Optimization of flexible components in reciprocating engines with cyclic dynamic loading

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OUTLINE

- Introduction & Motivations
- Finite element approach of multibody system dynamics
- Formulation of the flexible multibody system optimization problem
- Numerical applications
- Conclusion & Perspectives
INTRODUCTION
Evolution of virtual prototyping

- Finite Element Method: Structural analysis of components
- Multibody System: Mechanism of rigid bodies
- Flexible Multibody Systems: System approach (MBS) & structural dynamics (FEM)

Courtesy of SAMTECH
Evolution of virtual prototyping

- Structural optimization
- Mechanism synthesis
- Optimization of flexible components in multibody systems:
Motivations

- Optimization of flexible components in multibody system dynamics
  - Define realistic dynamic loadings
  - Take care of the coupling between large overall rigid-body motions and deformations

- Common approach: Equivalent static loads approach + Rigid (or component mode approach) MBS
  - Component interactions are ignored
  - Global vibration behavior and modeling of high frequency loadings are poor

- Here « Fully Integrated Method »
  - MBS approach based on non-linear FEM (SAMCEF Mecano)
  - Coupling with optimization (Boss Quattro)
Goals of this work

- Investigation on the formulation of the MBS optimization problems under dynamic loading
  - The formulation is critical for these types of problem
    ➔ Convergence, robustness, stability, ...

  ➔ Understanding the physical meaning of the problem and elaborate an appropriate formulation
  ➔ Choice of dynamic constraints
FINITE ELEMENT APPROACH OF MULTIBODY SYSTEM DYNAMICS
EQUATION OF FEM-MBS DYNAMICS

- Motion of the flexible body (FEM) is represented by absolute nodal coordinates \( q \) (Geradin & Cardona, 2001)

- Dynamic equations of multibody system

\[
M\ddot{q} = g(\dot{q}, q, t) = g^{\text{ext}} - g^{\text{int}}
\]

- Subject to kinematic constraints of the motion

\[
\Phi(q, t) = 0
\]

- Solution based on an augmented Lagrangian approach of total energy

\[
\begin{bmatrix}
M\ddot{q} + B^T (k\lambda + p\Phi) = g(\dot{q}, q, t) \\
k\Phi(q, t) = 0
\end{bmatrix} \quad B = \frac{\partial \Phi}{\partial q}
\]

\[ q'(0) = q'_0 \text{ and } \dot{q}'(0) = \dot{q}_0 \]
**TIME INTEGRATION**

- The set of nonlinear DAE solved using the generalized-α method by Chung and Hulbert (1993)
- Define pseudo acceleration $\mathbf{a}$:

$$(1 - \alpha_m)\mathbf{a}_{n+1} + \alpha_m \mathbf{a}_n = (1 - \alpha_f)\ddot{\mathbf{q}}_{n+1} + \alpha_f \ddot{\mathbf{q}}_n$$

- Newmark integration formulae

$${\dot{\mathbf{q}}}_{n+1} = {\dot{\mathbf{q}}}_n + h(1 - \gamma)\mathbf{a}_n + h\gamma \mathbf{a}_{n+1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h{\dot{\mathbf{q}}}_{n+1} + h^2(1/2 - \beta)\mathbf{a}_n + h\beta \mathbf{a}_{n+1}$$

- Solve iteratively the dynamic equation system (Newton-Raphson)

$$\begin{bmatrix}
M\Delta\ddot{\mathbf{q}} + C_t\Delta\dot{\mathbf{q}} + K_t\Delta\mathbf{q} + \mathbf{B}^T\Delta\lambda = \Delta\mathbf{r} \\
\mathbf{r} = M\ddot{\mathbf{q}} - \mathbf{g} + \mathbf{B}^T\lambda
\end{bmatrix}$$
FORMULATIONS OF FLEXIBLE MULTIBODY SYSTEM OPTIMIZATION PROBLEM
General form of the optimization problem

- Design problem is cast into a mathematical programming problem

\[
\min_{\mathbf{x}} g_0(\mathbf{x})
\]

\[\begin{align*}
&\text{s.t. } \quad g_j(\mathbf{x}) \leq \bar{g}_j, \quad j = 1, \ldots, m \\
&\quad \underline{x}_i \leq x_i \leq \bar{x}_i, \quad i = 1, \ldots, n
\end{align*}\]

