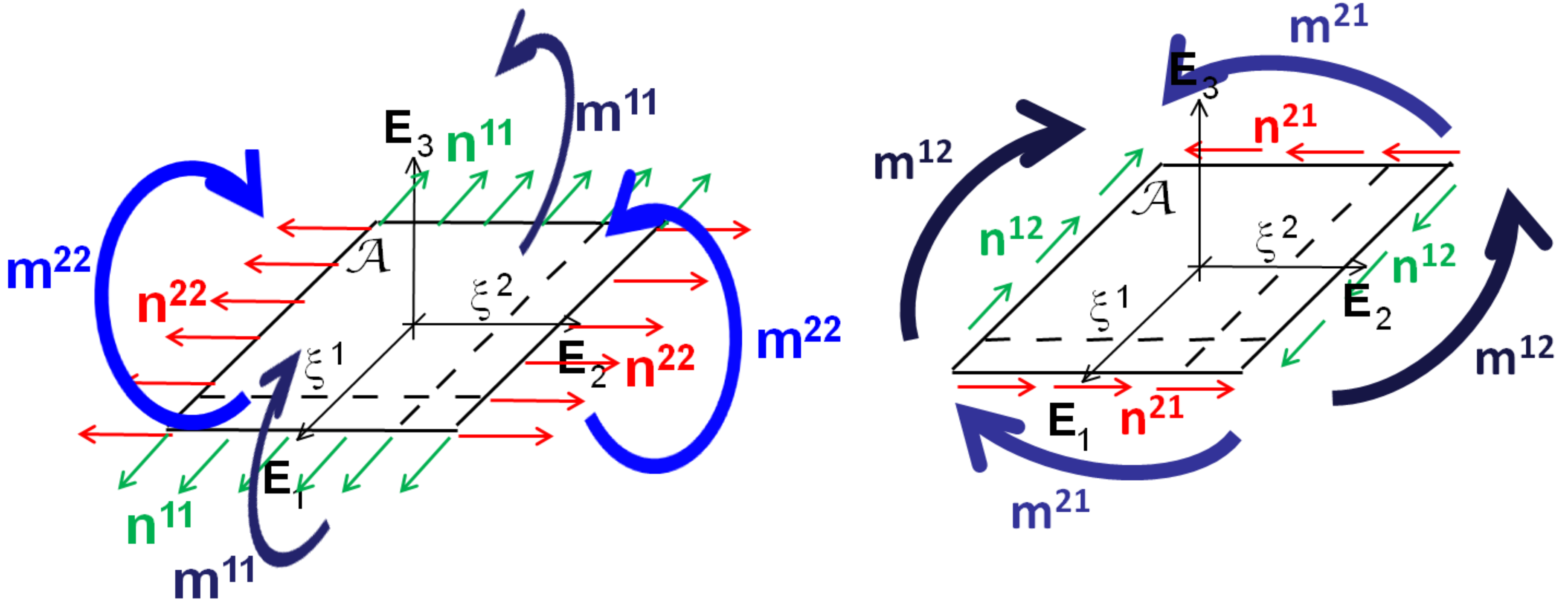
DG Kirchhoff-Love shell formulation

- Conservation equations
  - Linear momentum $$\rho\ddot{\boldsymbol{\Phi}} - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = 0$$
  - Angular momentum $$\boldsymbol{\Phi}\wedge\boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = \boldsymbol{\Phi}\wedge 0$$

- Integration on thickness
  - Linear momentum $$\overline{\rho}\ddot{\boldsymbol{\varphi}} - \frac{1}{\bar{j}}\left(\bar{j}\boldsymbol{n}^{\alpha}\right)_{,\alpha} = 0$$
  - Angular momentum $$\frac{1}{\bar{j}}\left(\bar{j}\tilde{\boldsymbol{m}}^{\alpha}\right)_{,\alpha} - \boldsymbol{l} + \lambda\boldsymbol{t} = 0$$

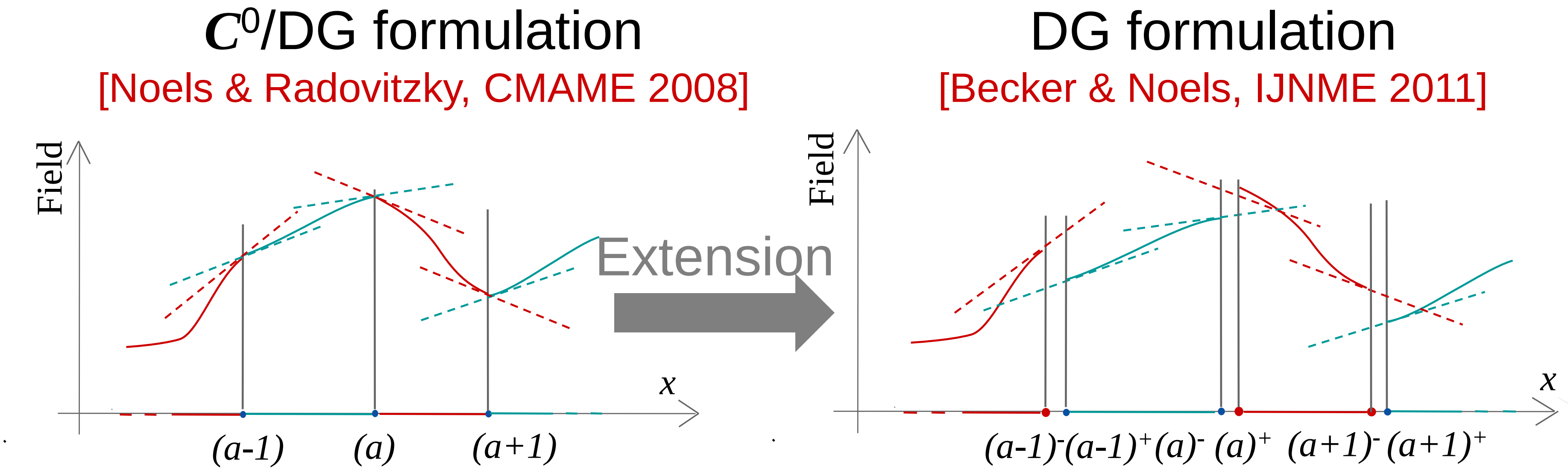
DG Kirchhoff-Love shell formulation

- Expression in term of resultant stresses
  - Stress Vector $$\boldsymbol{n}^{\alpha} = \frac{1}{\bar{j}} \int_{h_{\min}}^{h_{\max}} \boldsymbol{\sigma} \boldsymbol{g}^{\alpha} \det\left(\boldsymbol{\nabla}\boldsymbol{\Phi}\right) d\xi^3 \,,$$
  - Torque Vector $$\boldsymbol{m}^{\alpha} = \frac{1}{\bar{j}} \boldsymbol{t} \wedge \int_{h_{\min}}^{h_{\max}} \xi^3 \boldsymbol{\sigma} \boldsymbol{g}^{\alpha} \det\left(\boldsymbol{\nabla}\boldsymbol{\Phi}\right) d\xi^3 = \boldsymbol{t} \wedge \tilde{\boldsymbol{m}}^{\alpha}$$



DG Kirchoff-Love shell formulation

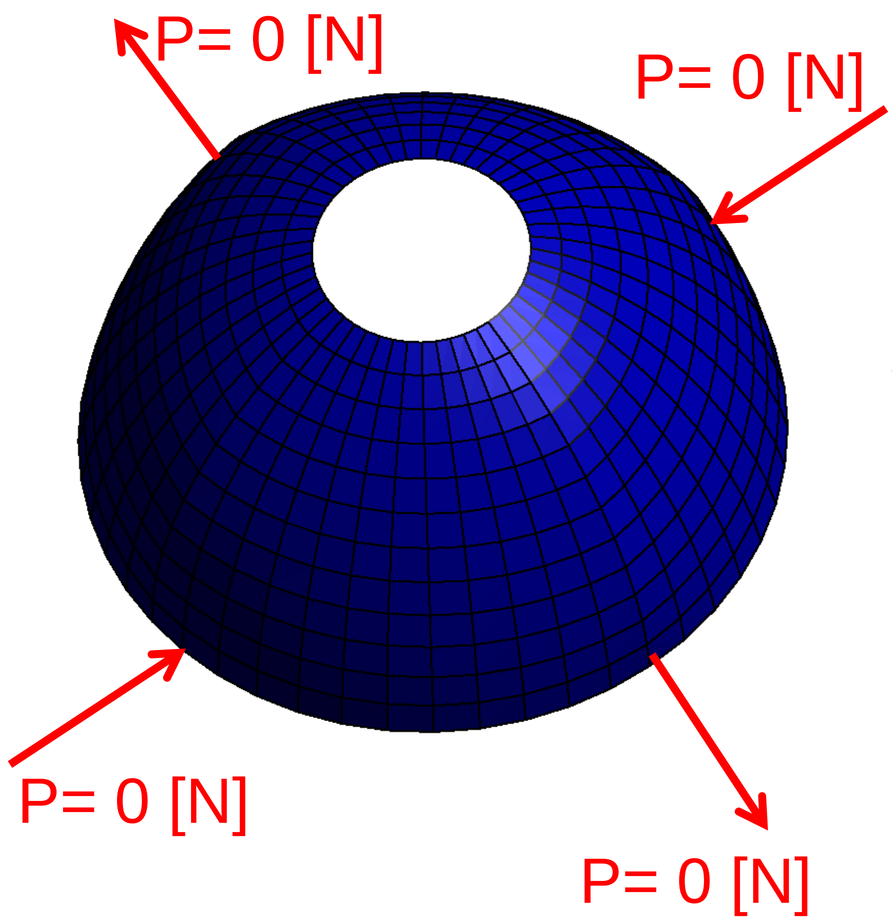
- Thin bodies C^1 continuity required
  - Test functions

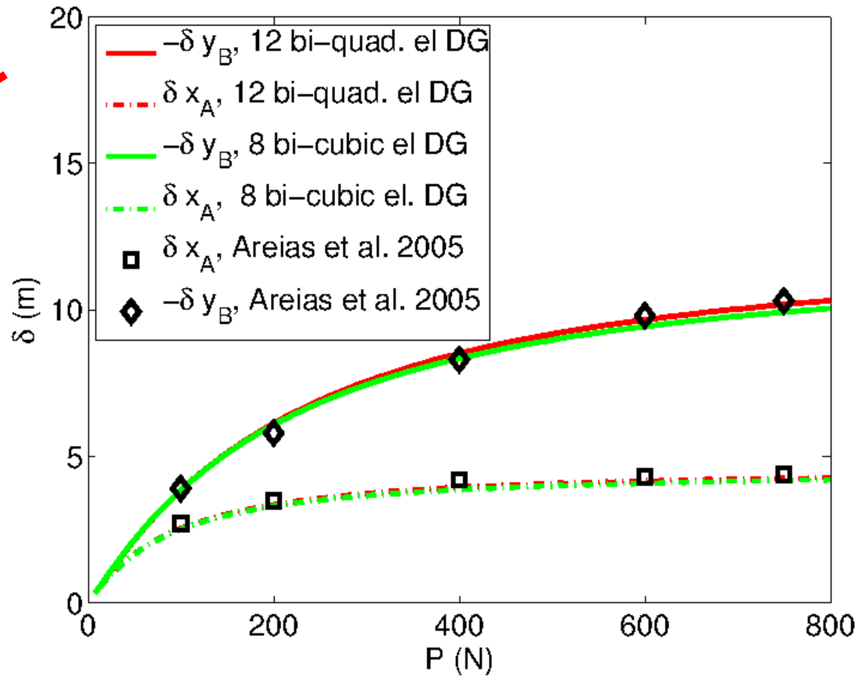


- New DG interface terms
  - Obtained as in literature (consistency, compatibility and stability)

DG Kirchhoff-Love shell formulation

- Pinched open hemisphere





DG Kirchhoff-Love shell formulation

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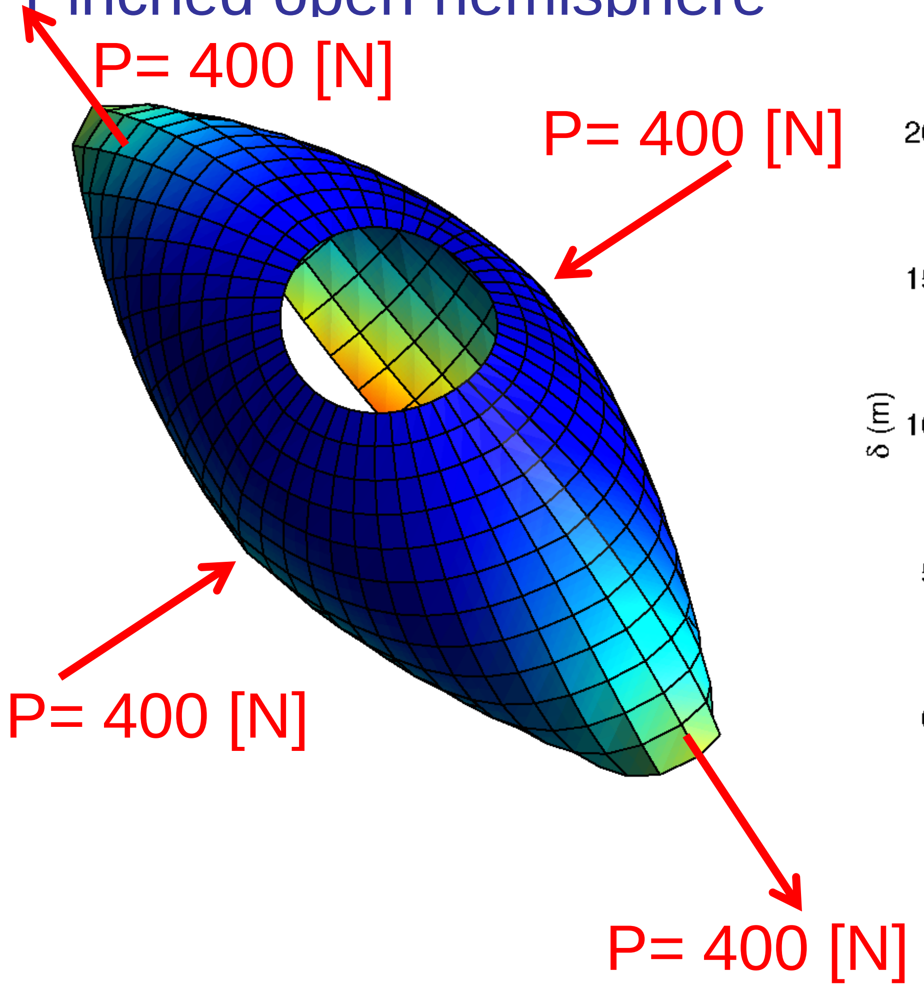
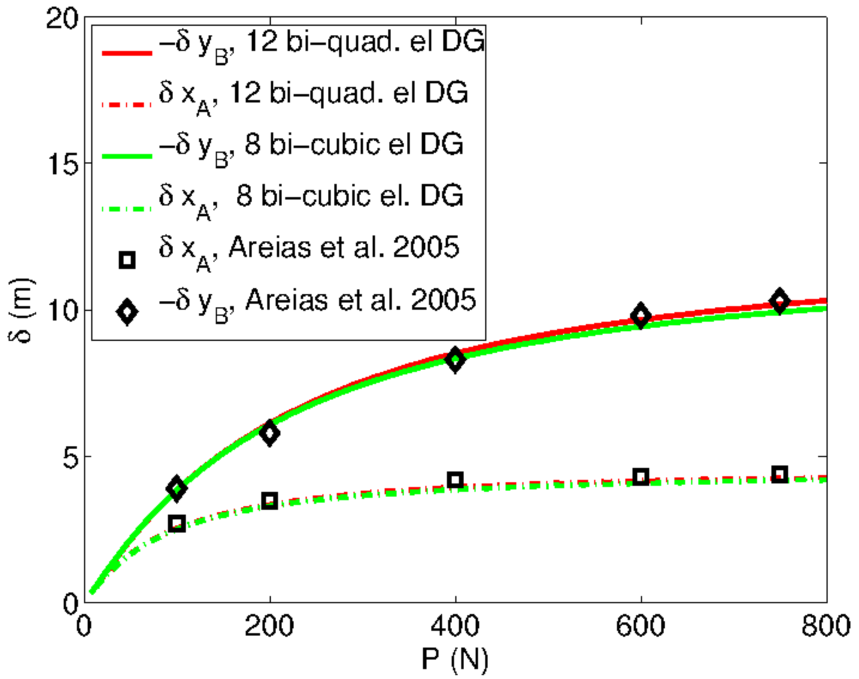
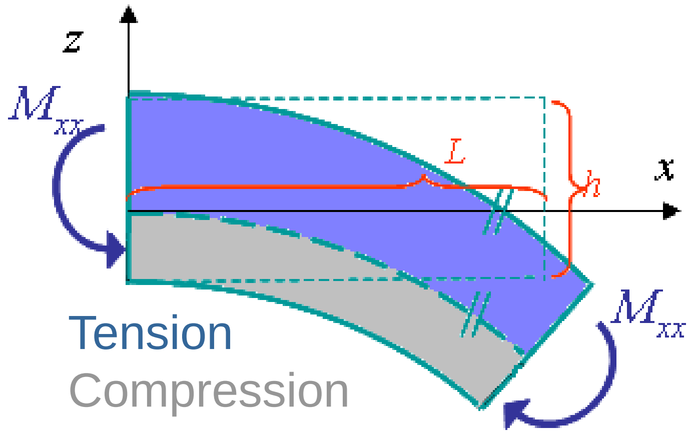


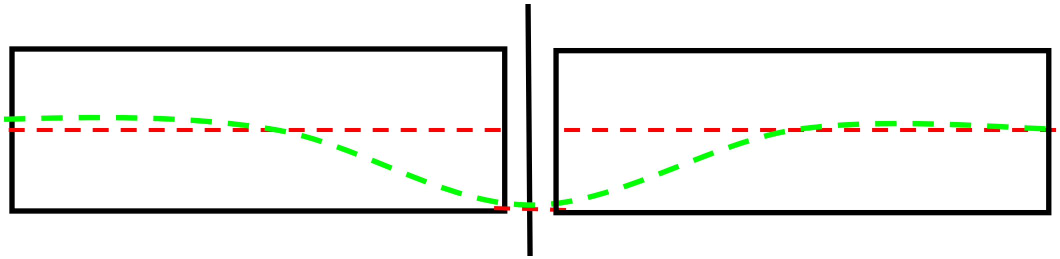


Extrinsic cohesive law for shell

- Main problem: through the thickness crack propagation
  - No element on thickness
  - No fracture in compression
  - Pure Bending



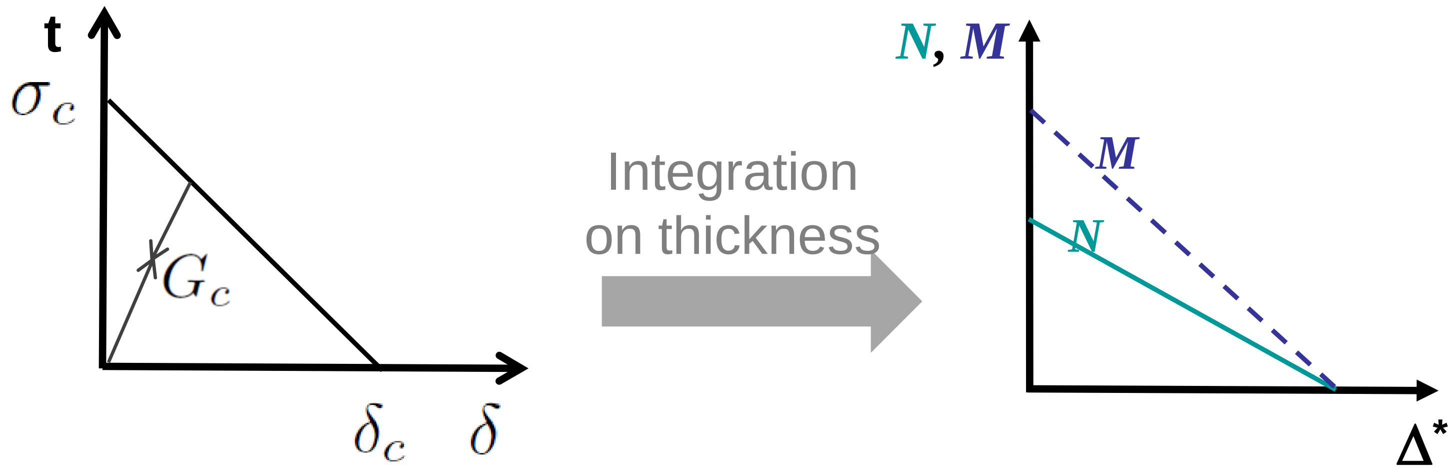
  - How to move the neutral axis ?



