UNIVERSITY of Liège

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Optimization of flexible components in reciprocating engines with cyclic dynamic loading

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OUTLINE

- Introduction & Motivations
- Finite element approach of multibody system dynamics
- Formulation of the flexible multibody system optimization problem
- Numerical applications
- Conclusion & Perspectives

INTRODUCTION

Evolution of virtual prototyping

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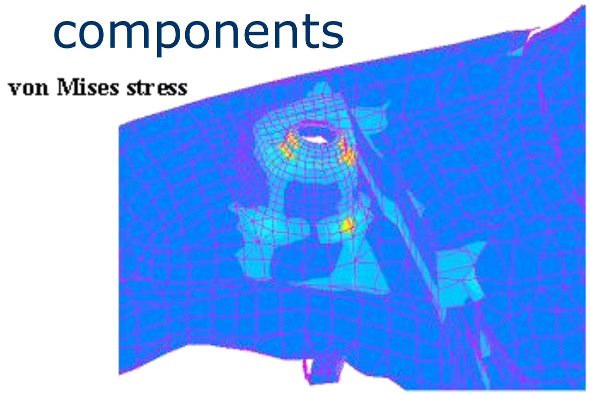
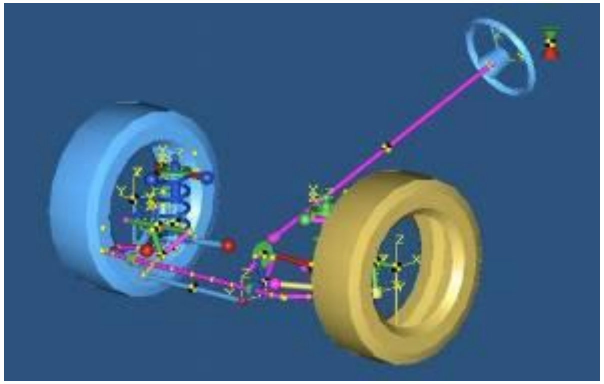
- Finite Element Method: Structural analysis of components

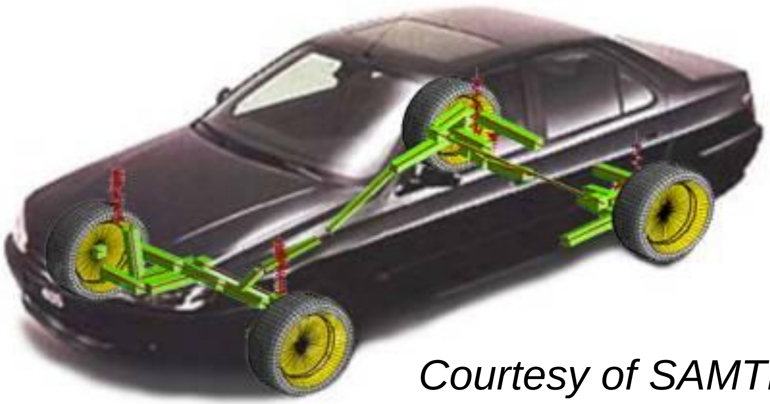


- Multibody System: Mechanism of rigid bodies



- Flexible Multibody Systems:

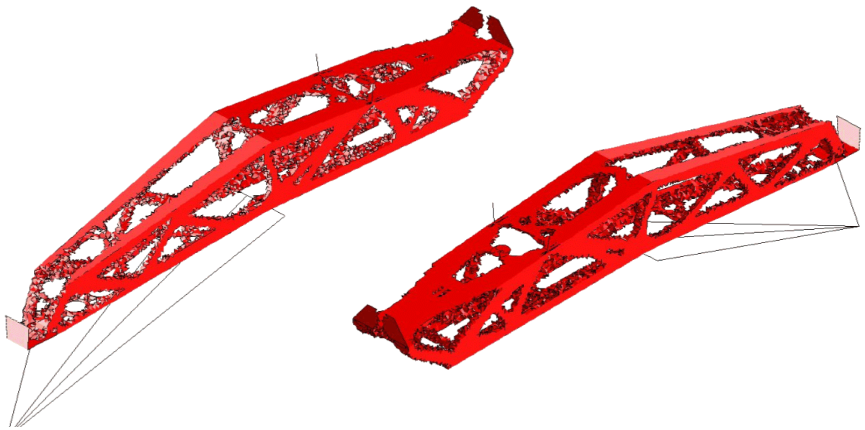
System approach (MBS)
& structural dynamics (FEM)



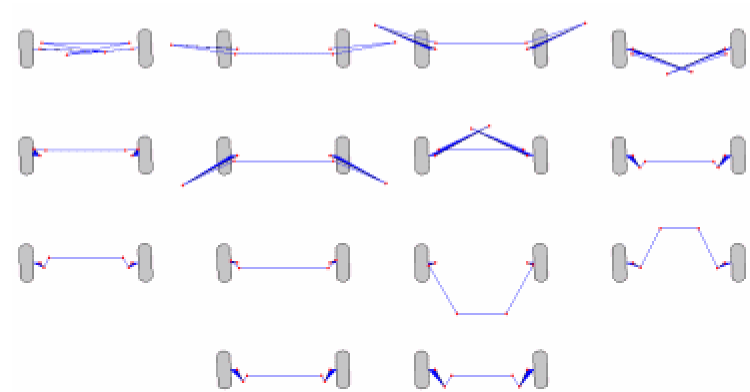
Courtesy of SAMTECH

Evolution of virtual prototyping

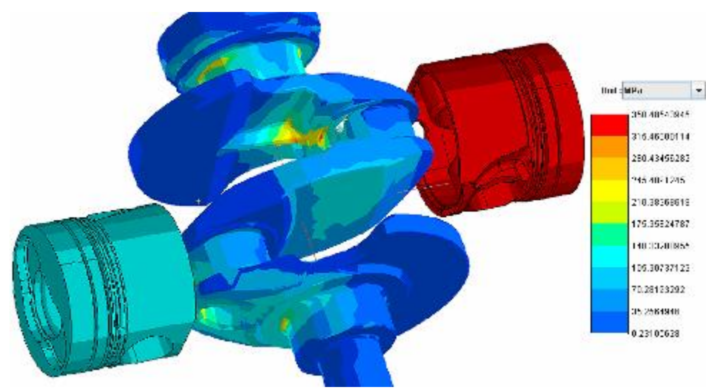
- Structural optimization



- Mechanism synthesis



- Optimization of flexible components in multibody systems:



Motivations

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- Optimization of flexible components in multibody system dynamics
  - Define realistic dynamic loadings
  - Take care of the coupling between large overall rigid-body motions and deformations
- Common approach: Equivalent static loads approach + Rigid (or component mode approach) MBS
  - Component interactions are ignored
  - Global vibration behavior and modeling of high frequency loadings are poor
- Here « Fully Integrated Method »
  - → MBS approach based on non-linear FEM (SAMCEF Mecano)
  - → Coupling with optimization (Boss Quattro)

Goals of this work

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- Investigation on the formulation of the MBS optimization problems under dynamic loading
  - The formulation is critical for these types of problem
    - ➔Convergence, robustness, stability, …

➔Understanding the physical meaning of the problem and elaborate an appropriate formulation

➔Choice of dynamic constraints

FINITE ELEMENT APPROACH OF MULTIBODY SYSTEM DYNAMICS

EQUATION OF FEM-MBS DYNAMICS

- Motion of the flexible body (FEM) is represented by absolute nodal coordinates **q** (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{g}^{\text{ext}} - \mathbf{g}^{\text{int}}$$

- Subject to kinematic constraints of the motion

$$\mathbf{\Phi}(\mathbf{q}, t) = 0$$

- Solution based on an augmented Lagrangian approach of total energy

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}^T(k\boldsymbol{\lambda} + p\mathbf{\Phi}) = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t) \\ k\mathbf{\Phi}(\mathbf{q}, t) = 0 \end{cases} \qquad \mathbf{B} = \frac{\partial \mathbf{\Phi}}{\partial \mathbf{q}}$$

$$\mathbf{q}'(0) = \mathbf{q}'_0 \text{ and } \dot{\mathbf{q}}'(0) = \dot{\mathbf{q}}_0$$

TIME INTEGRATION

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- The set of nonlinear DAE solved using the generalized-α method by Chung and Hulbert (1993)
- Define pseudo acceleration **a**:

$$(1-\alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1-\alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n$$

- Newmark integration formulae

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h(1-\gamma)\mathbf{a}_n + h\gamma\mathbf{a}_{n+1}$$
$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_{n+1} + h^2(1/2-\beta)\mathbf{a}_n + h\beta\mathbf{a}_{n+1}$$

- Solve iteratively the dynamic equation system (Newton-Raphson)

$$\begin{cases} \mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\Delta\dot{\mathbf{q}} + \mathbf{K}_t\Delta\mathbf{q} + \mathbf{B}^T\Delta\boldsymbol{\lambda} = \Delta\mathbf{r} \\ \mathbf{B} = \mathbf{0} \end{cases} \qquad \mathbf{r} = \mathbf{M}\ddot{\mathbf{q}} - \mathbf{g} + \mathbf{B}^T\boldsymbol{\lambda}$$

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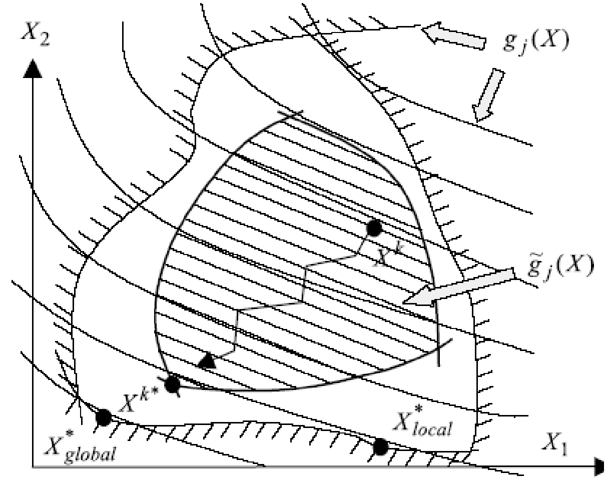
FORMULATIONS OF FLEXIBLE MULTIBODY SYSTEM OPTIMIZATION PROBLEM

General form of the optimization problem

- Design problem is cast into a mathematical programming problem

$$\min_{\mathbf{x}} g_0(\mathbf{x})$$

$$s.t. \begin{cases} g_j(\mathbf{x}) \leq \overline{g}_j, & j = 1, \ldots, m \\ \underline{x}_i \leq x_i \leq \overline{x}_i, & i = 1, \ldots, n \end{cases}$$



- Provides a general and robust framework to the solution procedure

- Efficient solver :
  - Sequential Convex Programming (Gradient based algorithm)

    ➔GCM (Bruyneel et al. 2002)

