

CONDITIONAL HETEROSCEDASTICITY IN PACIFIC-BASIN STOCK MARKET RETURNS

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Abstract

This paper examines the statistical behavior of daily stock returns in several Pacific-Basin stock exchanges, which have grown tremendously in recent years. Empirical evidence reveals that return-generating models which empirically fit the data are processes with conditional heteroscedastic innovations. Particularly, the generalized autoregressive conditional heteroscedastic GARCH(1,1) process turns out to be the best in most cases.

Introduction

The purpose of this paper is to examine the empirical distribution of stock market returns in several Pacific-Basin stock exchanges and to find out which statistical model fits their distributions best. The distributional properties of stock returns have important implications for mean-variance portfolio theory, theoretical models of capital asset-pricing, and pricing of contingent claims. For instance, empirical tests of asset-pricing models and the efficient market hypothesis make statistical inferences based on distributional assumptions of stock returns, and estimation of the variance is essential in the Black-Scholes option-pricing formula.

The simplest and most practical assumption underlying the distribution of security returns is multivariate normality with stationary parameters over time. As the normal distribution is stable under addition, any portfolio of stocks will also be normally distributed. If it is also assumed that investors are risk averse, mean-variance portfolio theory can be derived. Moreover, the assumptions of normality and parameter stability are implied by most of the statistical methods usually employed in empirical financial studies. Tests of normality hypothesis were first reported by Osborne (1964). He investigated returns of stock indices over several time periods and concluded they approximately follow normal processes. Other studies, however, questioned the normality assumption and showed that distributions of stock returns have fatter tails and are more peaked than the normal distribution. Mandelbrot (1963) and Fama (1965) observed that the family of stable Paretian distributions whose members exhibit heavy tails, conforms better to the distribution of stock returns.

Since then, there have been many models of stock returns describing processes that could generate distributions having heavier tails than normal distributions. For instance, Paretz (1972), and Blattberg and Gonedes (1974) showed that the scaled-f distribution, which can be derived as a continuous variance mixture of normal distributions, better fits daily stock returns than infinite variance stable Paretian distributions. Models using different mixtures of normal to generate distributions that would account for the higher magnitude of kurtosis, are, among others, the Poisson mixtures of Press (1967) and the discrete mixtures of Kon (1984). Furthermore, Clark (1973), Epps and Epps (1976), Merton (1982), and Tauchen and Pitts (1983) put forward models where the distribution of variance is a function of the arrival of the information rate, the trading activity, and the trading volume. Such models are, however, too complex to be used in empirical applications.

There is as yet no agreement regarding the best stochastic return-generating model. The general conclusion, however, seems to be that stock returns are approximately uncorrelated, but not independent, and described by distributions with fatter tails. One of the most recently proposed class of return-generating process in the literature that can capture the temporal dependence of stock-return series is the class of autoregressive conditional heteroscedastic processes introduced by Engle (1982) and its generalized version by Bollerslev (1986). These processes allow for volatility clustering, that is, large changes are followed by large changes, and small by small, which has long been recognized as an important feature of stock-returns behavior. Empirical studies showed indeed that such processes are successful in modeling various time series (see, for example, French, Schwert, and Stambaugh 1987; Baillie and Bollerslev 1989; Hsieh 1989; Baillie and DeGennaro 1990).

In this paper, we show whether such models can adequately describe the stock-price behavior of Pacific-B asin capital markets. To that end, we have selected seven countries: Australia, Hong Kong, Japan, Korea, New Zealand, Singapore, and Thailand. The study of stock-price behavior in these

markets is interesting for it can provide further evidence in favor of or against the use of this type of model to describe stock-price behavior. The structure of the paper is as follows. The second section presents the data and the third section examines the statistical properties of the return series. In the fourth section, we apply an autoregressive model to the return series. The fifth section concerns the presentation and estimation of autoregressive conditional heteroscedastic processes. Conclusions follow in the last section.