Extrinsic cohesive law for shell

- Shell equations are integrated on thickness

- Same for cohesive tensions

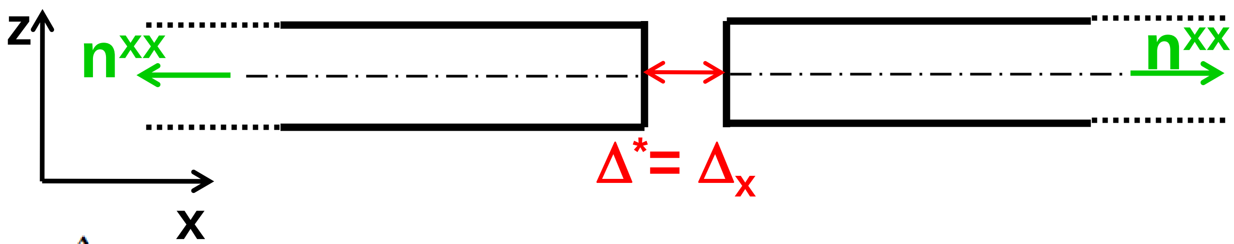


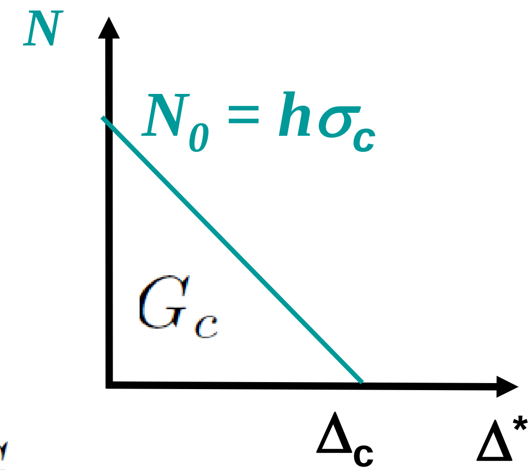
  – How to combine reduced stress (N) and reduced torque (M) ?

- Examine beam case first
  – Only mode I

Extrinsic cohesive law for beam

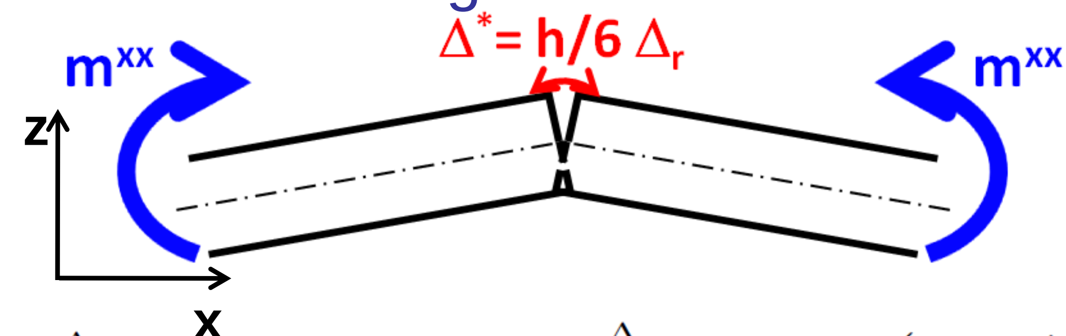
- Energy released = hG_c
- Pure tension

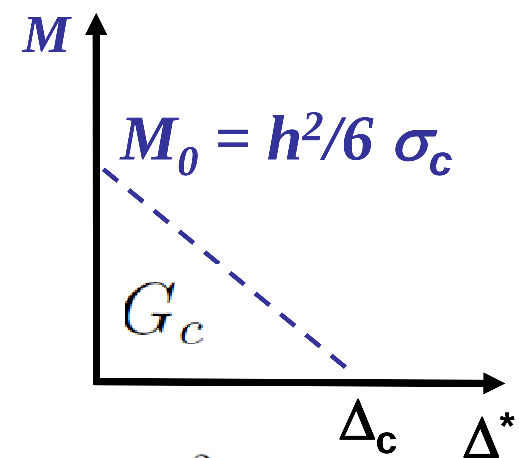




$$\int_0^{\Delta_c} N(\Delta_x)\,d\Delta_x = \frac{N_0\Delta_c}{2} = \frac{2h\sigma_c G_c}{2\sigma_c} = hG_c$$

- Pure bending

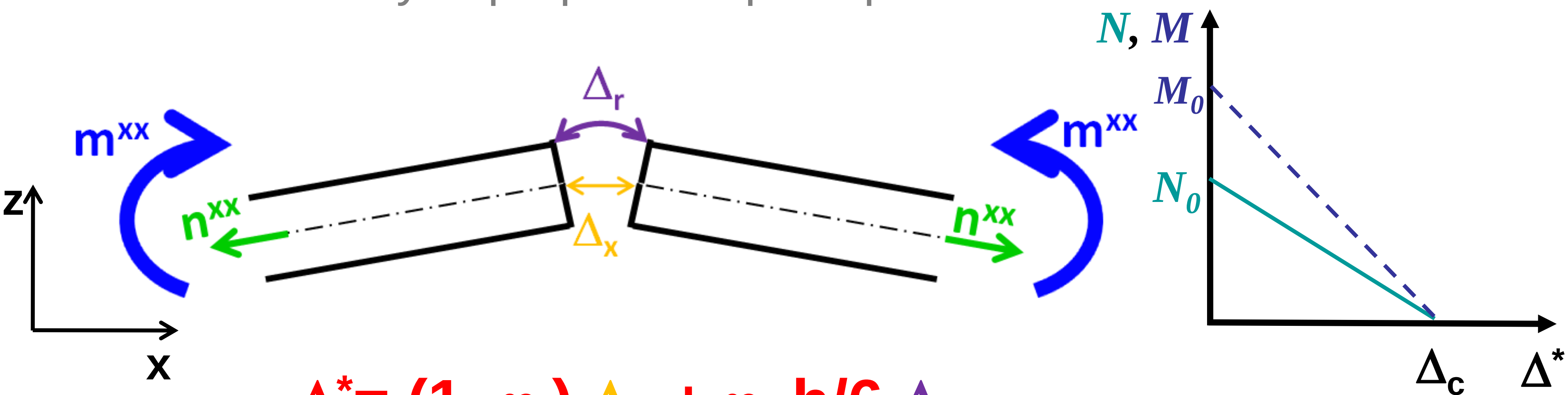




$$\int_0^{\Delta_{rc}} M(\Delta^*)\,d\Delta_{r'} = \int_0^{\Delta_c} \pm\frac{6}{h}M_0\left(1-\frac{\Delta^*}{\Delta_c}\right)d\Delta^* = \frac{6}{h}\,\frac{h^2\sigma_c}{6}\,\frac{\Delta_c}{2} = hG_c$$

Extrinsic cohesive law for beam

- Energy released = hG_c
- Coupled case
  - Obtained by superposition principle



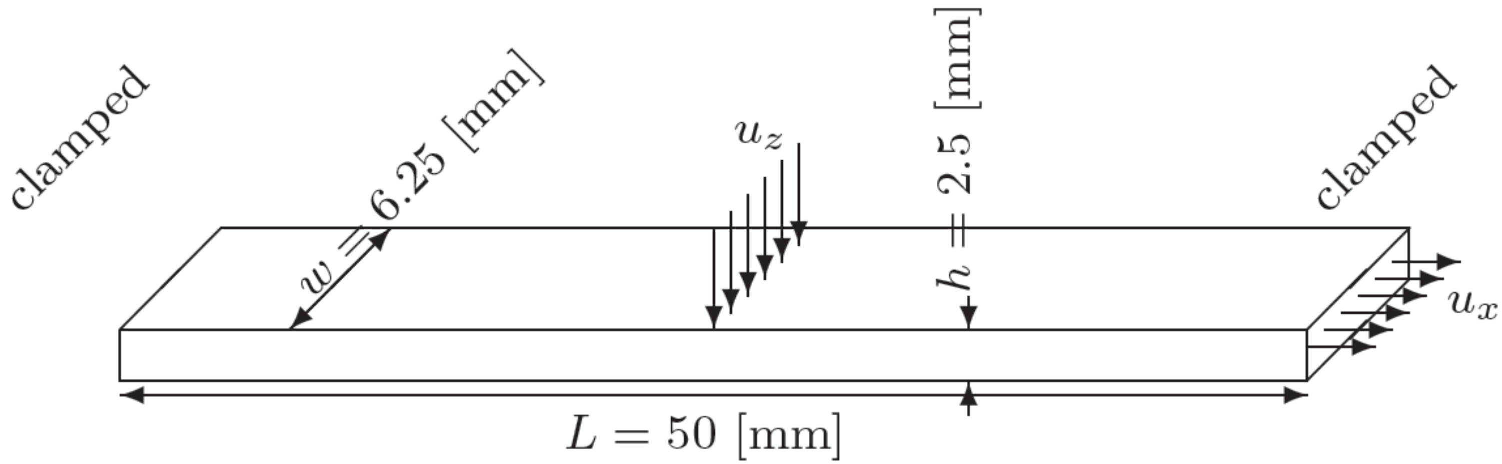
$$\Delta^* = (1-\eta_I)\,\Delta_x + \eta_I\, h/6\, \Delta_r$$

  - With the coupling parameter η_I

$$\eta_I = \frac{|6/hM_0|}{N_0 + |6/hM_0|}$$

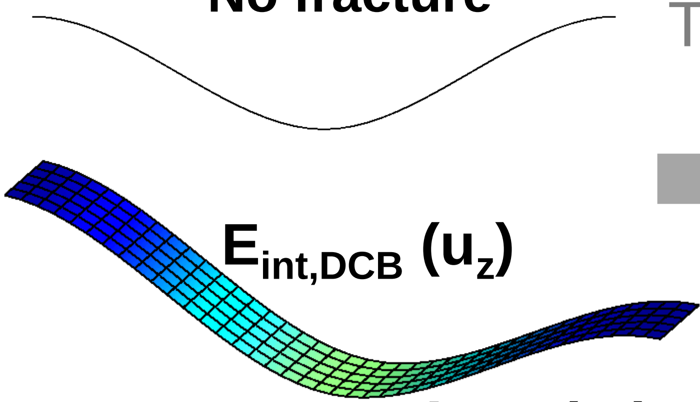
Extrinsic cohesive law for beam

- Coupled case



  - Increase u_z until fracture

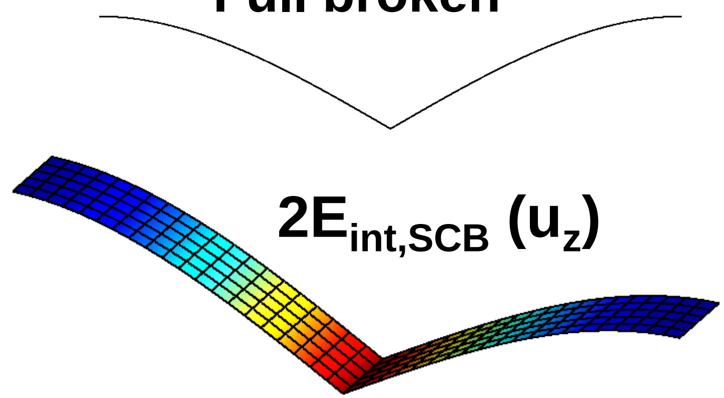
No fracture



$E_{int,DCB}(u_z)$

Transition ?

Full broken



$2E_{int,SCB}(u_z)$

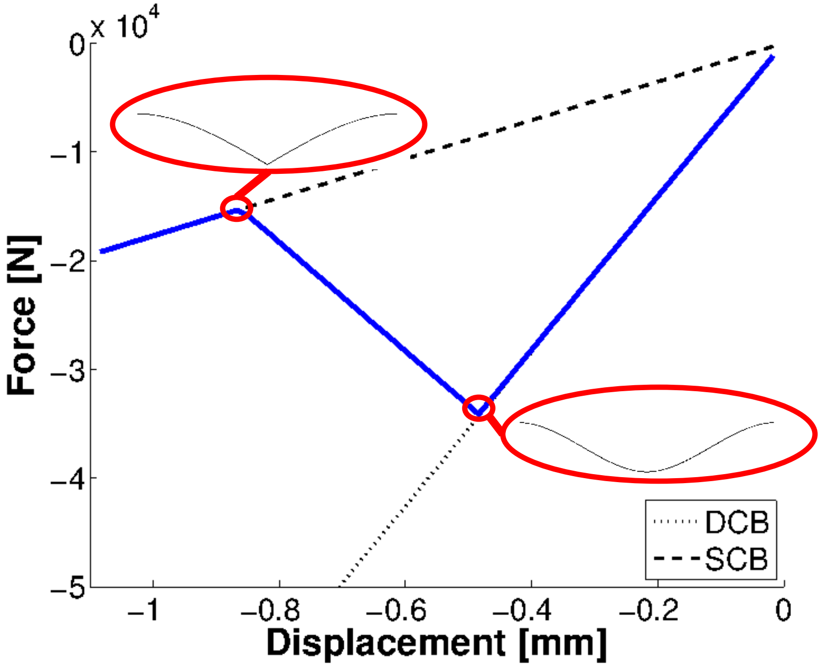
$$\Delta E_{int}(u_z) = E_{int,DCB}(u_z) - 2E_{int,SCB}(u_z)$$

Extrinsic cohesive law for beam

- Coupled case
  - Geometry effect (no pre-strain)

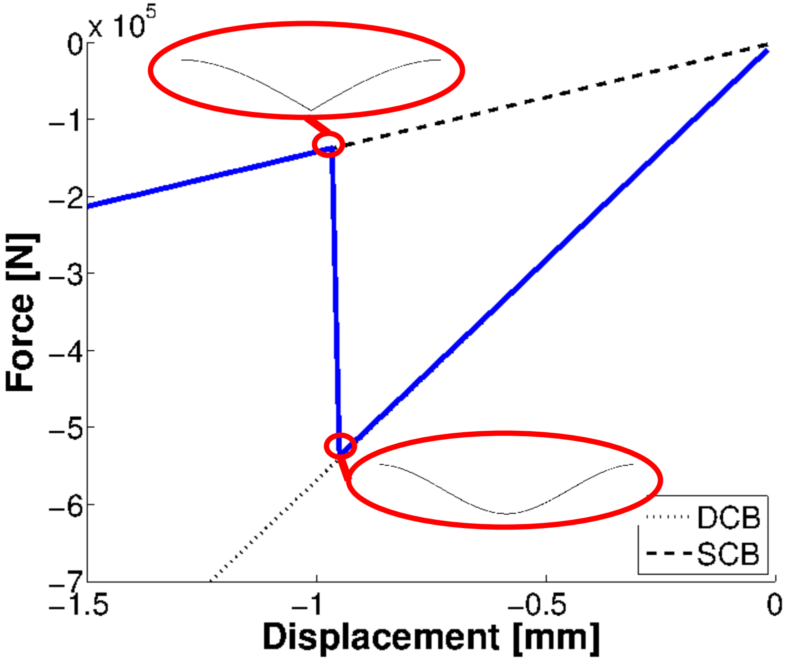
Stable transition

$\Delta E_{int}(u_z) < hG_c$



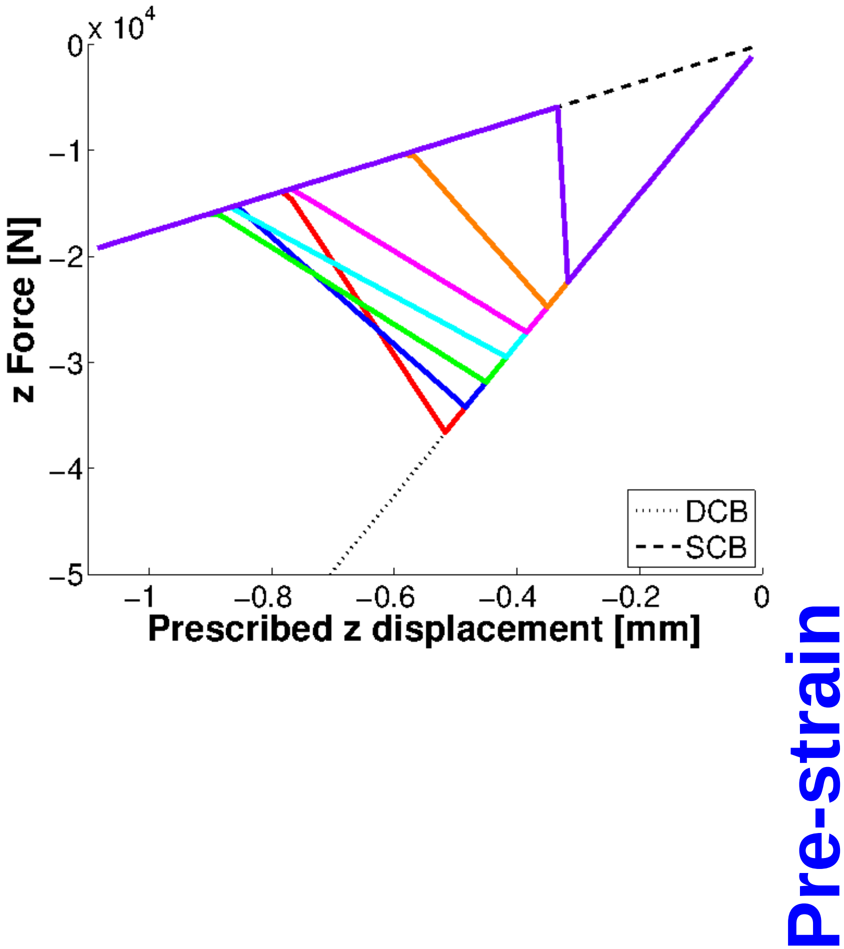
Unstable transition

$\Delta E_{int}(u_z) > hG_c$



Extrinsic cohesive law for beam

- Coupled case
  - Pre-strain effect



hG_c = 22.00

$u_{x,pres}$	η_I	ΔE_{int}	E_{released}
$-2e^{-5}$	1.0692	14.8043	21.98
0.	1	12.33	21.98
$2e^{-5}$	0.93	11.39	21.98
$4e^{-5}$	0.86	11.99	21.98
$6e^{-5}$	0.79	14.11	21.98
$8e^{-5}$	0.72	17.76	21.99
$10e^{-5}$	0.66	22.95	—