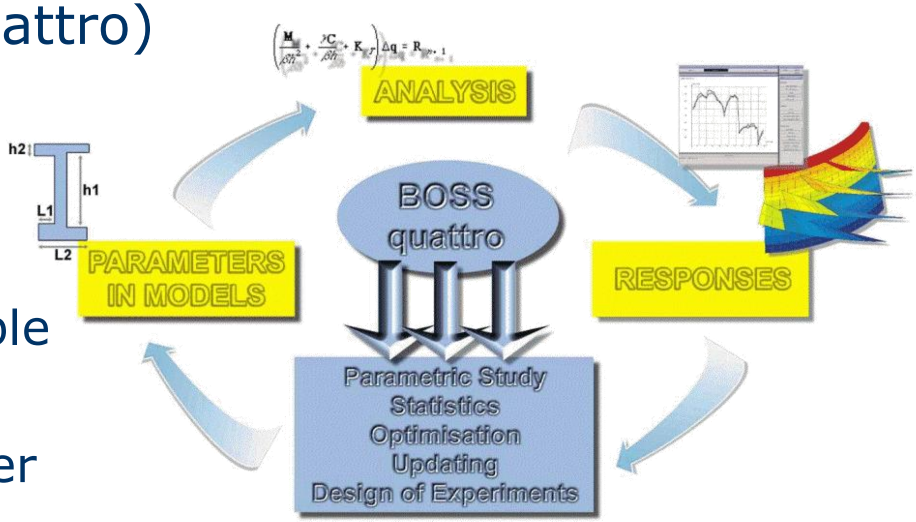
Sensitivity analysis

- Gradient-based optimization methods require the first order derivatives of the responses

- Finite differences (Boss Quattro)

$$\frac{\partial \mathrm{f}}{\partial x} \approx \frac{\mathrm{f}(x+\delta x)-\mathrm{f}(x)}{\delta x}$$

Perturbation of design variable
Additional call to MBS code
➔ Boss Quattro task manager



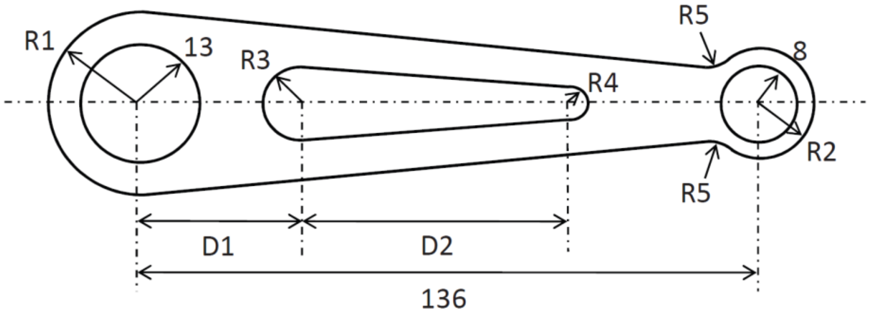
- Semi-analytical approach

$$\frac{\partial \mathbf{r}}{\partial x} \approx \frac{\mathbf{r}(x+\delta x)-\mathbf{r}(x)}{\delta x} \qquad \frac{\partial \mathbf{\Phi}}{\partial x} \approx \frac{\mathbf{\Phi}(x+\delta x)-\mathbf{\Phi}(x)}{\delta x}$$

Design variables

➔ Sizing variables: Plate thickness, bar and beams cross sections, lumped properties (stiffness, mass, etc.)

➔ Shape variables: Geometrical parameters of flexible body shape



➔ Topology: Pseudo density variables, e.g. SIMP $E=\mu^3 E°$ (Bruls et al. 2007)

- BUT NOT
  - Synthesis variables of mechanisms (Hansen, 2002)
  - Links and joints connectivity (Kawamoto et al. 2004)

Formulation (1)

- Ensure a certain rigidity/precision:

  Generally, a formulation based on the *Maximization of the stiffness* or the *Minimization of the compliance.*
  Under dynamic loading:

$$C_{(i)}\left(\mathbf{x},t\right)=\int_{V_E}\boldsymbol{\varepsilon}^T\left(\mathbf{x},t\right)\mathbf{D}\,\boldsymbol{\varepsilon}\left(\mathbf{x},t\right)\;dV$$

- For mechanical systems, an averaged compliance of all bodies estimated over a sufficiently long integration time T has been suggested. (Brüls *et al*., 2007)

$$\overline{C}\left(\mathbf{x}\right)=\frac{1}{T}\int_0^T\sum_i C_{(i)}\left(\mathbf{x},\tau\right)d\tau$$

- Advantage: quantity always positive
- Results: not so conclusive

Formulation (2)

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- Better to work with a more specific formulation to the treated problem

➔Depends on the mechanism and on design considerations

- Comparison with an ideal behavior (if known)
  - ➔A function *Δl* measures the difference

$$\Delta l\left(\mathbf{x}, t_i\right) \leq \Delta l_{max}, \qquad \forall\, i = 1, \ldots, n$$

- Can be considered as a constraint or as the objective function

Formulation (3)

- Definition of the function *Δl*:
  - Ideal behavior: $\mathbf{r}_{rigid}(t)$
  - Actual behavior: $\mathbf{r}(t)$
- A basic definition

$$\Delta l\left(\mathbf{x},t\right)=\mathbf{r}\left(\mathbf{x},t\right)-\mathbf{r}_{rigid}\left(t\right)$$

- Advantages:
  - Precise knowledge of the difference at each time step
  - High level of control on the design
- BUT
  - Large number of constraints ➔ Complex optimization problem

Formulation (4)