The data

For this study, we selected the indices of seven stock markets in the Pacific-Basin rim. The daily market indices were collected from DATASTREAM for the period January 1, 1980 to September 30, 1990, and they are the All Ordinary Shares (Australia), the Hang Seng Index (Hong Kong), the Nikkei Down Jones (Japan), the Barclays Industrial (New Zealand), the Straits Times (Singapore), the Composite (South Korea), and the Bangkok Book Club (Thailand). The daily returns of these market indices are continuously compounded, they are calculated as the difference in logarithm of the index value for two consecutive days, $R_t = \log(P_t) - \log(P_{t-1})$.

Statistical analysis

This section contains a detailed analysis of the distributional and time-series properties of the stock-market-indices returns in the sample. A range of descriptive statistics are presented in Table 1, as well as the maximized likelihood function value when a normal distribution is imposed on data. The results confirm the well-known fact that daily stock returns are not normally distributed, but are leptokurtic and skewed, whatever the country concerned. All distributions are negatively highly skewed, indicating that they are nonsymmetric, and they all exhibit a high level of kurtosis, meaning they are more peaked and have fatter tails than normal distributions. In order to test the hypothesis whether returns are strict white noise, that is, random walk, the Box-Pierce test statistics up to lag 25 are calculated and presented in the table.

Table 1. *Sample Statistics on Daily>Returns Series*

	Australia	Hong Kong	Japan	Korea	New Zealand	Singapore	Thailand
<i>Returns statistics</i>							
Sample size	2685	2645	2607	2615	2631	2665	2573
Mean ($\times 10^3$)	.3758	.4335	.4454	.6173	.5338	.3473	.5642
$T(\text{mean} = 0)$	1.6734	1.0946	<u>2.2741</u>	<u>2.4763</u>	<u>2.4860</u>	1.3366	<u>2.4784</u>
Variance ($\times 10^3$)	.1354	.4148	.1000	.1625	.1213	.1799	.1333
Skewness	<u>-5.9885</u>	<u>-4.0017</u>	<u>-1.6467</u>	<u>-.9357</u>	<u>-1.9180</u>	<u>-3.1190</u>	<u>-1.2128</u>
Kurtosis	<u>144.0611</u>	<u>72.1474</u>	<u>33.9016</u>	<u>16.9507</u>	<u>27.7900</u>	<u>47.9978</u>	<u>17.2857</u>
Log-Likelihood	10615.4	8977.1	10702.4	10100.8	10547.2	10158.1	10193.1
First order autocorrelation	<u>.1207</u>	.0235	.0604	<u>.0744</u>	<u>.1595</u>	<u>.1600</u>	<u>.2438</u>
Q(25)	<u>189.12</u>	<u>57.13</u>	<u>91.94</u>	<u>48.31</u>	<u>248.78</u>	<u>135.88</u>	<u>296.81</u>
Q*(25)	36.58	23.85	29.89	31.86	<u>55.54</u>	26.05	41.18

Note: The tests statistically significant at the 5% level are underlined Numbers in parentheses are heteroscedasticity-consistent standard errors.

This is a joint test that the first k autocorrelation coefficients are zero. Under the null hypothesis, that the sample autocorrelations are not asymptotically correlated, the Box-Pierce statistic, $Q = n \sum_{i=1}^k \hat{\rho}_i^2$

has chi-square distribution with k degrees of freedom, where $\hat{\rho}(i)$ is the i -th autocorrelation. The values of Q are all significant at the 5% level for all countries. This implies that the null hypothesis of strict white noise is rejected, reflecting a rather long range of dependency in the returns series.