Unstable > 22.00

Extrinsic cohesive law for shell

- No mode III
  - Kirchhoff-Love assumption: out-of-plate shearing is negligible
- Applied beam model on 2 fracture modes
  - Mode I : tension and bending $\rightarrow$ $\Delta_I^\star$
  - Mode II: shearing and torsion $\rightarrow$ $\Delta_{II}^\star$
- Combination: Camacho & Ortiz model [Camacho & Ortiz Int. J Solid Structures 1996]
  - Fracture criterion

$$\sigma_{\mathrm{eff}} = \begin{cases} \sqrt{\sigma^2 + \beta^{-2}\tau^2} & \text{if } \sigma \geq 0 \\ \frac{1}{\beta}\overline{|\tau| - \mu\,|\sigma|} & \text{if } \sigma < 0 \end{cases}$$

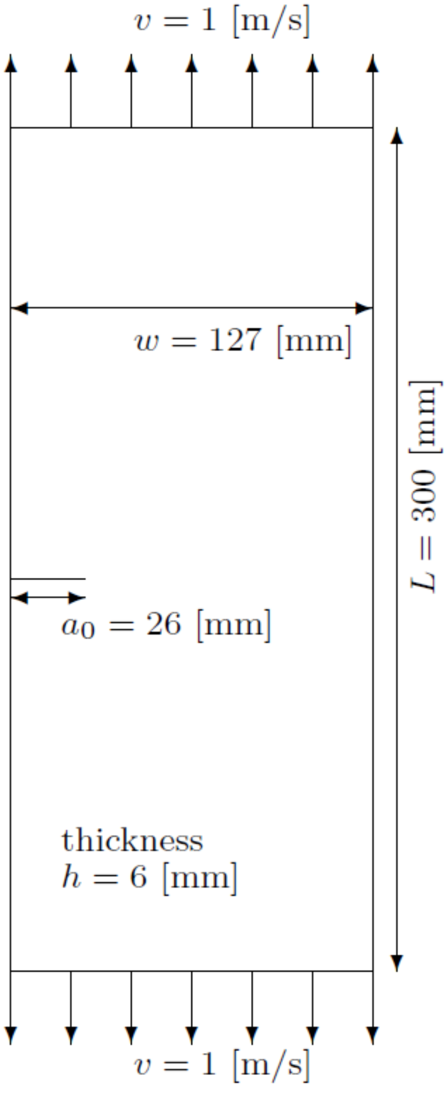
  - Resultant opening

$$\Delta^\star = \sqrt{\overline{\Delta_I^\star}^2 + \beta^2 {\Delta_{II}^\star}^2}$$

with, $\beta = K_{IIc}/K_{Ic}$

DG/ ECL Applications

- Mode I dynamic crack propagation



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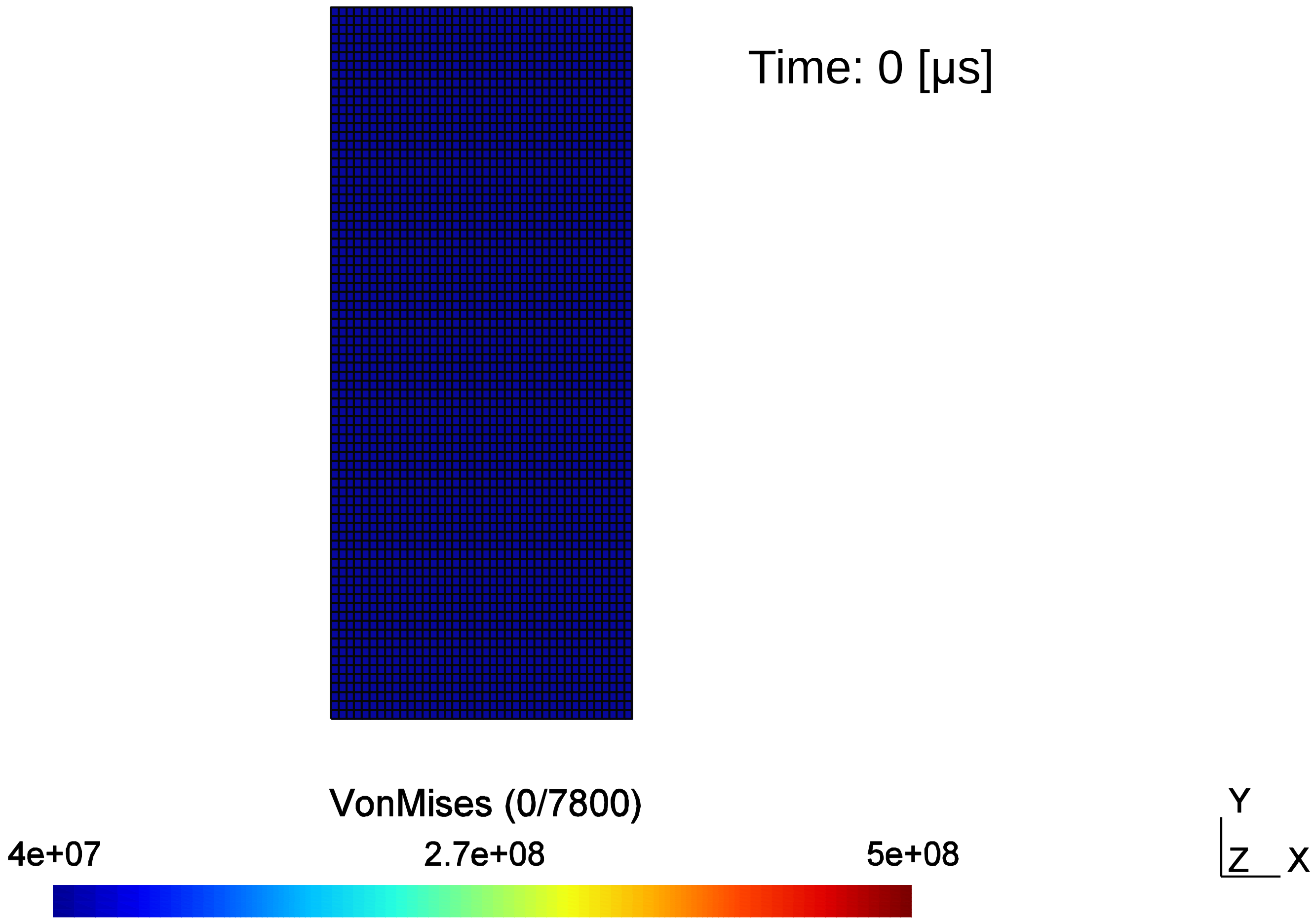

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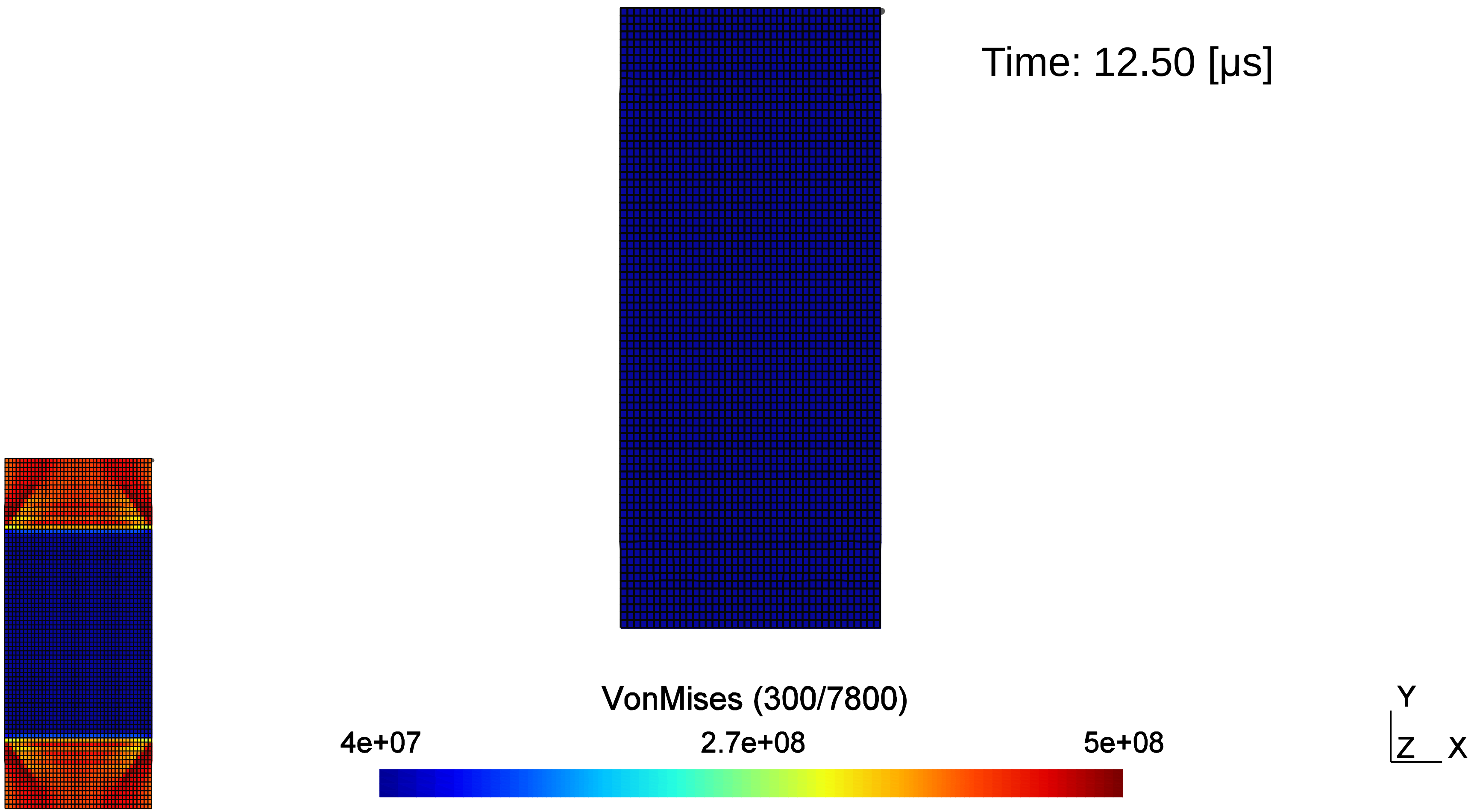

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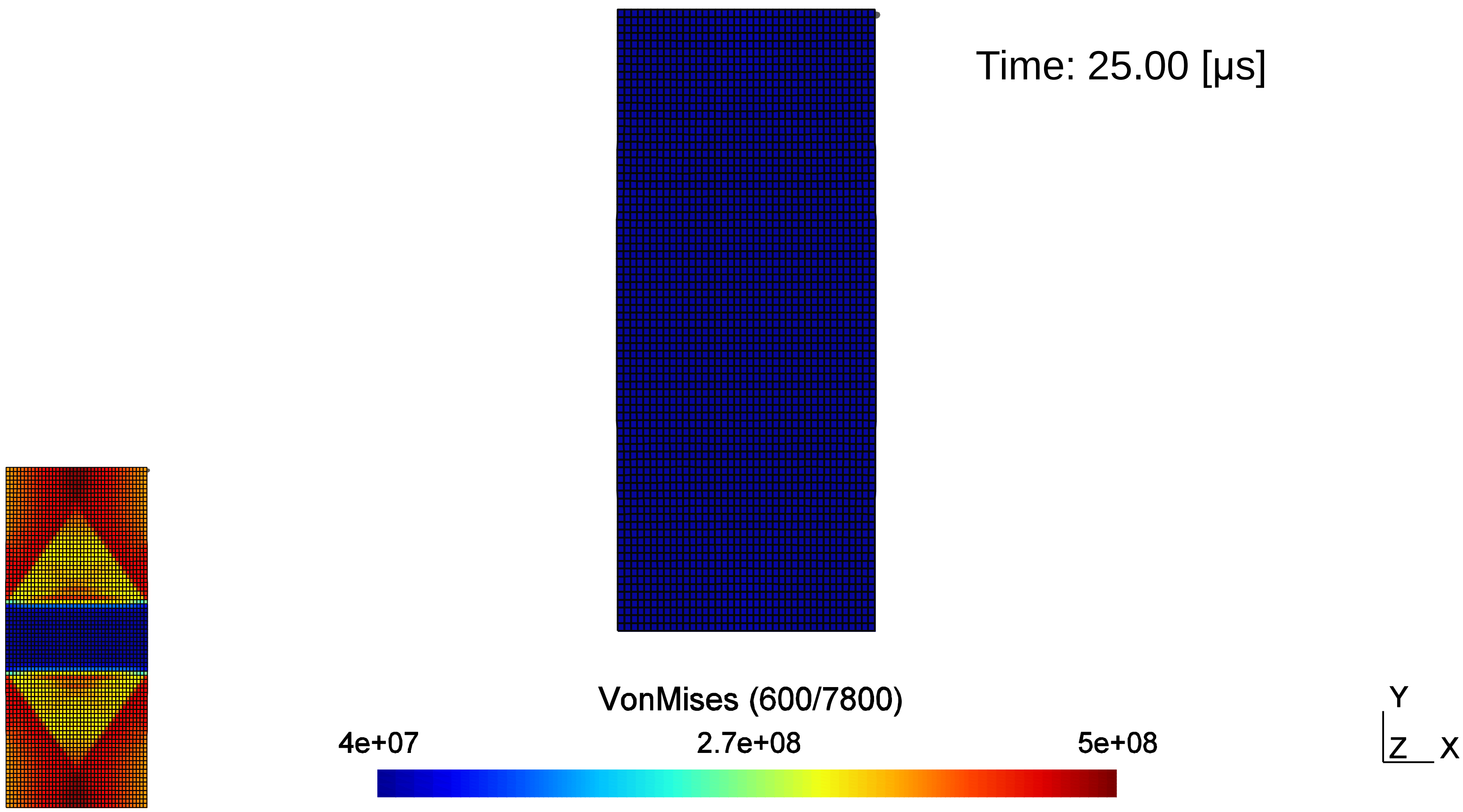

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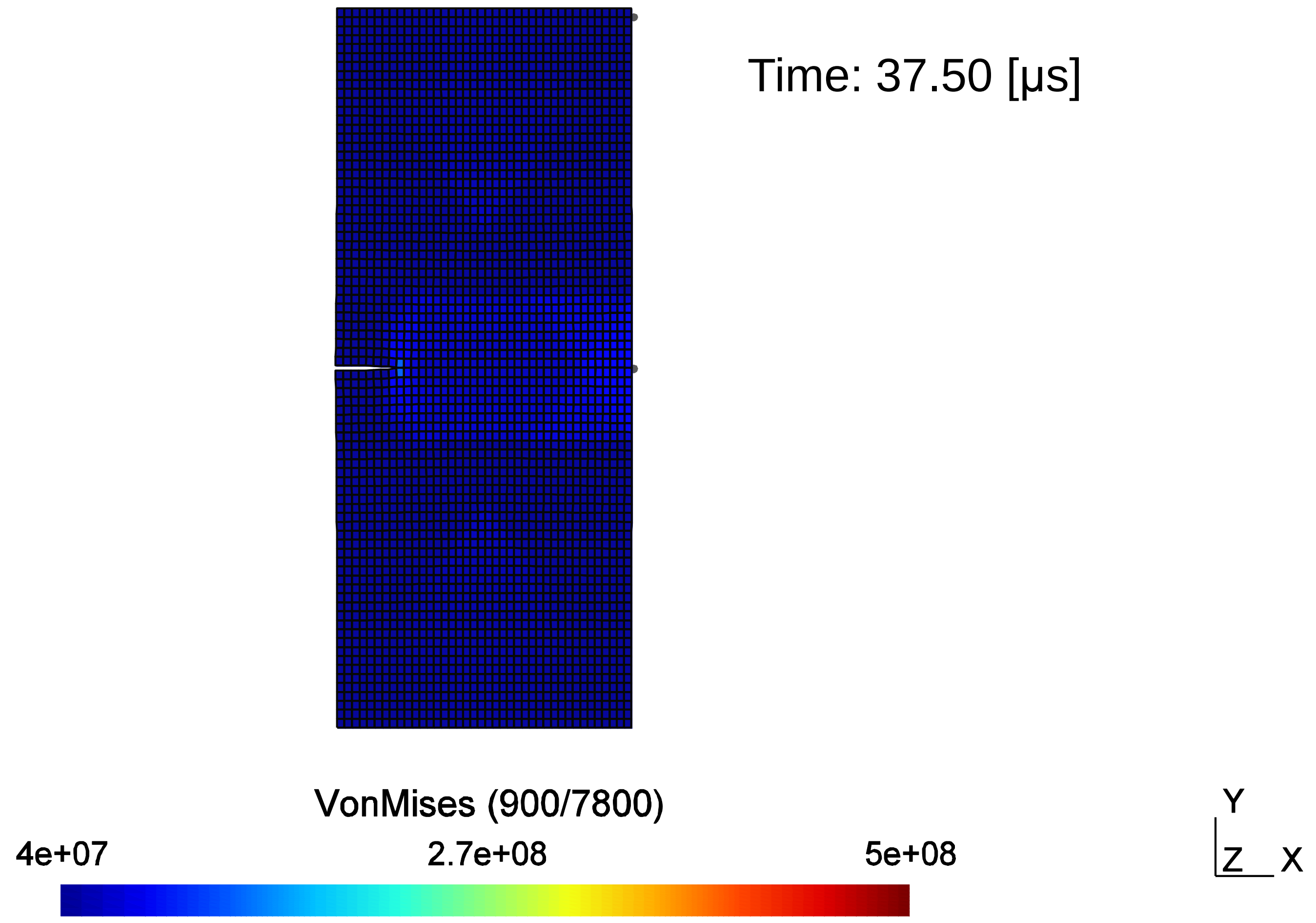