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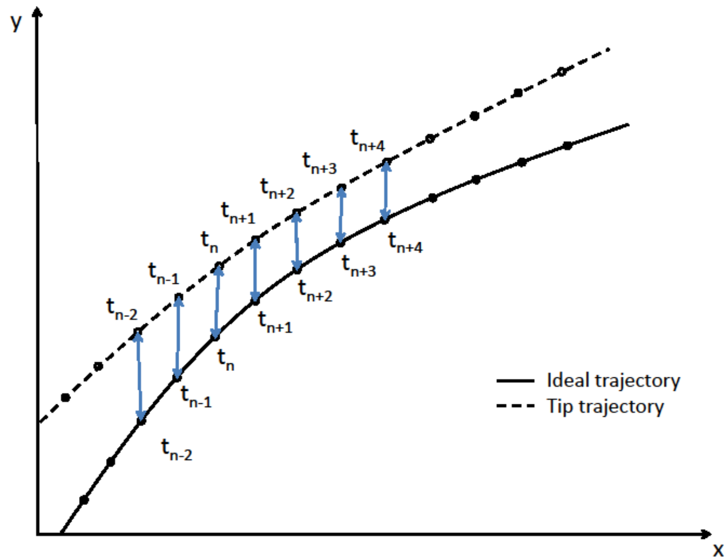
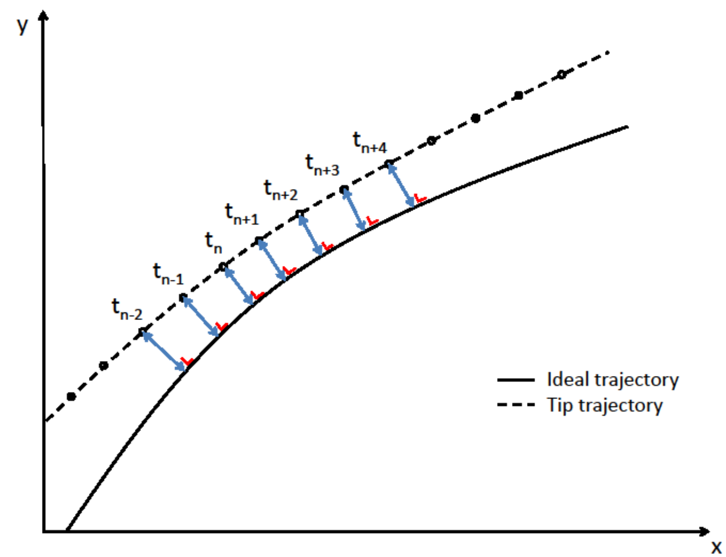
- Function *Δl*: not a trivial definition
- Example: "*The tip of a robot have to follow a desired trajectory*"



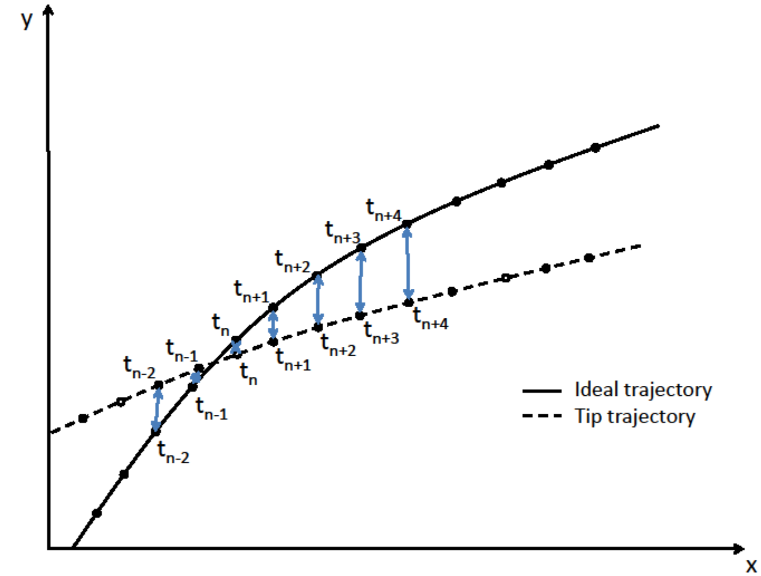


➔Influences the convergence

➔Need to be investigated

Formulation (5)

- Negative and positive values for the function *Δl*



- Only positive values – The distinction is not important
- Mathematical treatment: Norm 2

$$\| \Delta l(\mathbf{x},t) \|^2 = \| \mathbf{r}(\mathbf{x},t) - \mathbf{r}_{rigid}(t) \|^2$$

Formulation (6)

- Local formulation towards global formulation
- Mathematical treatments can reduce the large number of constraints

- *Max function*

$$\Delta l\left(\mathbf{x},t\right) \leq \Delta l_{max} \text{ becomes } \max_{\mathbf{t}} \Delta l\left(\mathbf{x},t\right) \leq \Delta l_{max}$$

  - *Non-smooth function*
  - *Control level on the design is decreased*
  - *BUT only 1 constraint*

Formulation (7)

- *An averaged function over the time T*

$$\Delta l\left(\mathbf{x},t\right) \leq \Delta l_{max} \text{ becomes } \frac{1}{T}\int_0^T \parallel \mathbf{r}\left(\mathbf{x},t\right) - \mathbf{r}_{rigid}\left(t\right) \parallel^2 \; dt \leq \Delta l_{max}$$

  - *Loose control of instantaneous difference*
  - *Relation between mean square deviation and max deviation?*
    - *How to define Δl_{max}*

Formulation (8)

- The stresses defined on elements

$$\boldsymbol{\sigma}\left(\mathbf{x},\mathbf{P},t_i\right)\leq\sigma_{max},\qquad\forall\, i=1,\ldots,n \text{ and } \forall\, P\in V_E.$$