However, it can be questioned whether this test accounts for the full probability distribution of the

returns series since heteroscedasticity can lead to the underestimation of the standard error, $1/n$, of each sample, and therefore to the overestimation of the t - and chi-square statistics. Diebold (1987) provides a heteroscedasticity-consistent estimate of the standard error for the i -th sample autocorrelation coefficient:

$$S(i) = \sqrt{\frac{1}{n} \left(1 + \frac{\gamma R^2(i)}{\sigma^4}\right)} \quad (1)$$

where $R^2(i)$ is the i -th sample autocovariance of the square data and σ is the sample standard deviation of the data. These adjusted standard errors are presented in Table 1 under their respective autocorrelation coefficients. It can be seen that only four autocorrelation coefficients are statistically different from zero, whereas they are all significant, except that of Hong Kong, when $1/n$ is used as standard error.

Using the adjusted standard errors, Diebold proposed an adjusted Box-Pierce statistic:

$$Q^* = \sum_{i=1}^k (\rho(i) / (S(i)^2)) \quad (2)$$

which is asymptotically chi-square distributed with k degrees of freedom, under the null hypothesis of no serial correlation in the data. The values of Q^* which are presented in Table 1 are much lower than the nonadjusted ones. They are significant at the 5% level for New Zealand only. So, even after adjusting for heteroscedasticity, there remains some significant long-term serial correlation in the series of returns for this country. A comparison between the values of Q and Q^* suggests that the rejection of serial independence using Q , based on the standard testing procedure, is due to the presence of heteroscedasticity in the returns series. However, the fact that the first lag autocorrelation is significant for six countries implies the rejection of white noise, that is, uncorrected process. Therefore, we have to eliminate the serial correlation in the return series before searching for appropriate models that could account for heteroscedasticity in the returns. One way to do this is to apply Autoregressive Moving Average (ARMA) models.

AUTOREGRESSIVE MODELS

The class of univariate ARMA models might adequately represent the behavior of the stock returns. In our case, an $AR(1)$ model appears to fit returns series best.

$$R_t = \phi_0 + \phi_1 R_{t-1} + \varepsilon_t \quad (3)$$

The estimates of the above regression model for each country are presented in Table 2. In order to observe whether the residuals ε_t obtained from Equation (3) are uncorrected, we applied the same tests for normality and serial correlation as for the returns series. As before, the values of the autocorrelation coefficients and their respective standard errors adjusted for heteroscedasticity are presented in Table 2. The first order autocorrelation coefficient for all countries is not significantly different from zero, based on both adjusted and nonadjusted standard errors. The estimate of ϕ_1 is statistically significant, except for Hong Kong. The Dickey-Fuller test for unit roots indicates that ϕ_1 is significantly less than one. The series of returns appear to follow a stationary random walk. As far as the assumption of normality of the residuals is concerned, it can be rejected. The residual series appear to be leptokurtic and skewed. The comparison between values of Q and Q^* is interesting in that while the former indicates significant serial correlation up to lag 25, the latter always rejects the presence of serial correlation. This shows that after controlling for heteroscedasticity, the $AR(1)$ transformation of the returns provides an uncorrelated series.

Table 2. *The Autoregressive Model*

	Australia	Hong Kong	Japan	Korea	New Zealand	Singapore	Thailand
ϕ_0	.0003	.0004	.0004	.0006	.0004	.0003	.0004
$t(\phi_0)$	1.4593	1.0581	<u>2.1339</u>	<u>2.6604</u>	<u>2.1058</u>	1.1355	1.9529
ϕ_1	.1207	.0235	.0603	.0745	.1595	.1600	.2438
$t(\phi_1)$	<u>6.2999</u>	1.2127	<u>3.1887</u>	<u>3.9613</u>	<u>8.2838</u>	<u>8.3646</u>	<u>12.7488</u>
Fuller's test	2360.8	2582.6	2449.4	2420.6	2211.4	2238.5	1945.7