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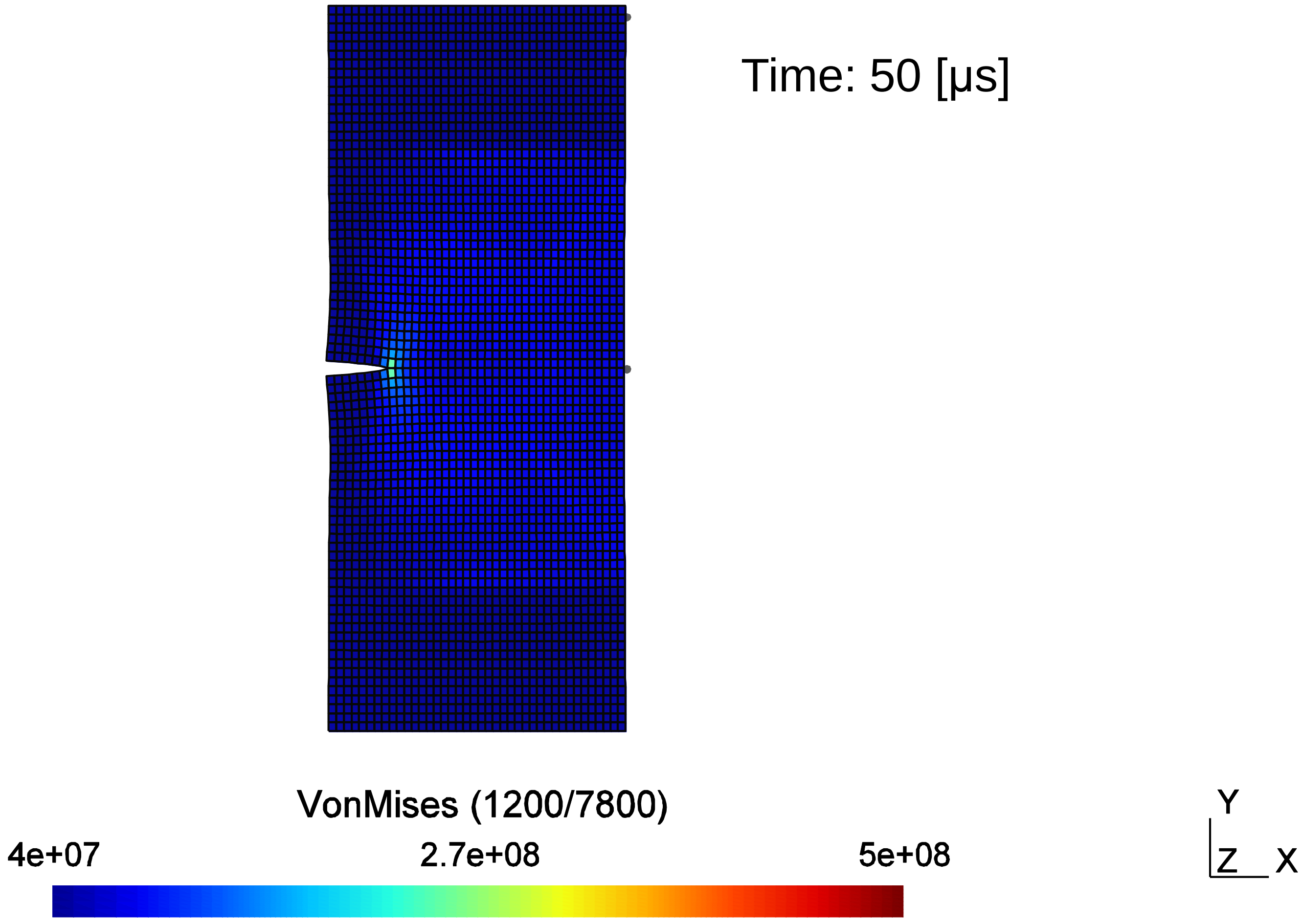

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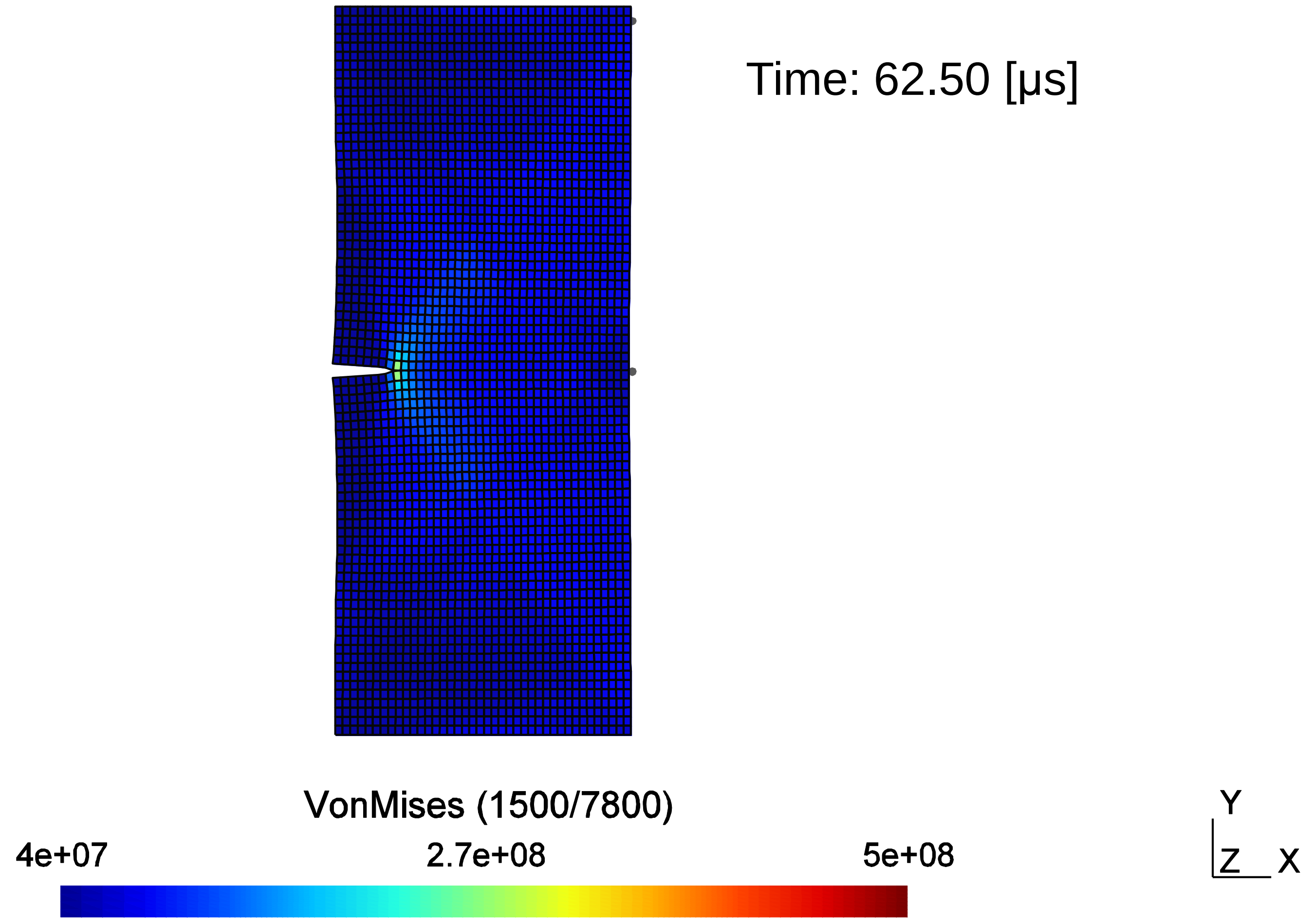

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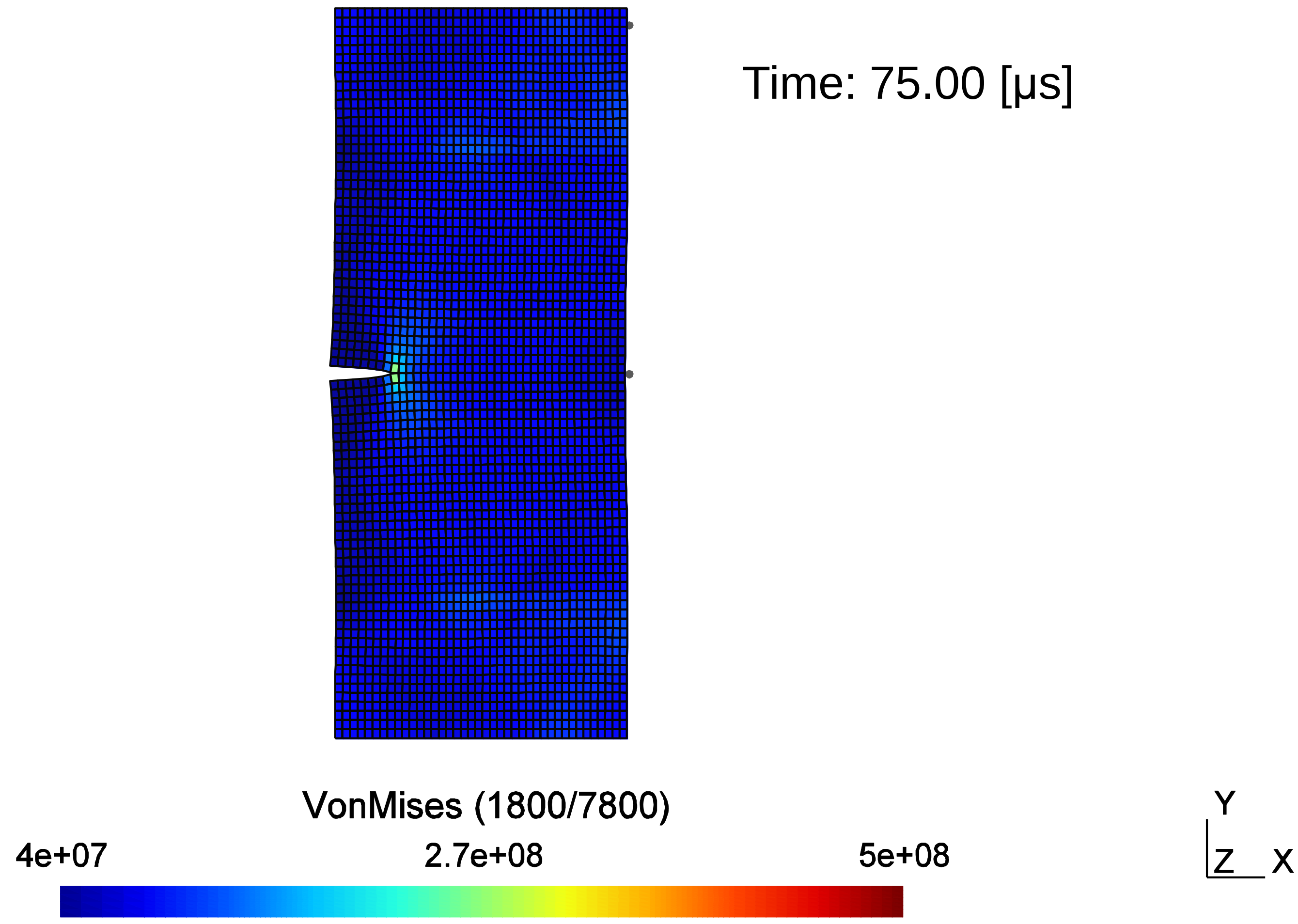

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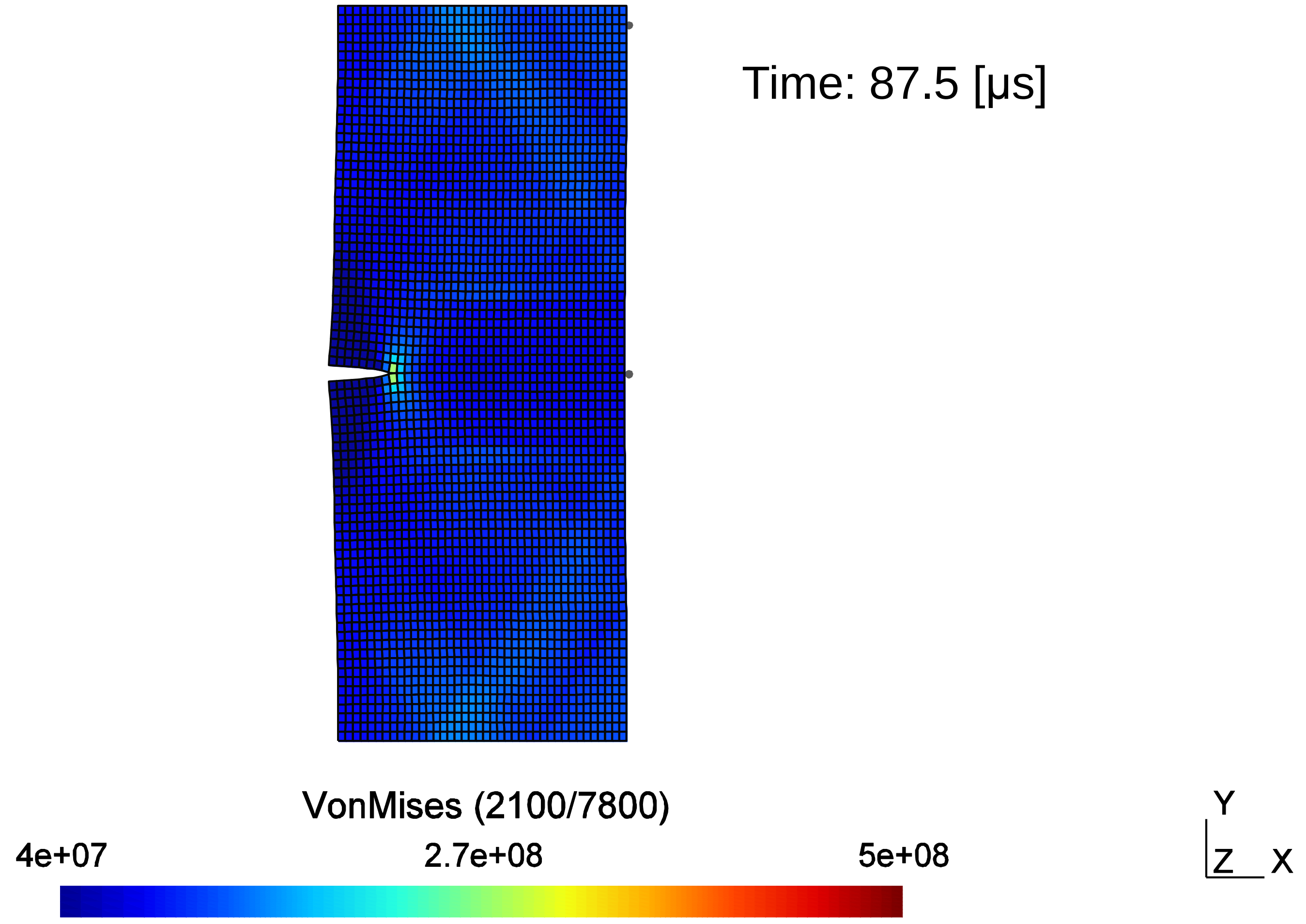

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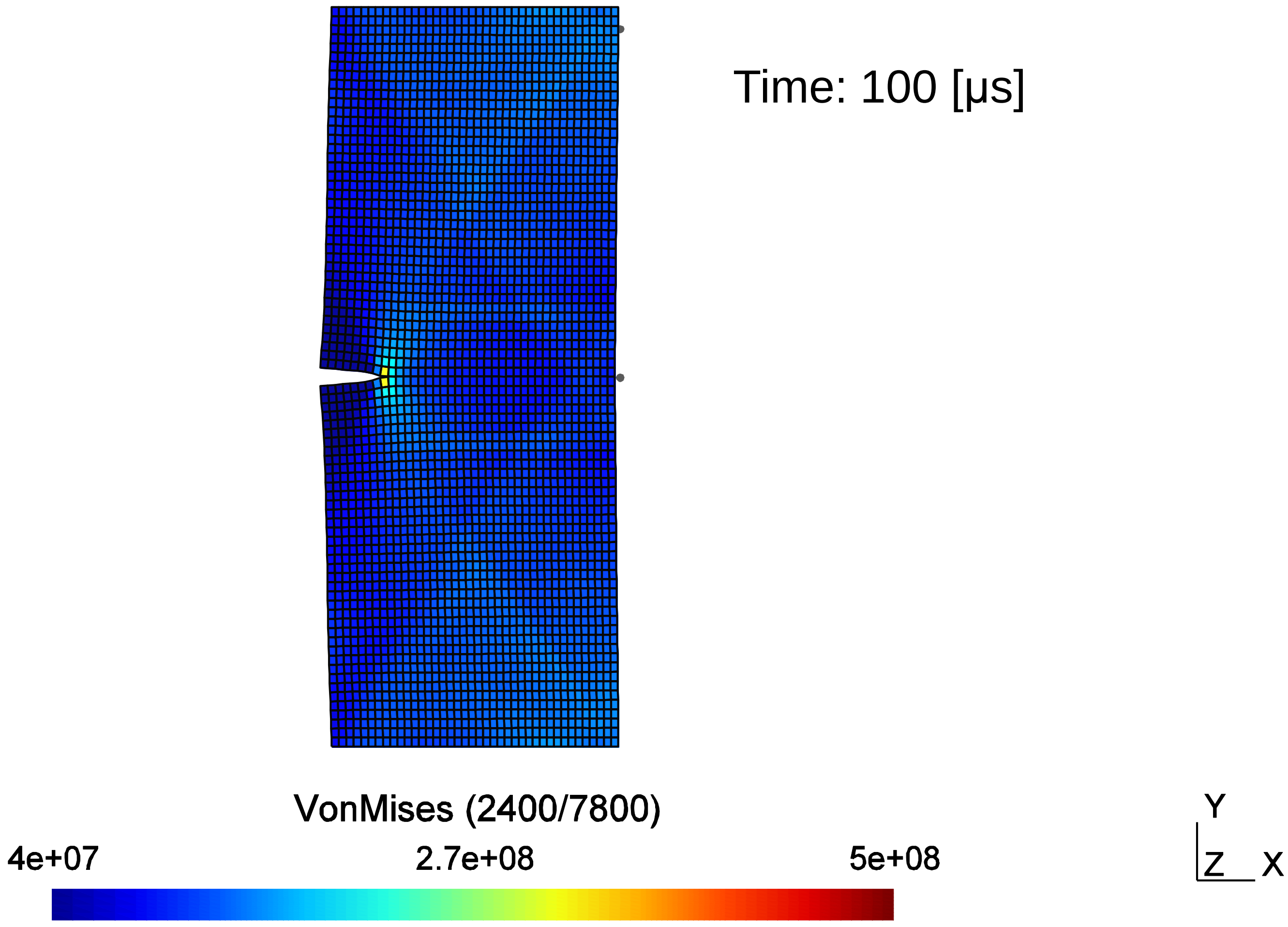