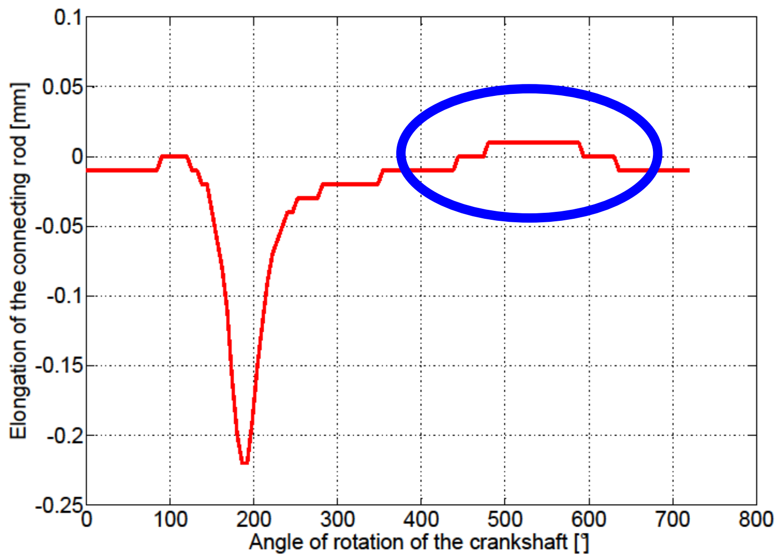
- The number of stress constraints=
  Number of elements * Number of time steps
  ➔ VERY LARGE number

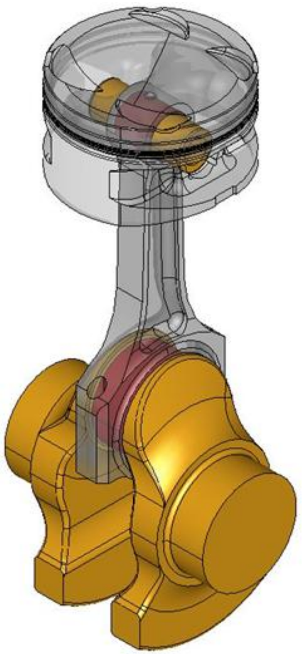
- Need to reduce this number of constraints

NUMERICAL APPLICATIONS

Connecting rod optimization

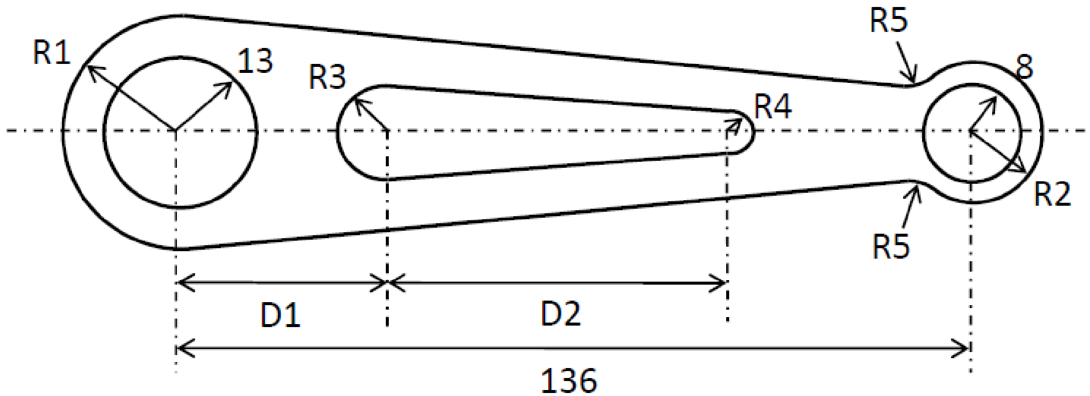
- The link between the piston and the crankshaft in a combustion engine.
- During the exhaust phase, the connecting rod elongates which can destroy the engine.
  ➔Collision between the piston and the valves.
- Minimization of the elongation

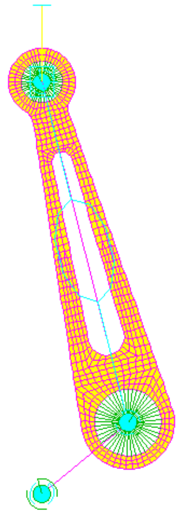




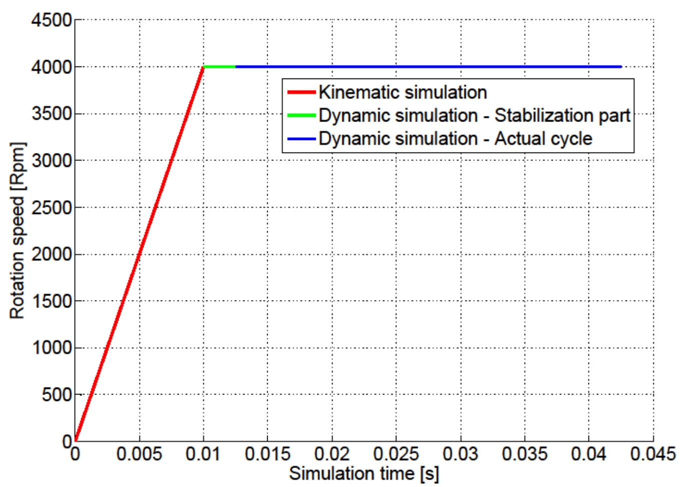
Modeling of the connecting rod

- 7 shape parameters: $\mathbf{x} = [D_1\ D_2\ R_1\ R_2\ R_3\ R_4\ R_5]^T$





- Simulation of a single complete cycle as the behavior is cyclic (720°)
- Rotation speed 4000 Rpm
- Gas pressure taken into account.