Residuals statistics

Mean ($\times 10^3$)	.0000	.0000	.0000	.0000	.0000	.0000	.0000
t(mean = 0)	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Variance ($\times 10^3$)	.1335	.4147	.0997	.1500	.1182	.1754	.1254
Skewness	<u>-5.7520</u>	<u>-.39343</u>	<u>-1.4590</u>	<u>.0687</u>	<u>-1.4676</u>	<u>-2.3729</u>	<u>-.7438</u>
Kurtosis	<u>140.3135</u>	<u>69.6704</u>	<u>34.1775</u>	<u>4.8406</u>	<u>25.9706</u>	<u>44.7506</u>	<u>16.6906</u>
First order autocorrelation	.0045	.0004	.0055	.0186	-.0015	-.0107	-.0119
	(.0621)	(.0599)	(.0717)	(.0316)	(.0628)	(.1061)	(.0602)
Q(25)	<u>136.35</u>	<u>54.98</u>	<u>83.19</u>	<u>40.13</u>	<u>126.73</u>	<u>40.96</u>	<u>85.75</u>
Q*(25)	27.24	23.31	28.97	30.41	30.74	17.20	18.48

Note: Statistical tests significant at the 5% level are underlined. Numbers in parentheses are heteroscedasticity-consistent standard errors.

Thus, it can be concluded that the residuals series is uncorrelated, but still exhibit heteroscedasticity.

Conditional heteroscedastic models

Various models have been developed to account for the presence of heteroscedasticity in stock-returns and market models (see Morgan 1976; Giaccoto and Ali 1982). While these models concentrate on unconditional heteroscedasticity, we use Engle's Autoregressive Conditional Heteroscedastic (ARCH) model, which focuses on conditional volatility movements. It is interesting to note that, according to Diebold, Im, and Lee (1988), the presence of ARCH effect appears to be generally independent of unconditional heteroscedasticity. Excess kurtosis observed in both returns and residuals series can be related to conditional heteroscedasticity, that is, its presence can be due to a time-varying pattern of the volatility. ARCH models and its extensions have been successfully applied, for instance, in foreign exchange markets by Baillie and Bollerslev (1989) and Hsieh (1989), and in stock markets by Akgiray (1989) and Baillie and De Gennaro (1990). The ARCH process imposes an autoregressive structure on the conditional variance which permits volatility shocks to persist over time. In this process, the conditional error distribution is normal, with a conditional variance that is a linear function of past squared innovations. The model, denoted by ARCH(p), is the following:

$$\varepsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 \quad (4)$$

with $p > 0$; $\alpha_i > 0$, $i = 0, \dots, p$, and where Ω_t is the information set of all information through time t , and the ε_t are obtained from a linear regression model. An important extension of the ARCH model is the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process of Bollerslev (1986), denoted by GARCH(p,q). In this model, the linear function of the conditional variance includes lagged conditional variances as well. Equation (4) in the case of a GARCH model becomes:

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j} \quad (5)$$

where also $q > 0$ and $\beta_j > 0$, $j=1, \dots, q$.

Before estimating (G)ARCH models, it is useful to test for the presence of ARCH properties on the returns series. This is the object of the next subsection.

Testing for ARCH Presence

In an ARCH process, the variance of a time series depends on past squared residuals of the process. Therefore, the appropriateness of an ARCH model can be tested by means of a LM test, that is, by

regressing the squared residuals against a constant and lagged squared residuals (Engle 1982).

$$\epsilon_t^2 = \gamma_0 + \sum_{i=1}^p \beta_i \epsilon_{t-i}^2 \quad (6)$$

Under the null hypothesis of no ARCH process, the coefficient of determination R^2 can be used to obtain the test statistic TR^2 which is distributed as a chi-square with i degrees of freedom. This LM test has been applied to our series up to lag 5 for all the seven returns series. The values we obtained for the TR^2 are reported in Table 3.

Table 3. LM Test Statistic

Lags	Australia	Hong Kong	Japan	Korea	New Zealand	Singapore	Thailand
ARCH (1)	30.32	73.09	369.74	461.78	378.13	1065.21	617.39
ARCH (2)	32.48	73.11	373.98	480.38	381.33	1068.83	654.98
ARCH (3)	49.81	76.75	407.92	486.51	396.53	1087.83	809.87
ARCH (4)	55.64	76.69	408.53	486.69	590.12	1090.96	813.77
ARCH (5)	61.34	76.85	411.58	490.91	591.99	1104.42	819.67

Note: All LM test statistics for ARCH (p) are significant at the 1% level.