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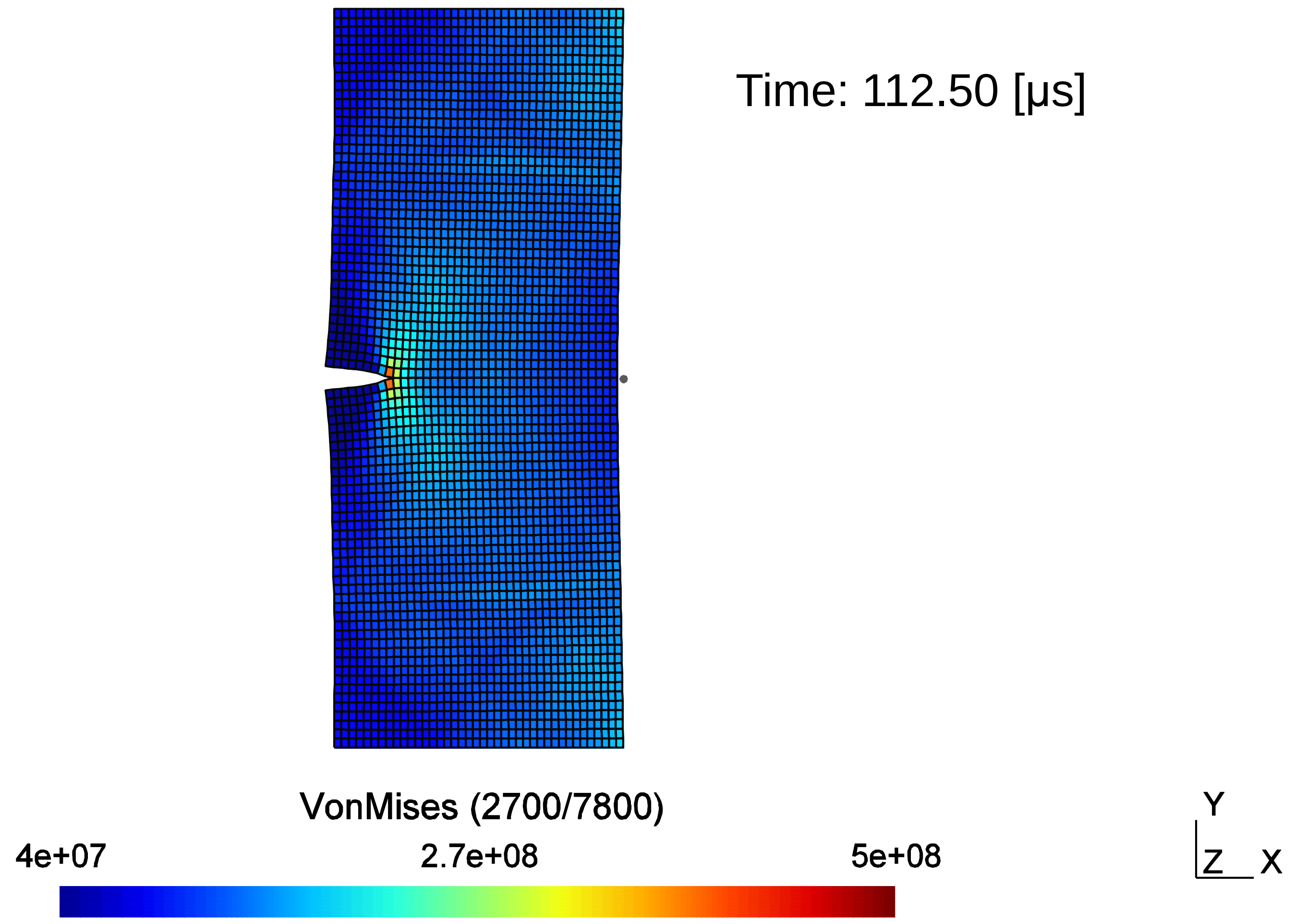

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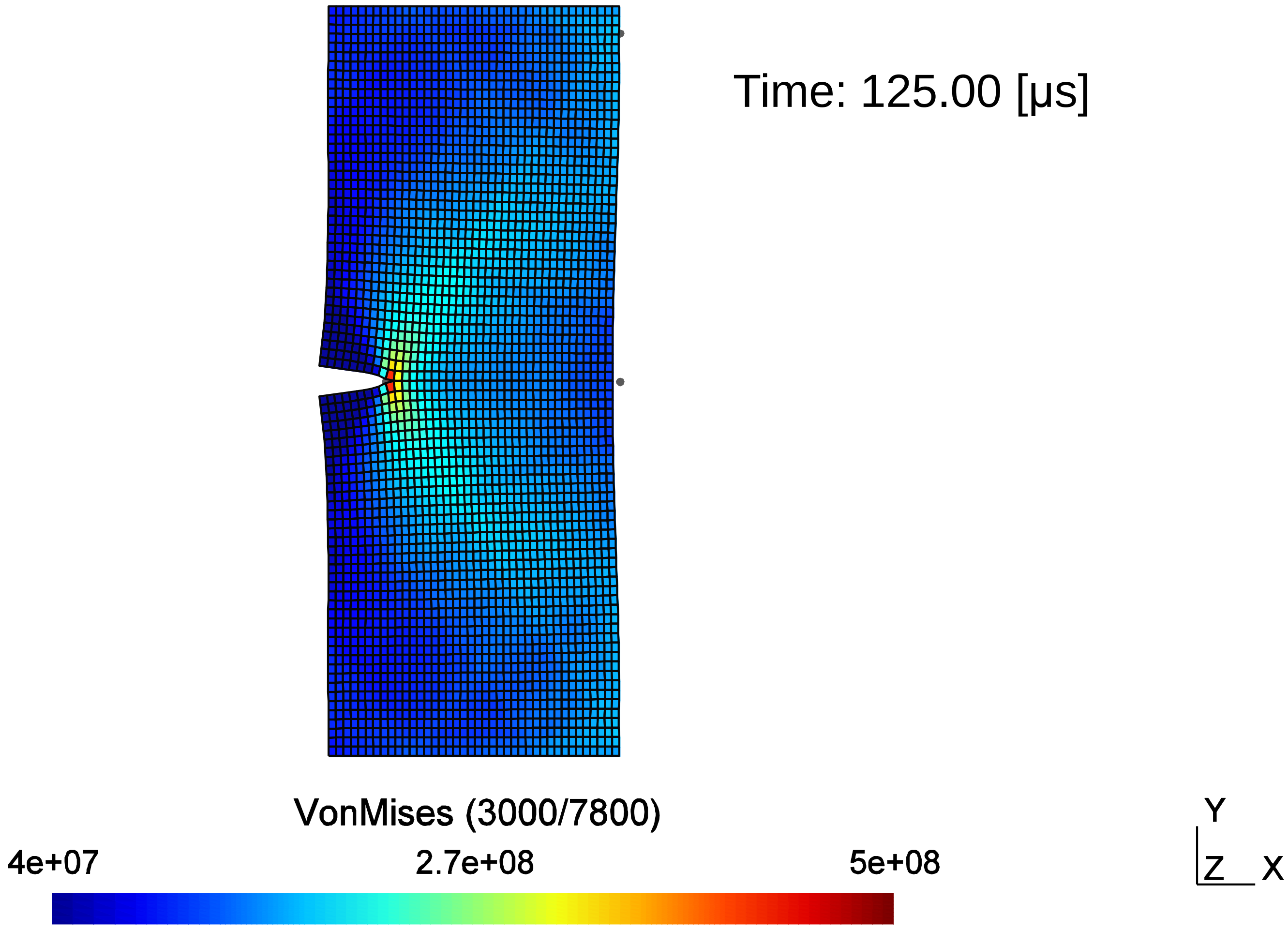

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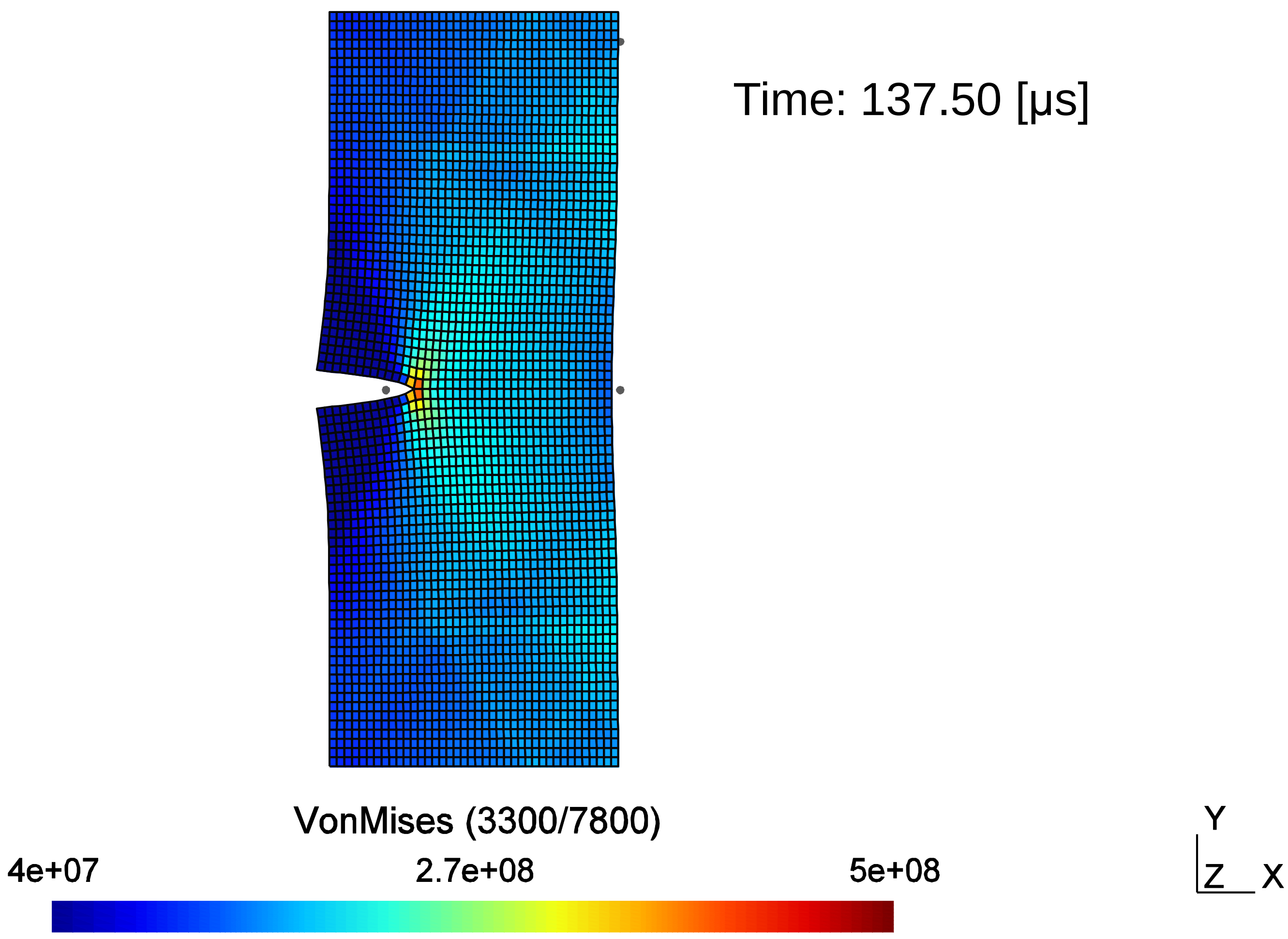

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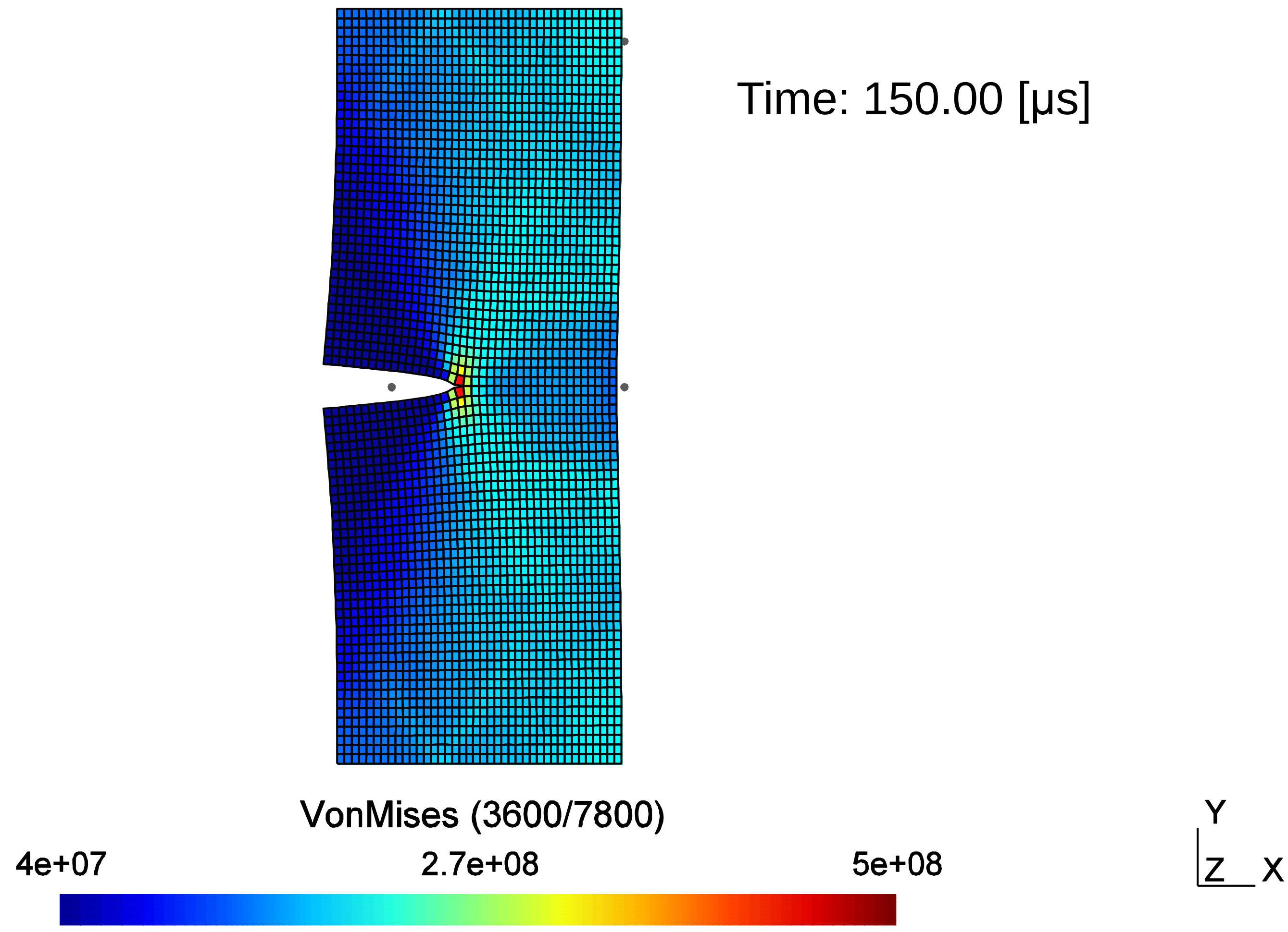

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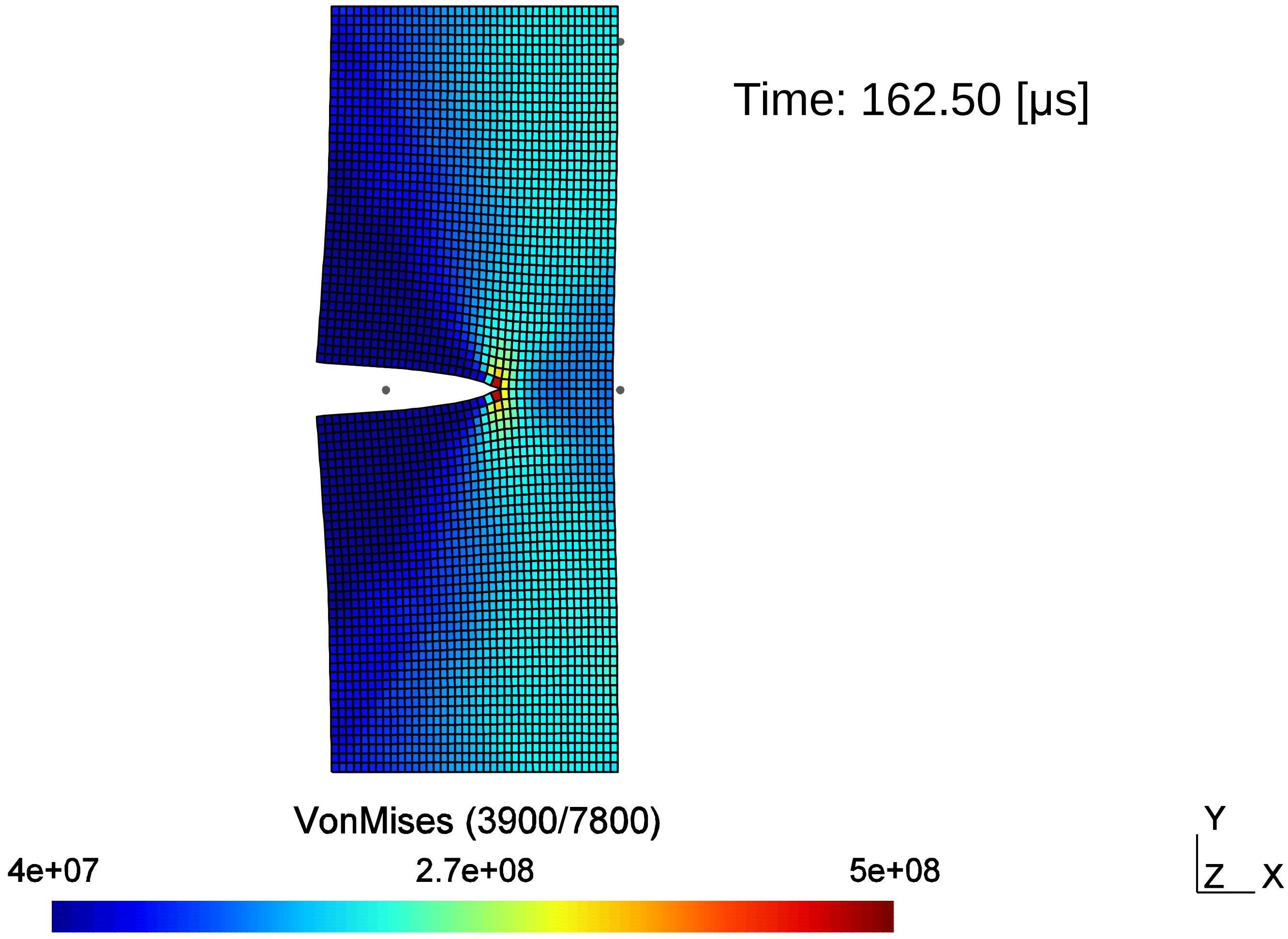