Shape optimization - elongation

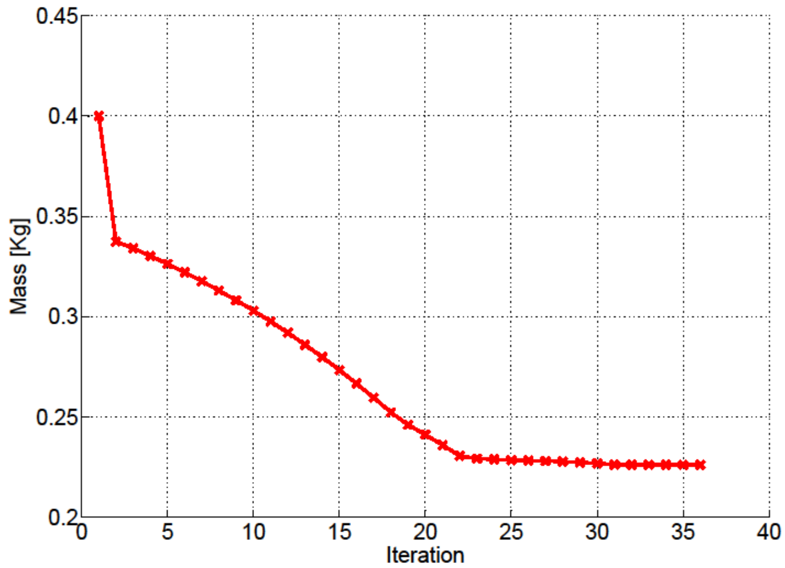
- The value of the function *Δl* is given by a distance indicator element.
- Only the elongation is important.
- Therefore, the problem for the definition of the function is simplified!

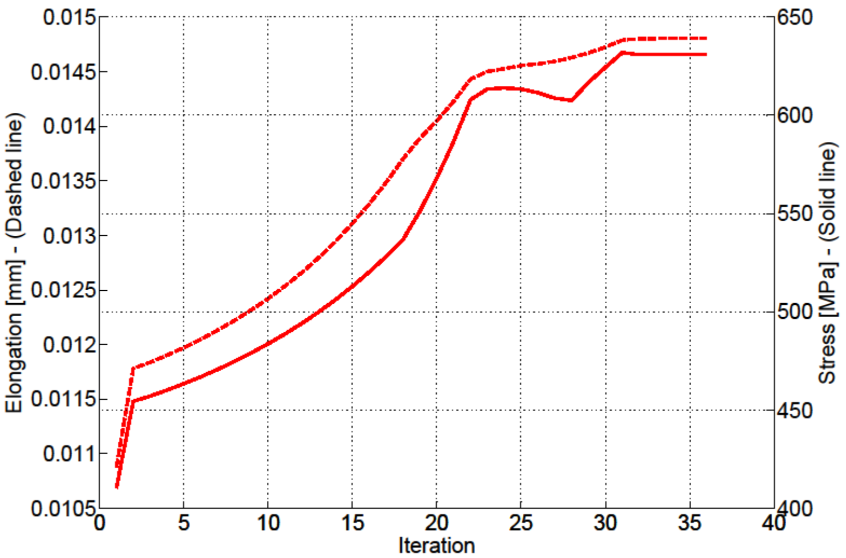
Shape optimization – elongation Local formulation

$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t. \;\; k\left(\Delta l(\mathbf{x}, t_i) \leq \Delta l_{max}\right)$$

$$\text{with } i = 1, \ldots, \text{nbr time step}$$





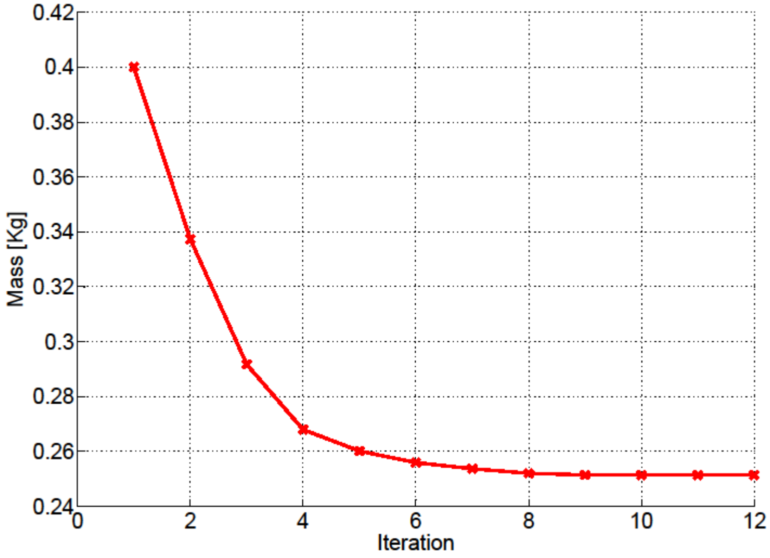
- Convergence in a stable and monotonous way
- High control level on the design
- Large number of constraints but convergence

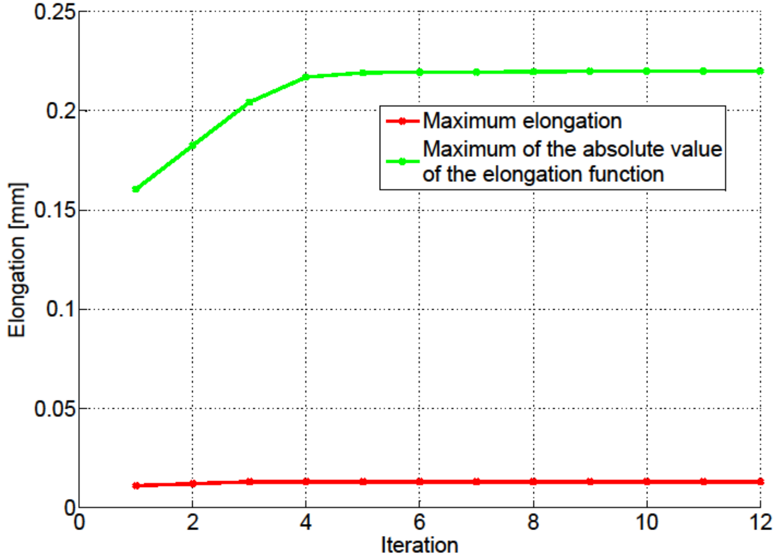
Shape optimization – elongation
Local formulation

$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t.\ \ k\left(\left|\Delta l\left(\mathbf{x}, t_i\right)\right| \leq \Delta l_{max}\right)$$

$$\text{with } i = 1, \ldots, \text{nbr time step}$$





- Faster convergence
- Definition of Δl_{max} do not correspond to the maximum elongation but to the maximum contraction.