- Provides a general and robust framework to the solution procedure

- Efficient solver:
  - Sequential Convex Programming (Gradient based algorithm)
  - GCM (Bruyneel et al. 2002)
Sensitivity analysis

- Gradient-based optimization methods require the first order derivatives of the responses

- Finite differences (Boss Quattro)

\[
\frac{\partial f}{\partial x} \approx \frac{f(x + \delta x) - f(x)}{\delta x}
\]

Perturbation of design variable
Additional call to MBS code
⇒ Boss Quattro task manager

- Semi-analytical approach

\[
\frac{\partial \mathbf{r}}{\partial x} \approx \frac{\mathbf{r}(x + \delta x) - \mathbf{r}(x)}{\delta x}
\]

\[
\frac{\partial \Phi}{\partial x} \approx \frac{\Phi(x + \delta x) - \Phi(x)}{\delta x}
\]
Design variables

- Sizing variables: Plate thickness, bar and beams cross sections, lumped properties (stiffness, mass, etc.)

- Shape variables: Geometrical parameters of flexible body shape

- Topology: Pseudo density variables, e.g. SIMP $E = \mu^3 E^\circ$ (Bruls et al. 2007)

BUT NOT
- Synthesis variables of mechanisms (Hansen, 2002)
- Links and joints connectivity (Kawamoto et al. 2004)
Formulation (1)

- Ensure a certain rigidity/precision:
  Generally, a formulation based on the *Maximization of the stiffness* or the *Minimization of the compliance*. Under dynamic loading:

  \[ C_{(i)}(x, t) = \int_{V_E} \varepsilon^T(x, t) D \varepsilon(x, t) \, dV \]

- For mechanical systems, an averaged compliance of all bodies estimated over a sufficiently long integration time \( T \) has been suggested. (Brüls et al., 2007)

  \[ \overline{C}(x) = \frac{1}{T} \int_0^T \sum_i C_{(i)}(x, \tau) \, d\tau \]

- Advantage: quantity always positive
- Results: not so conclusive
Formulation (2)

- Better to work with a more specific formulation to the treated problem
  - Depends on the mechanism and on design considerations

- Comparison with an ideal behavior (if known)
  - A function $\Delta l$ measures the difference

$$\Delta l (x, t_i) \leq \Delta l_{max}, \quad \forall \ i = 1, \ldots, n$$

- Can be considered as a constraint or as the objective function
Formulation (3)

- Definition of the function $\Delta l$:
  - Ideal behavior: $r_{\text{rigid}}(t)$
  - Actual behavior: $r(t)$

- A basic definition

$$\Delta l(x, t) = r(x, t) - r_{\text{rigid}}(t)$$

- Advantages:
  - Precise knowledge of the difference at each time step
  - High level of control on the design

- BUT

  - Large number of constraints $\Rightarrow$ Complex optimization problem
Formulation (4)

- Function $\Delta l$: not a trivial definition
- Example: “The tip of a robot have to follow a desired trajectory”

→ Influences the convergence
→ Need to be investigated
Formulation (5)

- Negative and positive values for the function $\Delta l$
- Only positive values – The distinction is not important
- Mathematical treatment: Norm 2

\[ \| \Delta l(x, t) \|^2 = \| r(x, t) - r_{rigid}(t) \|^2 \]
Introduction (6)

- Local formulation towards global formulation
- Mathematical treatments can reduce the large number of constraints

Max function

\[ \Delta l(x, t) \leq \Delta l_{max} \text{ becomes } \max_t \Delta l(x, t) \leq \Delta l_{max} \]

- Non-smooth function
- Control level on the design is decreased
- BUT only 1 constraint
Formulation (7)

- An averaged function over the time $T$

$$\Delta l(x, t) \leq \Delta l_{max} \text{ becomes } \frac{1}{T} \int_0^T \| r(x, t) - r_{rigid}(t) \|^2 \, dt \leq \Delta l_{max}$$

- Loose control of instantaneous difference
- Relation between mean square deviation and max deviation?
  - How to define $\Delta l_{max}$
Formulation (8)

- The stresses defined on elements

\[ \sigma(x, P, t_i) \leq \sigma_{max}, \quad \forall \ i = 1, \ldots, n \text{ and } \forall \ P \in V_E. \]

- The number of stress constraints =
  Number of elements * Number of time steps
  \( \Rightarrow \) VERY LARGE number

- Need to reduce this number of constraints
NUMERICAL APPLICATIONS
Connecting rod optimization

- The link between the piston and the crankshaft in a combustion engine.