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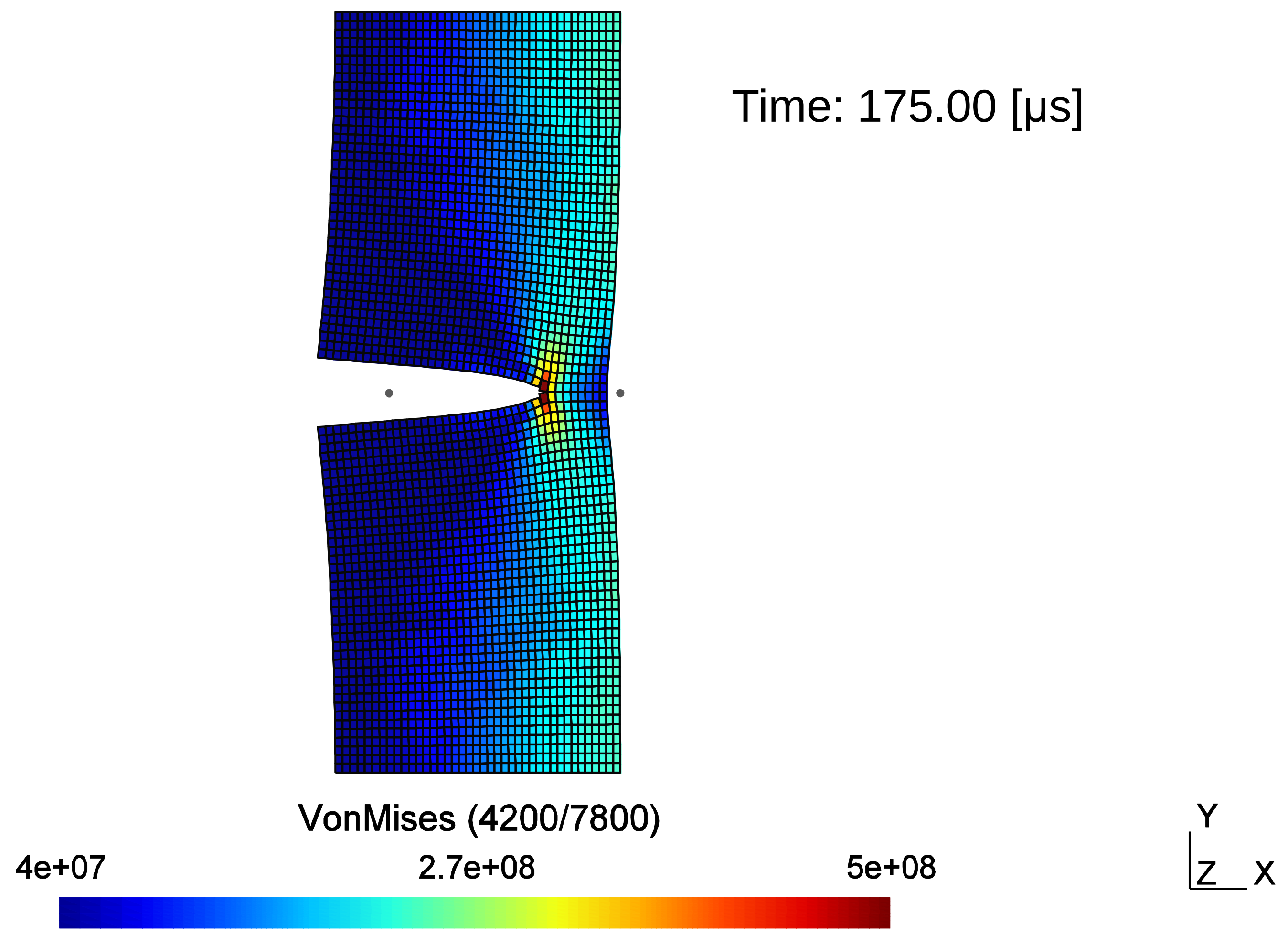

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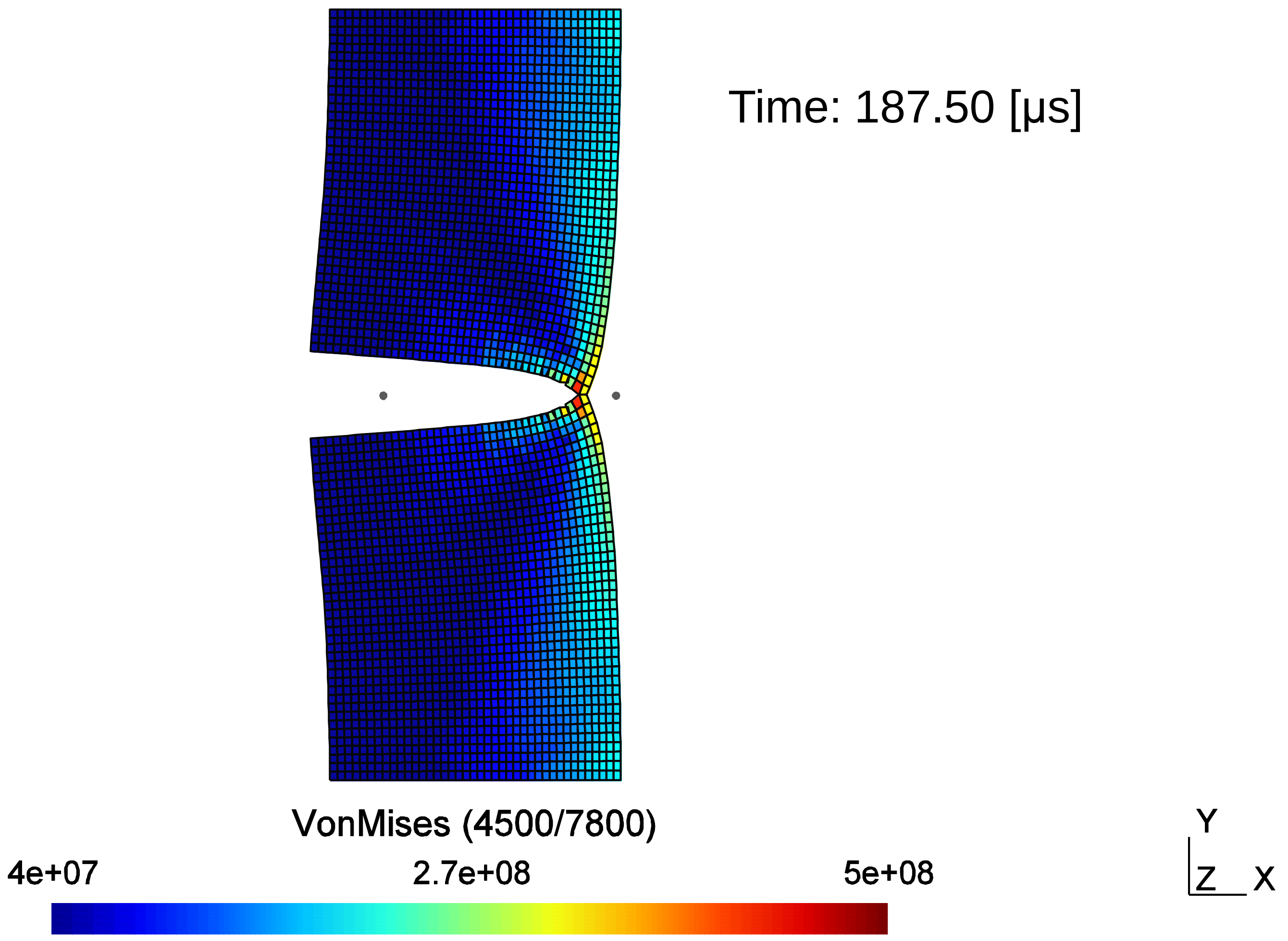

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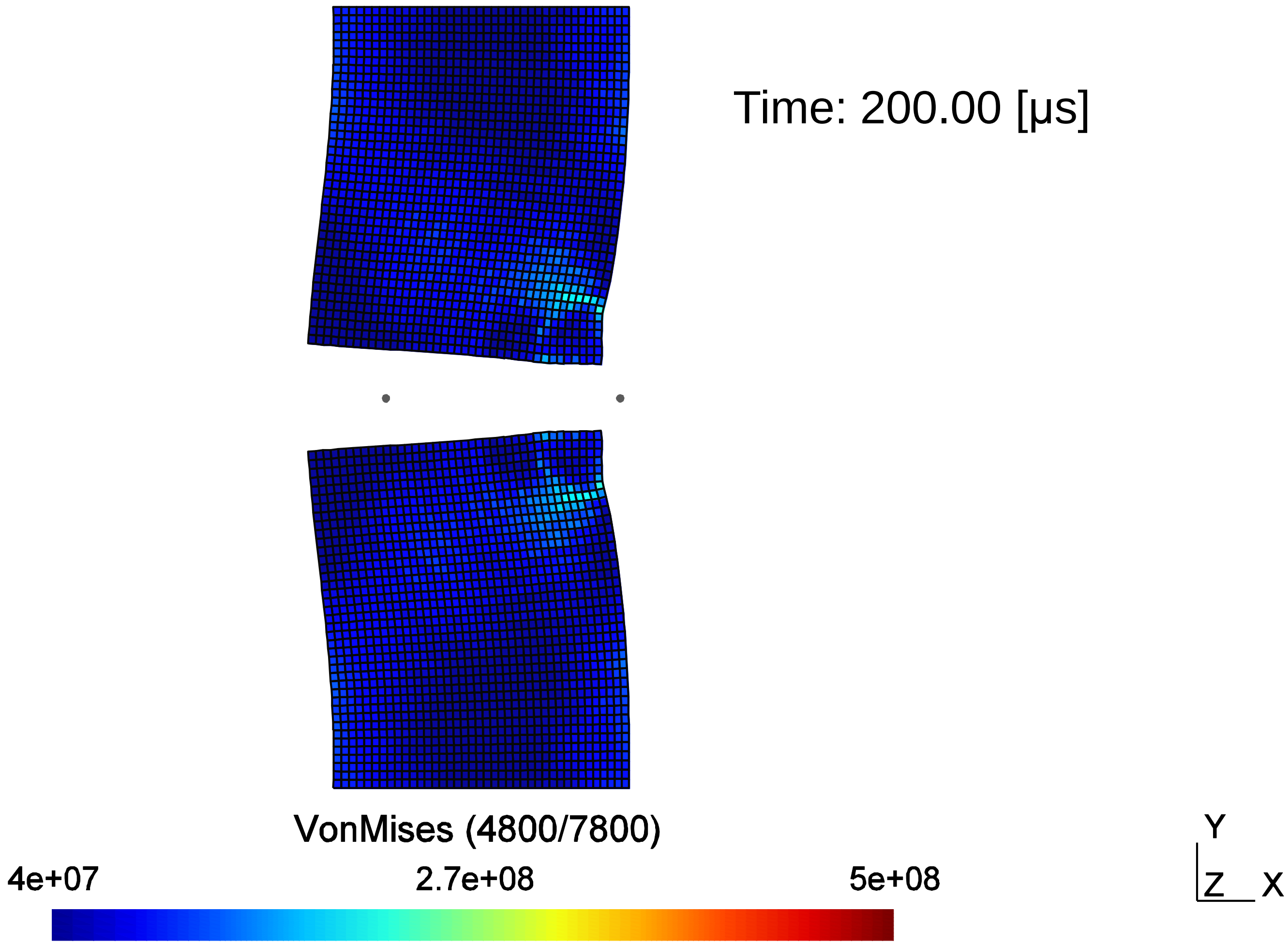
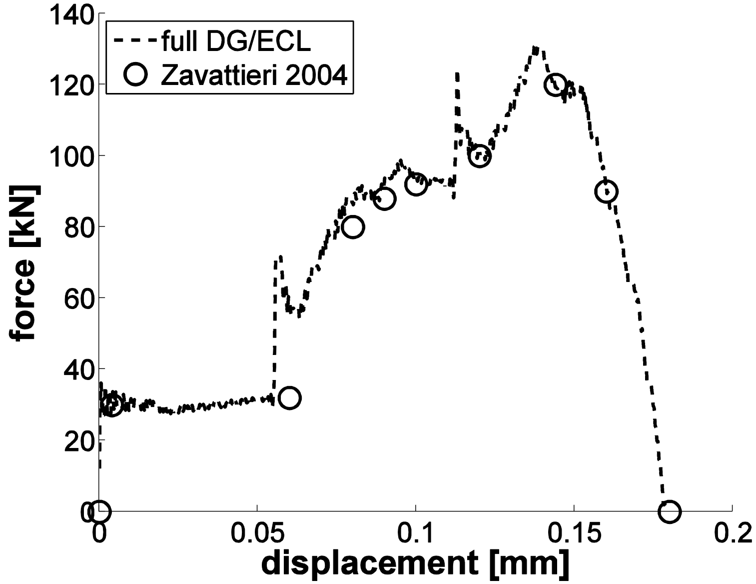
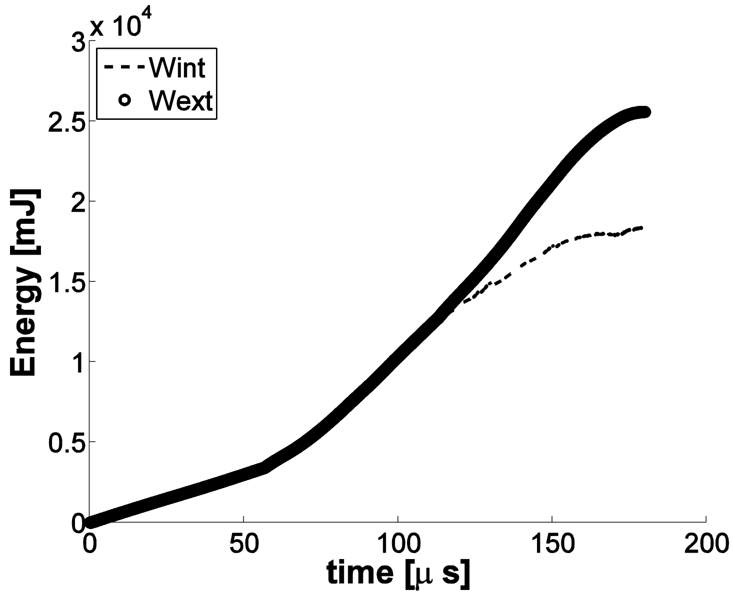
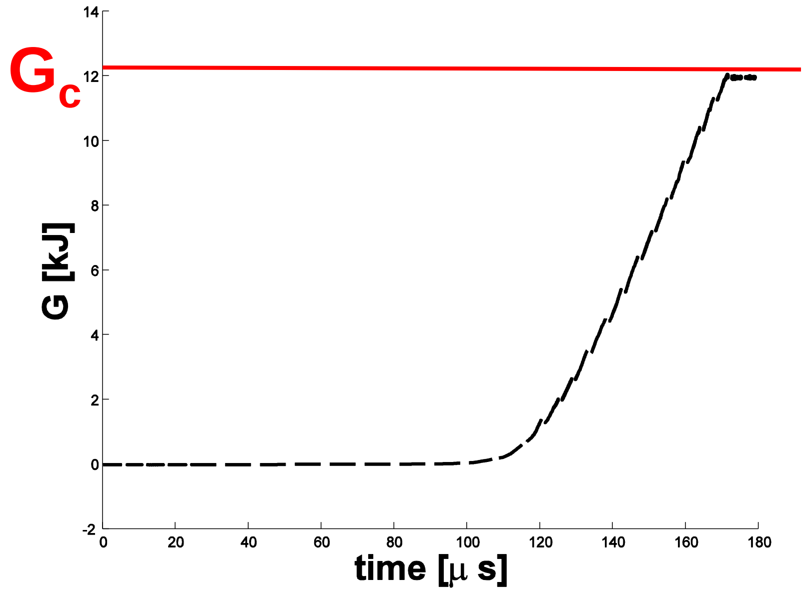