Shape optimization – elongation
Global formulation

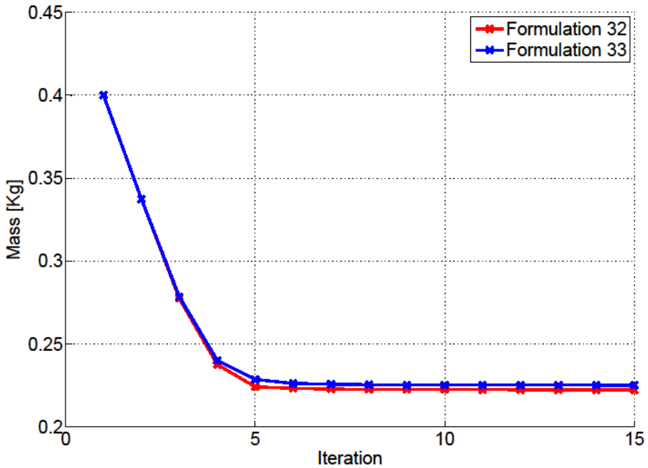
$$\min_{\mathbf{x}} m(\mathbf{x})$$

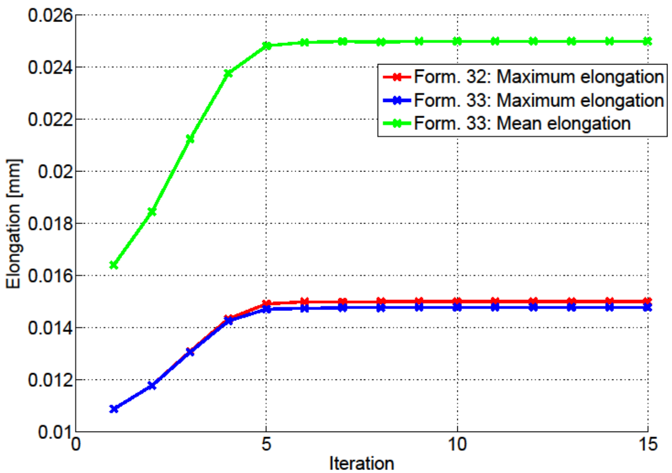
$$s.t.\ \ k\left(\max_{t_i}\left[\Delta l(\mathbf{x}, t_i)\right] \leq \Delta l_{max}\right)$$

$$\text{with } i = 1, \ldots, \text{nbr time step}$$

$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t.\ \ k\left(\frac{1}{t_{max}} \sum_{i=0}^{t_{max}} \left|\Delta l(\mathbf{x}, t_i)\right| \leq \Delta l_{max}\right)$$





- Similar convergence, stability and monotony
- Max form.: good convergence despite the non-smooth character
- Mean form.: Difficulty to define the bound

Shape optimization - stress

- Stress constraints at each time step:
  - 80400 stress constraints !

- Connecting rod: critical instant when the explosion occurs ➔ Selection of this critical instant for the stress constraints

- Test of two meshes
  - Coarse mesh: 600 stress constraints
  - Fine mesh: 3832 stress constraints

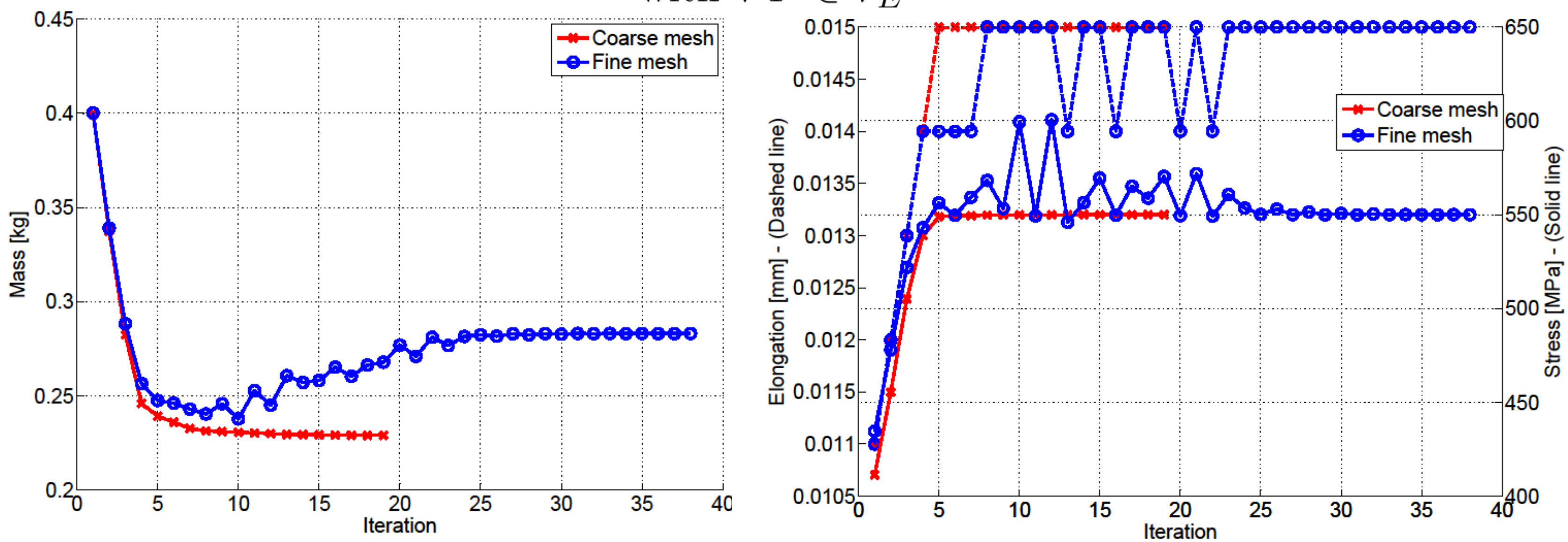
  ➔ How does the optimization process react?