They are all statistically significant at the 1% level, which strongly indicates the presence of an ARCH process in the series.

Estimating (G)ARCH Models

The parameters of a (G)ARCH model are obtained through a maximum likelihood estimation. Given the return series and initial values of e_l and h_l for $l=0, \dots, r$ and with $r = \max(p, q)$, the log-likelihood function we have to maximize for a normal distribution is the following:

$$L(\phi | p, q) = -\frac{1}{2}T \ln(2\pi) + \sum_{t=r}^T \ln\left(\frac{1}{\sqrt{h_t}}\right) \exp\left(\frac{-\epsilon_t^2}{2h_t}\right) \quad (7)$$

where T is the number of observations; h_t , the conditional variance, is defined by Equations (4) and (5) for the ARCH and GARCH models respectively; and ϵ_t^2 are the residuals obtained from the appropriate linear regression model according to the country in consideration.

As the values of p and q have to be prespecified in the model, we tested several combinations of p and q . The values of the maximized likelihood functions for all pairs of p and q are presented in Table 4.

We conducted a likelihood ratio test to compare the null hypothesis of a normal distribution with the alternative hypothesis of a (G)ARCH process. The degrees of freedom equal to the difference in the number of parameters under the two hypotheses. Table 4 gives the values of the LR test for each model. It can be observed that the value of the LR test for all (G)ARCH models is statistically significant at the 1% level, which means that all of these models are more likely to fit the data than is the normal distribution. In order to distinguish between an improvement in the likelihood function due to a better fit and an improvement due to an increase in the

order-selection criterion, where K is the number of parameters in the model.

Table 4. Maximum Log Likelihoods for (G)ARCH Models

Model	p,q	Log Likelihood	LRTest	Schwarz Criterion	Log Likelihood	LRTest	Schwarz Criterion	Log Likelihood	LR Test	Schwarz Criterion
Australia										
Normal	—	10615.45			8977.12			10702.39		
ARCH	(1,0)	11189.30	1147.70	-22375.17	9282.45	610.66	-18561.47	11119.54	834.30	-22235.66
ARCH	(2,0)	11230.27	1229.64	-22453.68	9408.03	861.82	-18804.21	11275.02	1145.26	-22543.21
ARCH	(3,0)	11235.82	1240.74	-22461.35	9456.68	959.12	-18903.09	11297.50	1190.22	-22584.75
GARCH	(1,1)	11254.02	1277.14	-22501.18	9607.39	1260.54	-19207.93	11336.41	1268.04	-22665.99
H										
GARCH	(2,1)		
H										
GARCH	(1,2)	11254.90	1278.90	-22499.51				11332.60	1260.42	-22654.95
H										
GARCH	(2,2)		
H										
Korea										
Normal	—	10100.83			10547.19			10158.10		
ARCH	(1,0)	10321.09	440.52	-20638.76	11004.27	914.16	-22005.12	10621.23	926.26	-21239.03
ARCH	(2,0)	10380.84	560.02	-20754.84	11048.89	1003.40	-22090.94	10660.65	1005.10	-21314.45
ARCH	(3,0)	10388.54	575.42	-20766.83	11086.93	1079.48	-22163.60	10719.01	1121.82	-21427.74
GARCH	(1,1)	10414.98	628.30	-20823.12	11153.47	1212.56	-23000.10	10731.55	1146.90	-21456.25
H										
GARCH	(2,1)	10408.48	615.30	-20806.71		
H										
GARCH	(1,2)	10408.80	615.94	-20807.35	11160.24	1226.10	-22300.22	10727.16	1138.12	-21444.04
H										
GARCH	(2,2)									
H										
Thailand										
Normal	—	10193.08								
ARCH	(1,0)	11049.69	1713.22	-22095.97						
ARC	(2,0)	11244.75	2103.34	-22482.68						
H										
ARC	(3,0)	10805.81	1225.46	-21601.39						
H										
GAR	(1,1)	11534.78	2683.40	-23062.74						

CH
 GAR (2,1) ...
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Note: ... indicates where the optimization routine failed.