DG/ ECL Applications

- Mode I dynamic crack propagation
  - Results



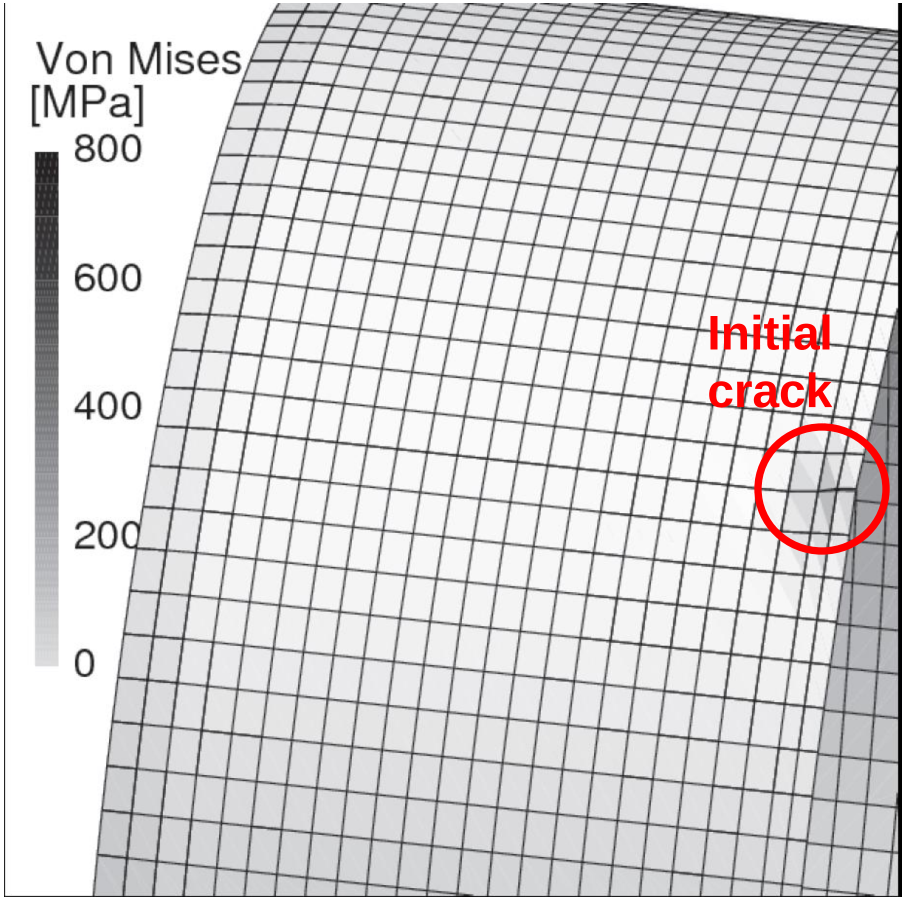


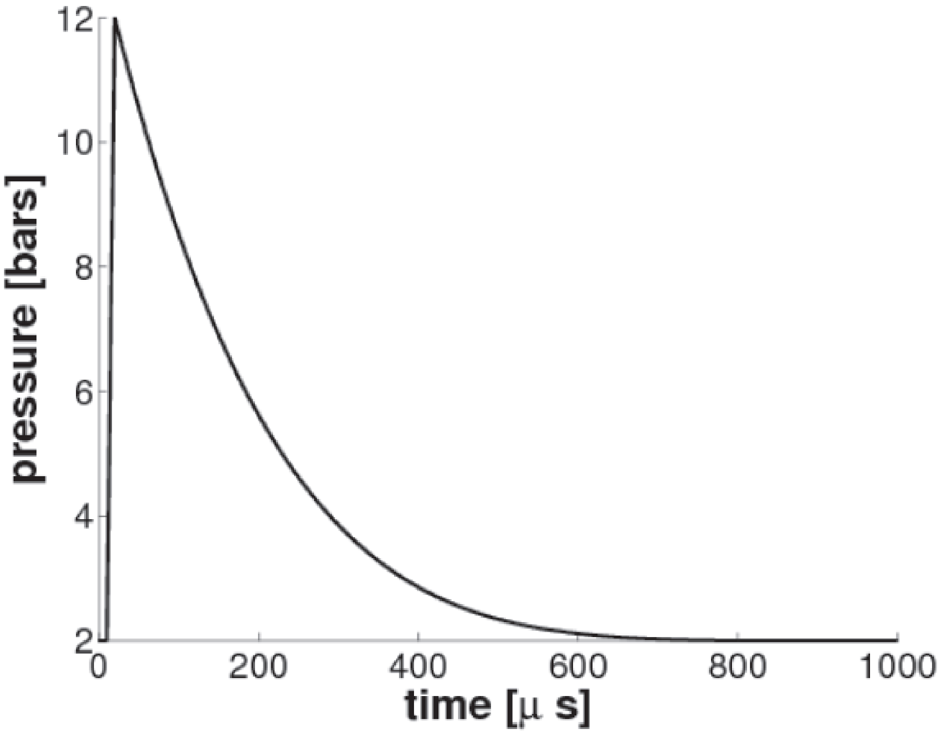


$$G = \frac{\Delta(W_{ext} - W_{int})}{bh\Delta a}$$

DG/ ECL Applications

- Blast of pressurized cylinder with an initial crack





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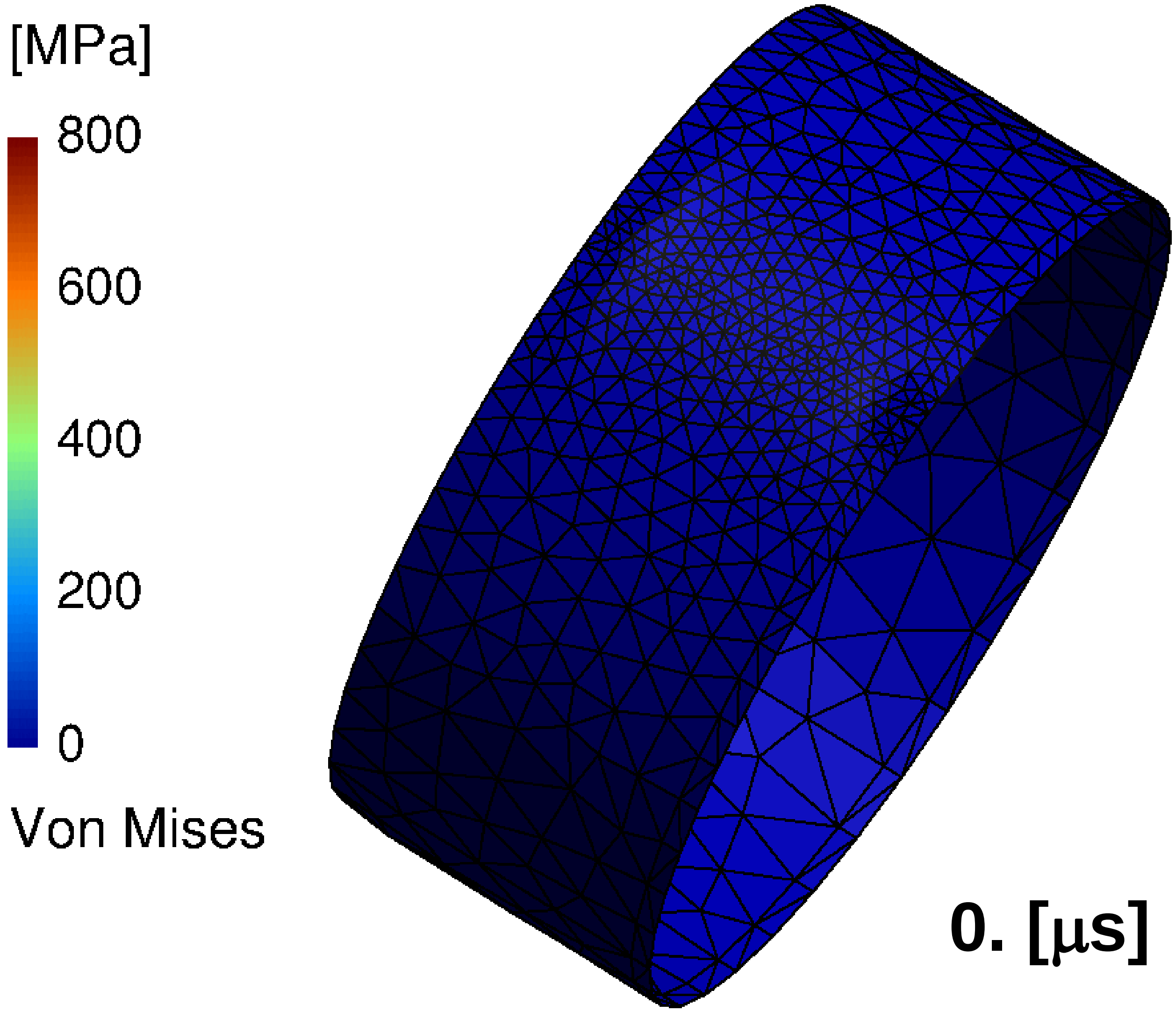

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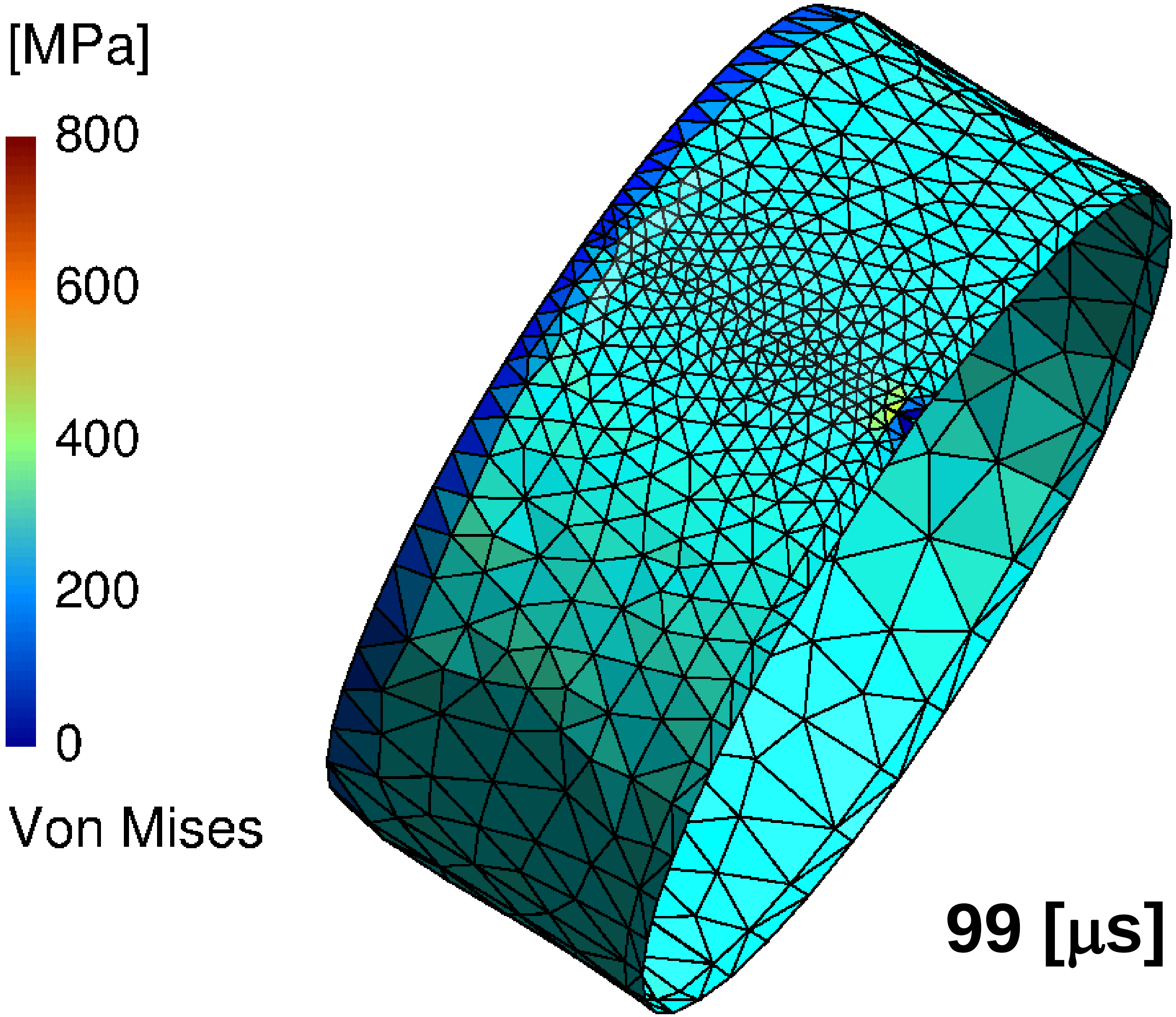
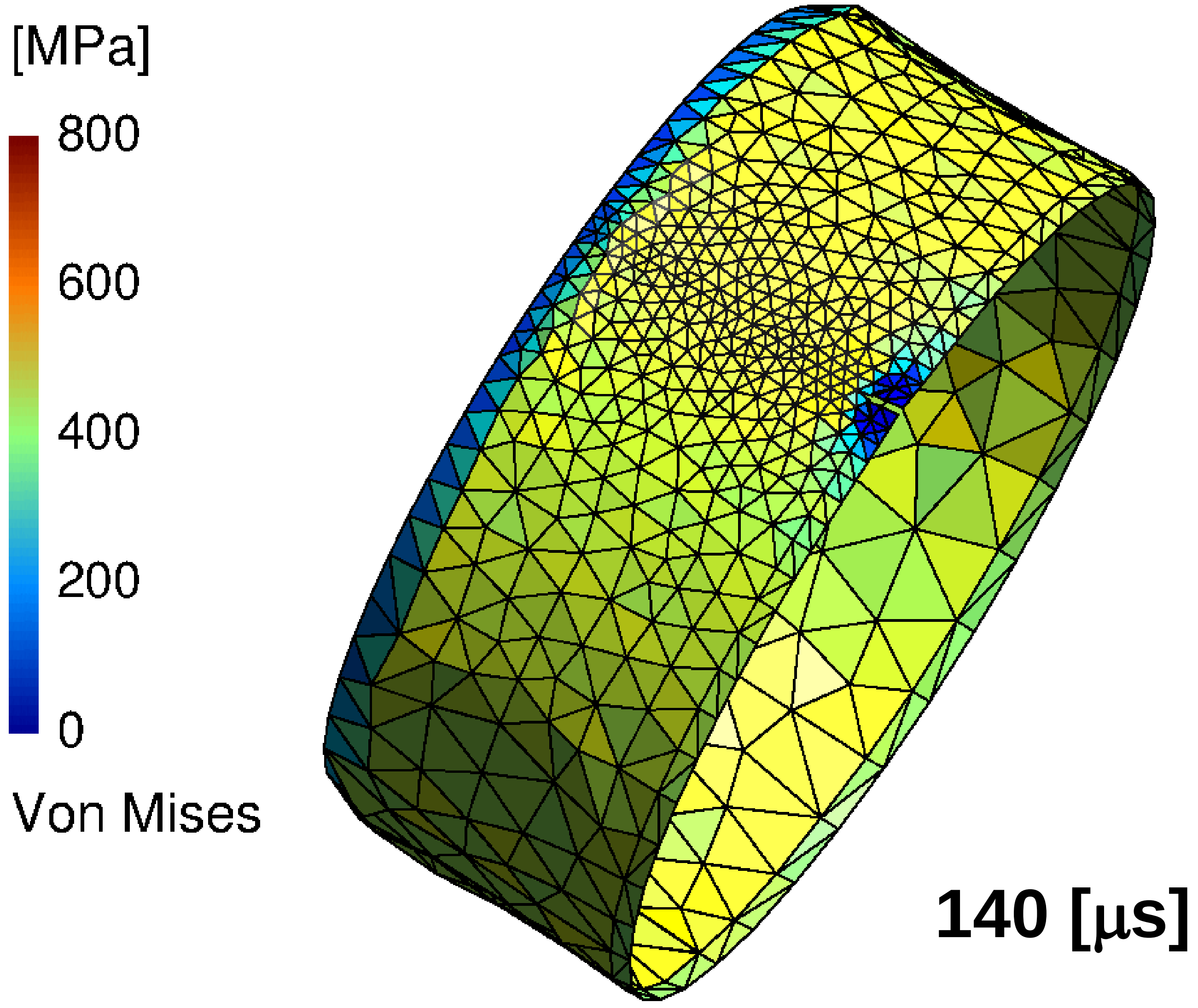


DG/ ECL Applications

- Blast of pressurized cylinder with an initial crack



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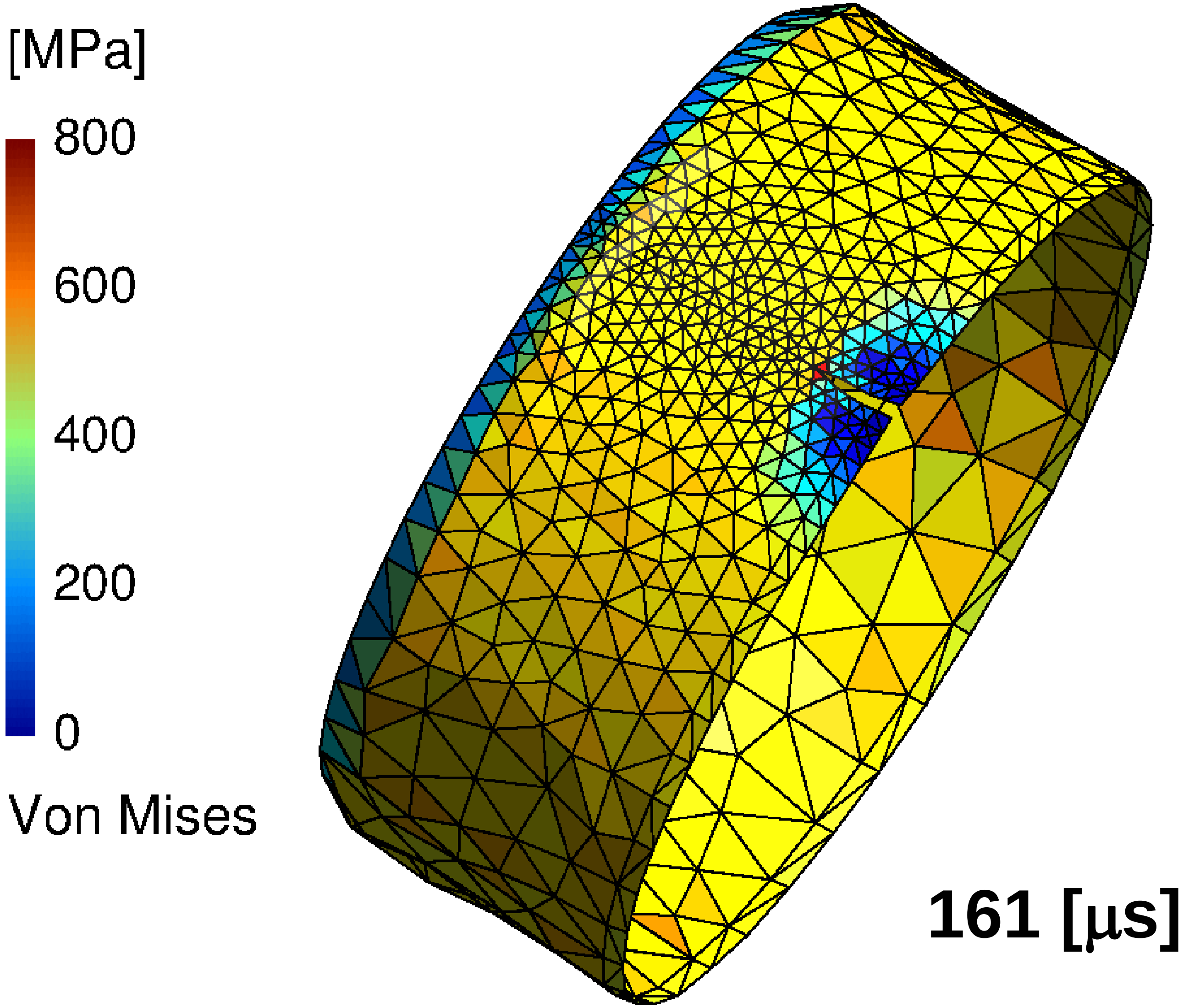