Shape optimization - stress

$$\min_{\mathbf{x}} m(\mathbf{x})$$

$$s.t.\ \boldsymbol{\sigma}(\mathbf{x}, \mathbf{P}, t_{crit}) \leq \sigma_{max}$$

$$\text{with } \forall P \in V_E$$



- Coarse mesh: stable and monotonous convergence
- Fine mesh:
  - Oscillations
  - Violation of the stress constraints during the optimization
  - Heavier as the stresses are better captured

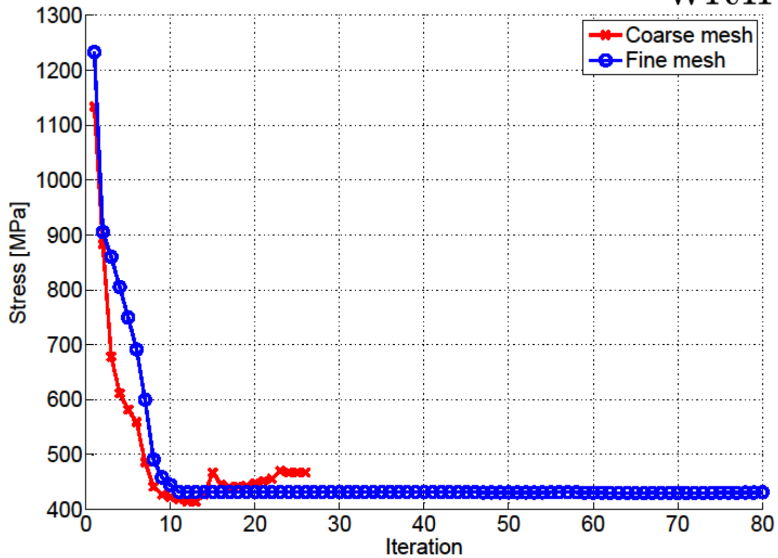
Shape optimization - stress

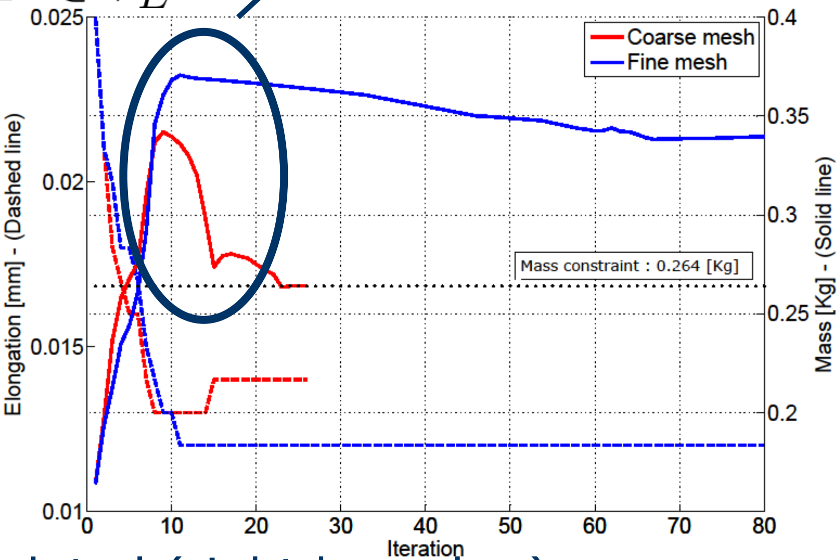
$$\min \boldsymbol{\sigma}\left(\mathbf{x}, \mathbf{P}, t_{crit}\right)$$

$$s.t.\ \ m\left(\mathbf{x}\right) \leq m_{max}$$

$$\text{with } \forall\, P \in V_E$$

Violation of the constraints





- Similar problem if it is well translated (right bounds…)
- Coarse mesh non stable and non monotonous convergence but it converges
- Fine mesh: not able to increase the stresses in order to respect the mass constraint.

Shape optimization – stress
Unfeasible starting point

- Feasible or unfeasible starting point
  - All the previous cases have a feasible starting point because gradient-based algorithm have more facilities to converge.

➔ Observation : *The convergence of the optimization process with an unfeasible starting point can be conclusive if the process is at least stable and monotonous with a feasible starting point.*

CONCLUSIONS

Conclusions

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- The formulation of the optimization process is a key point to obtain convergence.
- The method has been extended to a more realistic application.
- Despite that a *Max function* is non-smooth, it seems to be a good formulation.
  Oral and Ider (1997) also used a non-smooth function and they concluded: "*It has been shown that the piecewise-smooth nature of this equivalent constraint does not cause a deficiency in the optimization process.*"
- Averaged formulations give a good convergence but they are not suitable: difficulties to define the bounds and non accurate control on the design
- Stress constraints have been included in the optimization process due to the identification of a critical time.

Perspectives

- Mixed formulations including global (average) constraints and some time step constraints
- Dynamic stress constraints when a critical time does not exist.
- Improve algorithms for dynamic problems!
  - Structural approximations: local / global: trust regions?
  - Reliability and robustness when starting from unfeasible design points
- Other design criteria for time domain analysis of dynamic systems.

THANK YOU VERY MUCH
FOR YOUR ATTENTION

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