Table 5. Model Estimates

	Australia	Hong Kong	Japan	Korea	Zealand	Singapore	Thailand
ϕ_0 (thousands)	.6516	1.3718	.7769	.4324	.6516	.6887	-.0908
$t(\phi_0)$	<u>4.3213</u>	<u>5.3079</u>	<u>5.9698</u>	<u>2.1753</u>	<u>4.3213</u>	<u>3.6294</u>	-.9716
ϕ_1	.3488	.0909	.1182	.0680	.3488	.2475	.2292
$t(\phi_1)$	<u>16.5473</u>	<u>4.3779</u>	<u>4.9872</u>	<u>2.9247</u>	<u>16.5473</u>	<u>10.8812</u>	<u>10.8484</u>
ρ_1 (thousands)	.0052	.0102	.0046	.0149	.0052	.0180	.0011
$t(\rho_1)$	<u>5.7262</u>	<u>6.1830</u>	<u>5.6141</u>	<u>6.1588</u>	<u>5.7262</u>	<u>7.7949</u>	<u>5.3935</u>
ρ_2	.2281	.2281	.3003	.2133	.2281	.1183	.2366
$t(\rho_2)$	<u>9.5348</u>	<u>14.2678</u>	<u>10.6646</u>	<u>8.2473</u>	<u>9.5348</u>	<u>9.5651</u>	<u>10.3057</u>
ρ_3	.7451	.7769	.6927	.7062	.7451	.6554	.7644
$t(\rho_3)$	<u>29.9080</u>	<u>59.4841</u>	<u>25.5277</u>	<u>22.6403</u>	<u>29.9079</u>	<u>21.6986</u>	<u>40.1522</u>
$\rho_1 + \rho_2 + \rho_3$	0.9732	1.0000	0.993	0.9195	0.9732	0.7737	1.0000

Note: t -statistics significant at the 5% level are underlined.

According to this criterion, the model with the lowest SIC value fits the data best. The SIC values are reported in Table 4. The GARCH(1,1) model has the lowest SIC values for all countries. Table 5 contains the results of the model best fitting the series of returns for each country. The sum of the conditional

variance equations measures the persistence of the volatility. Engle (1982) and Bollerslev (1986) have shown that if this sum is equal to one, the GARCH process becomes an integrated GARCH or IGARCH process. Such an integrated model implies the persistence of a forecast of the conditional variance over all future horizons and also an infinite variance of the unconditional distribution of ϵ_t . Therefore, a restricted test on the sum of the parameters must be carried out. We calculated the sum of

and the IGARCH(1,1) model for New Zealand, Thailand, Hong Kong, and Singapore. The conditional variance follows an integrated process. The IGARCH(1,1) model has therefore been estimated for these two countries.

Conclusions

This paper examines the return properties of seven Pacific-Basin stock markets. It provides empirical support that the class of autoregressive conditional heteroscedasticity models is generally consistent with the stochastic behavior of daily stock returns. Descriptive statistics and normality tests reveal that the distribution of returns is not normal, whatever the country concerned. It has further been shown that the residuals obtained after applying an AR(1) model, which accounts for the presence of first-order autocorrelation in the returns, exhibit nonnormality and heteroscedasticity. Then we tested various models belonging to the class of autoregressive conditional heteroscedasticity models. Among them the GARCH(1,1) fits the data best for Australia, Japan, Korea, New Zealand, and Singapore, and the IGARCH(1,1) for Hong Kong and Thailand.

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