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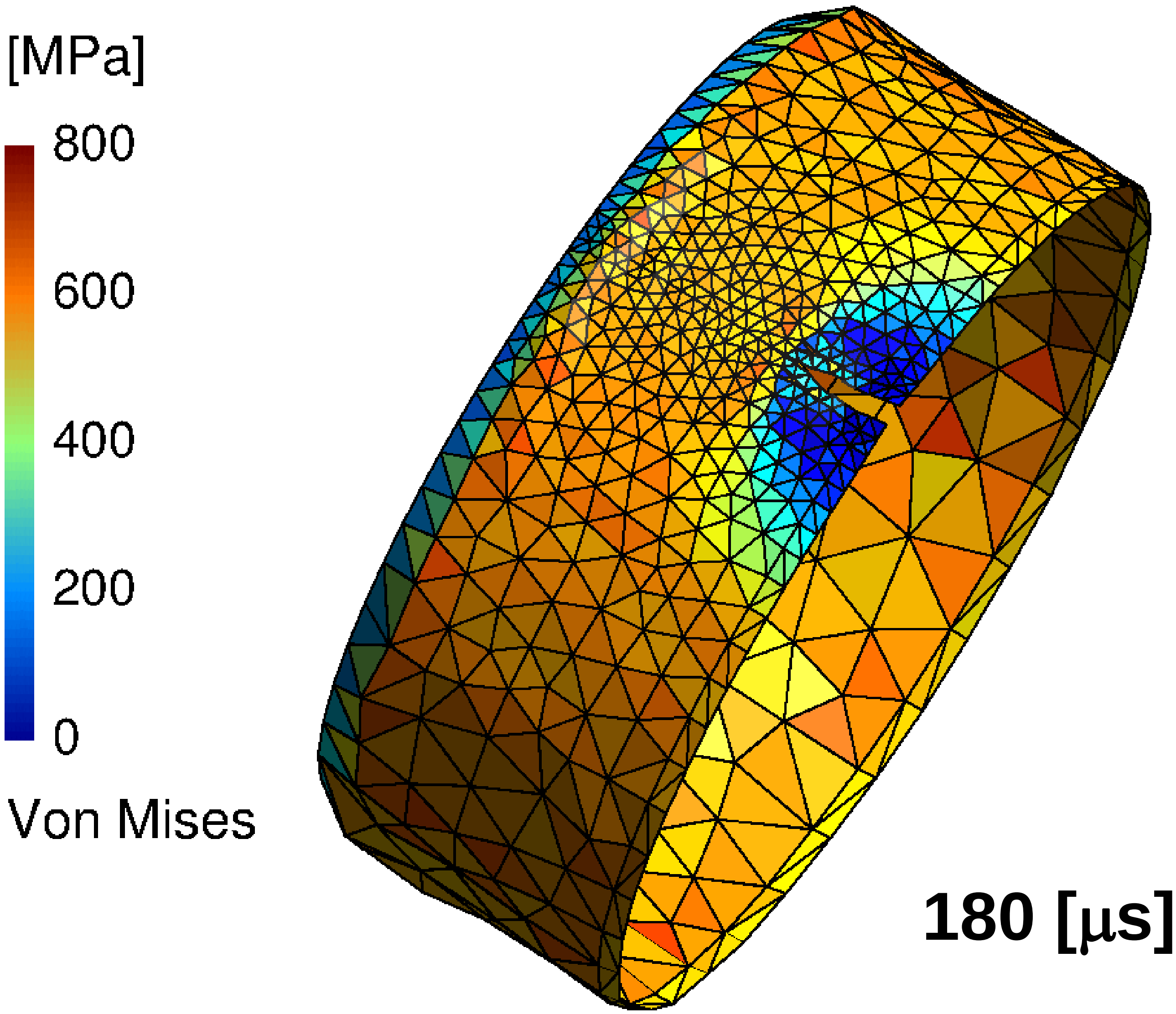

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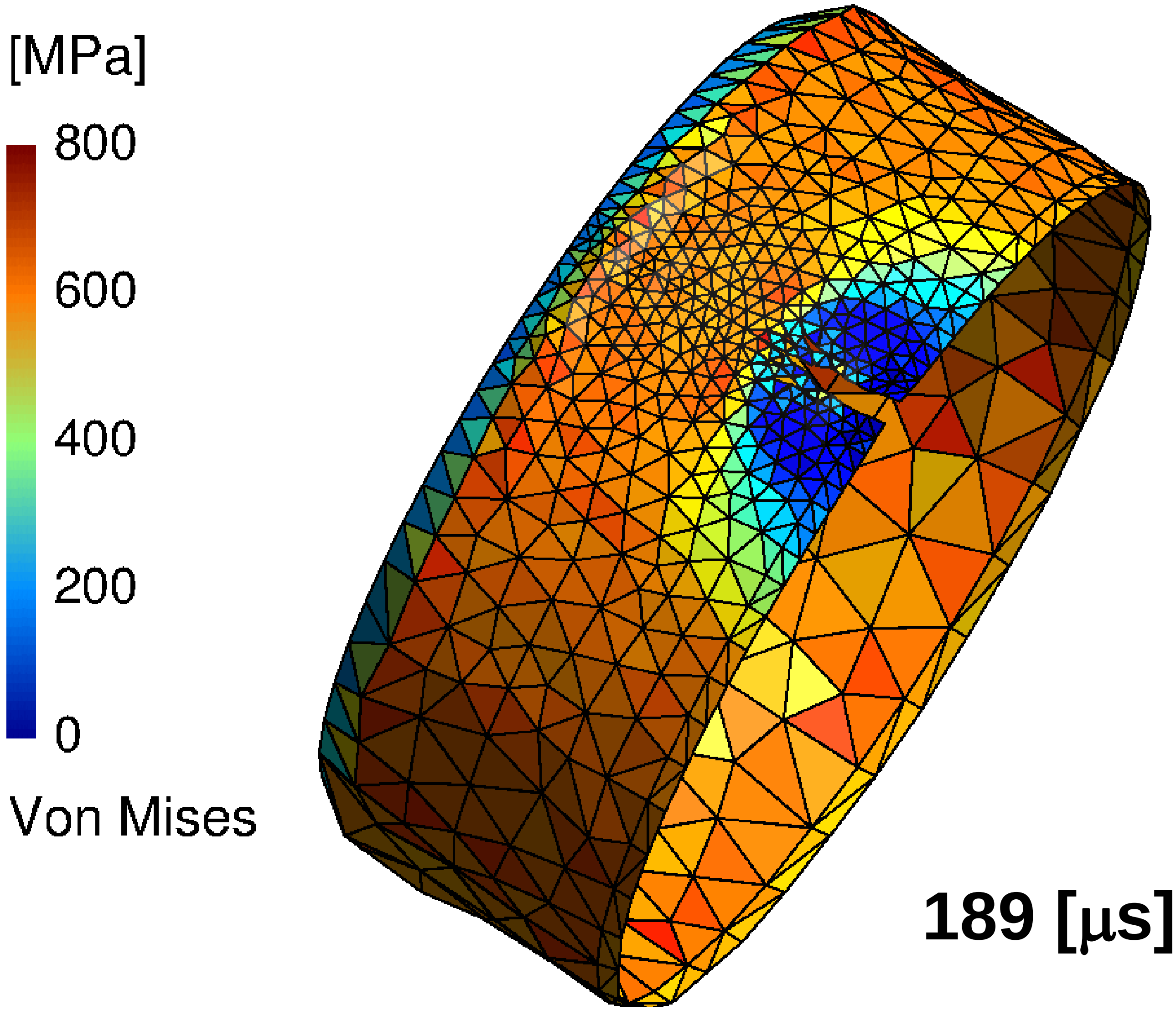

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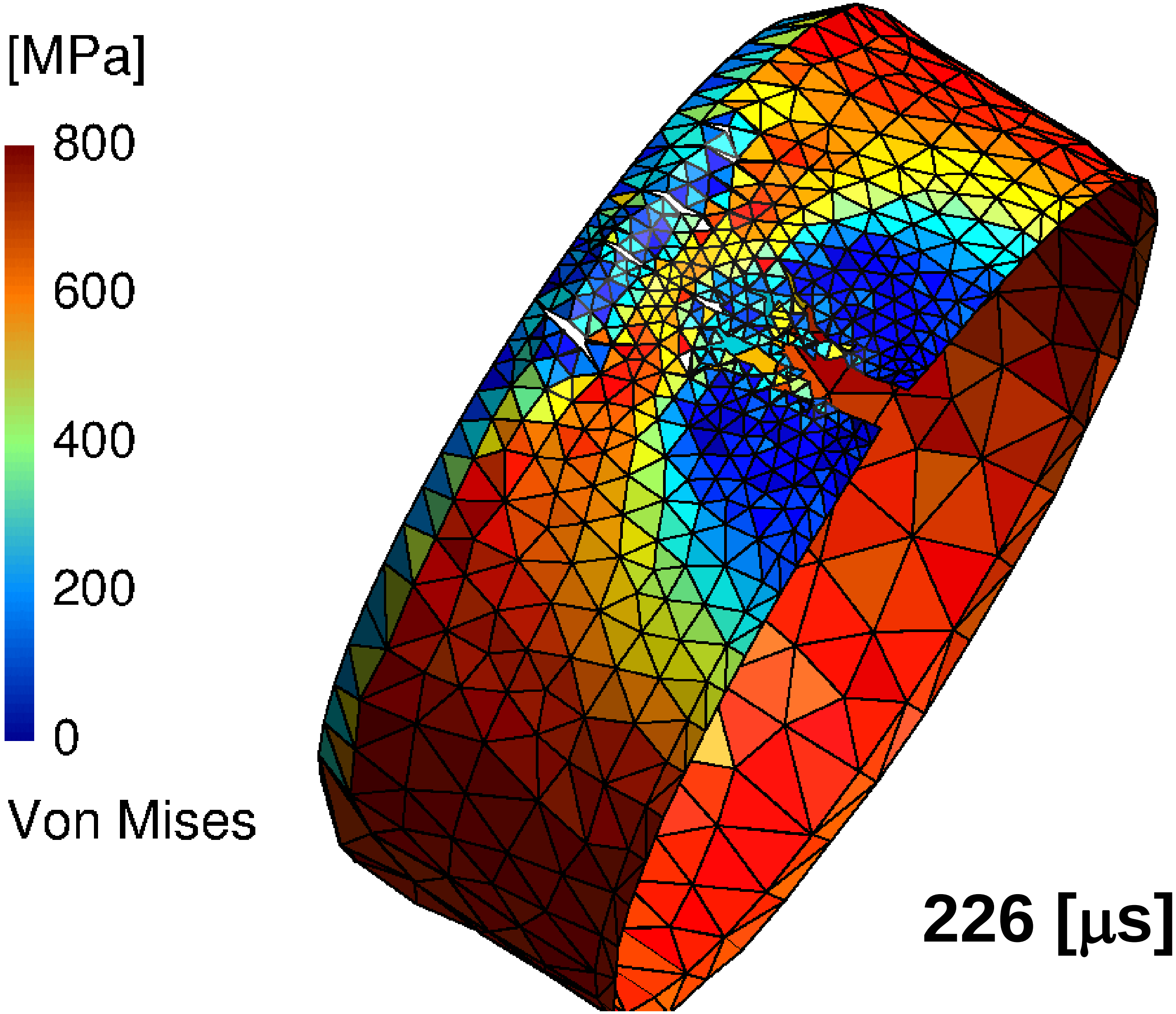

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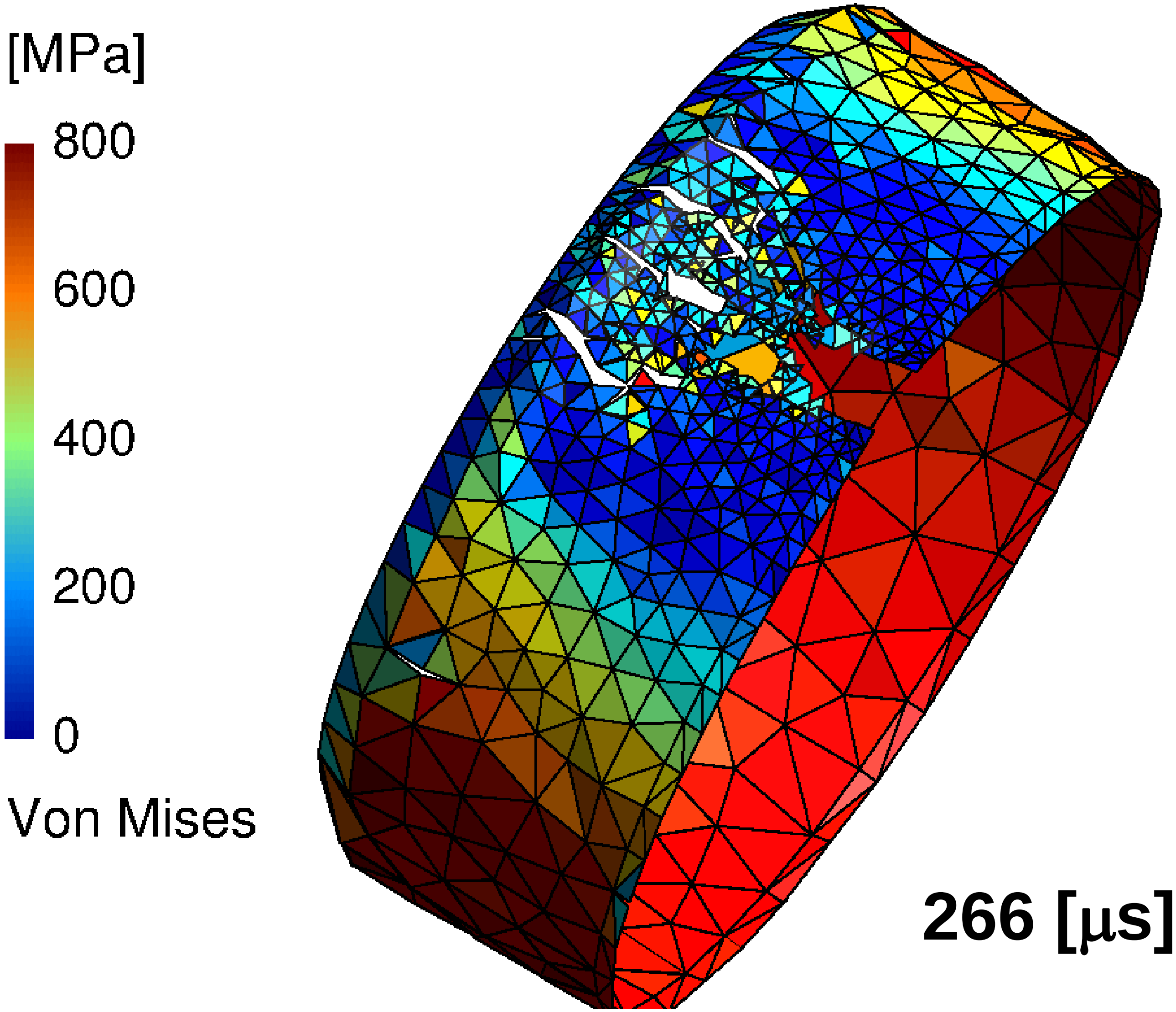
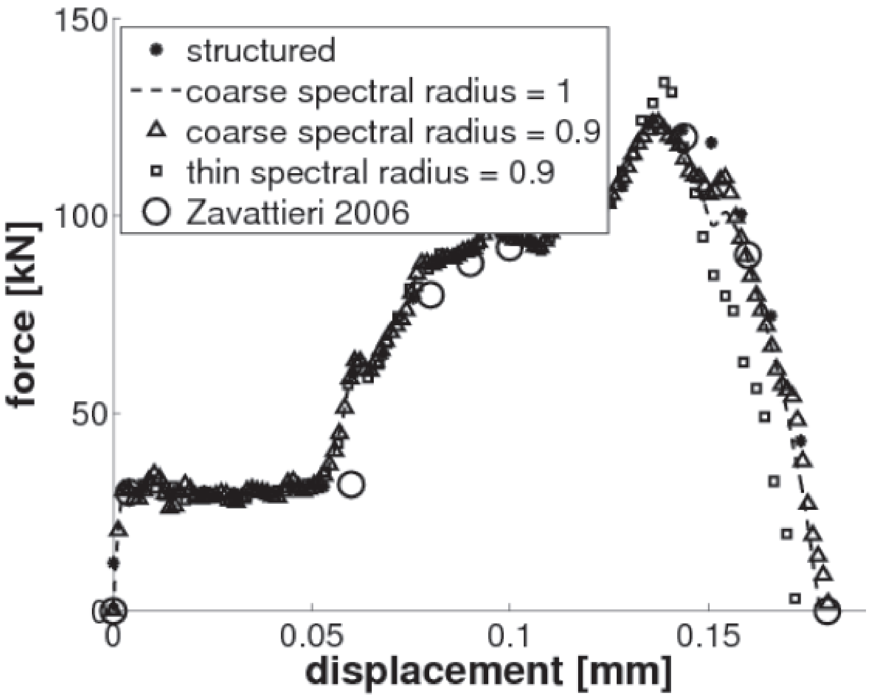


Conclusions

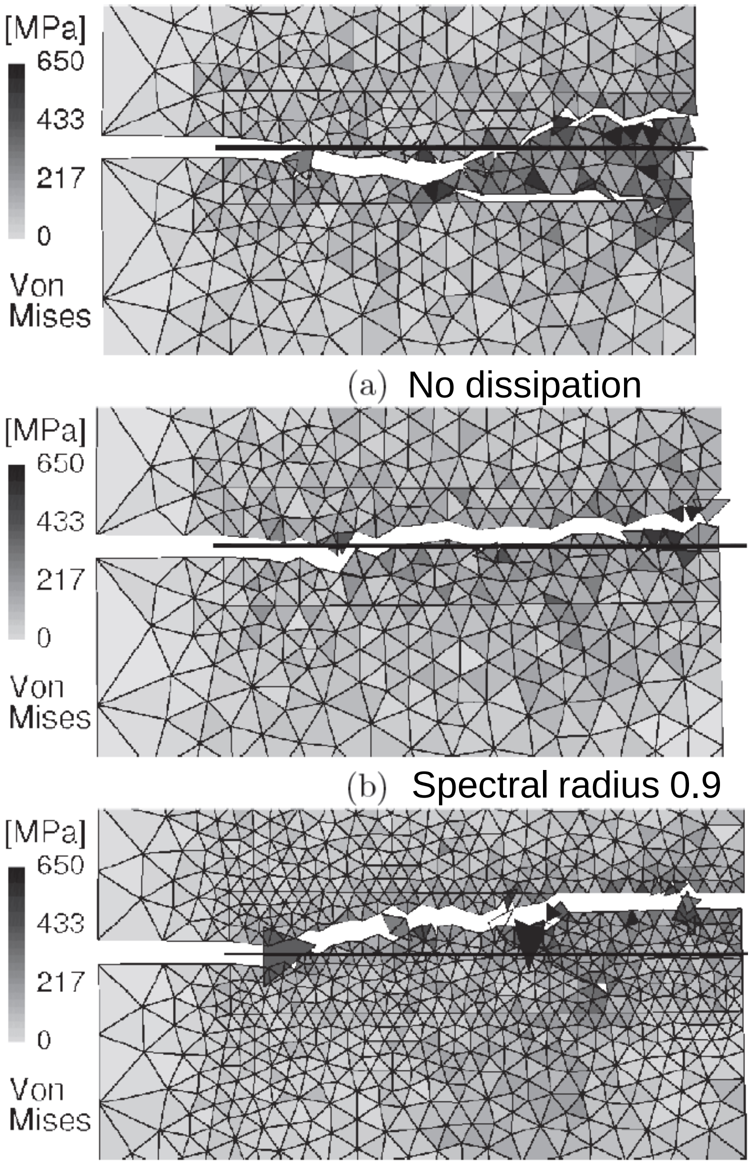
- DG methods
  - Easy combined with extrinsic cohesive method
  - Extended to shell
  - Appealing for parallelization (ongoing work)
- Cohesive with shell
  - Complex modelization of through the thickness crack
  - Application on resultant stresses
- Extension to large deformations
  - Ductile materials ?

DG/ ECL Applications

- Mode I dynamic crack propagation
  – Result unstructured meshes



  – Independent of mesh
    - Max relative error (thin mesh worst path)
      – 2.7% on maximal force
      – 4.3 % on time where F=0



(a) No dissipation

(b) Spectral radius 0.9

(c) Spectral radius 0.9