- During the exhaust phase, the connecting rod elongates which can destroy the engine.
  ➔ Collision between the piston and the valves.

- Minimization of the elongation
Modeling of the connecting rod

- 7 shape parameters: \[ x = [D_1 \ D_2 \ R_1 \ R_2 \ R_3 \ R_4 \ R_5]^T \]

- Simulation of a single complete cycle as the behavior is cyclic (720°)
- Rotation speed 4000 Rpm
- Gas pressure taken into account.
Shape optimization - elongation

- The value of the function $\Delta l$ is given by a distance indicator element.

- Only the elongation is important.

- Therefore, the problem for the definition of the function is simplified!
**Shape optimization – elongation**

**Local formulation**

\[
\min_x m(x) \\
\text{s.t. } k(\Delta l(x, t_i) \leq \Delta l_{max}) \\
\text{with } i = 1, \ldots, \text{nbr time step}
\]

- Convergence in a stable and monotonous way
- High control level on the design
- Large number of constraints but convergence
Shape optimization – elongation
Local formulation

\[
\min_x m(x)
\]

\[
s.t. \quad k(|\Delta l(x, t_i)| \leq \Delta l_{max})
\]

with \( i = 1, \ldots, \text{nbr time step} \)

- Faster convergence
- Definition of \( \Delta l_{max} \) do not correspond to the maximum elongation but to the maximum contraction.
Shape optimization – elongation
Global formulation

\[
\min_{\mathbf{x}} m(\mathbf{x})
\]

\[
\text{s.t. } k \left( \max_{t_i} [\Delta l(\mathbf{x}, t_i)] \leq \Delta l_{\text{max}} \right)
\]

with \( i = 1, \ldots, \text{nbr time step} \)

- Similar convergence, stability and monotony
- Max form.: good convergence despite the non-smooth character
- Mean form.: Difficulty to define the bound
Shape optimization - stress

- Stress constraints at each time step:
  - 80400 stress constraints!

- Connecting rod: critical instant when the explosion occurs ⇒ Selection of this critical instant for the stress constraints

- Test of two meshes
  - Coarse mesh: 600 stress constraints
  - Fine mesh: 3832 stress constraints
  ⇒ How does the optimization process react?
**Shape optimization - stress**

\[
\min_x m(x) \\
\text{s.t. } \sigma(x, P, t_{\text{crit}}) \leq \sigma_{\text{max}}
\]

with \( \forall P \in V_E \)

- Coarse mesh: stable and monotonous convergence
- Fine mesh:
  - Oscillations
  - Violation of the stress constraints during the optimization
  - Heavier as the stresses are better captured
Shape optimization - stress

\[
\min \sigma(x, P, t_{\text{crit}})
\]

\[
s.t. \quad m(x) \leq m_{\text{max}}
\]

with \( \forall P \in V_E \)

- Similar problem if it is well translated (right bounds...)
- Coarse mesh non stable and non monotonous convergence but it converges
- Fine mesh: not able to increase the stresses in order to respect the mass constraint.

Violation of the constraints
Shape optimization – stress
Unfeasible starting point

- Feasible or unfeasible starting point
  - All the previous cases have a feasible starting point because gradient-based algorithm have more facilities to converge.

→ Observation: The convergence of the optimization process with an unfeasible starting point can be conclusive if the process is at least stable and monotonous with a feasible starting point.
CONCLUSIONS
Conclusions

- The formulation of the optimization process is a key point to obtain convergence.

- The method has been extended to a more realistic application.

- Despite that a Max function is non-smooth, it seems to be a good formulation. Oral and Ider (1997) also used a non-smooth function and they concluded: "It has been shown that the piecewise-smooth nature of this equivalent constraint does not cause a deficiency in the optimization process."

- Averaged formulations give a good convergence but they are not suitable: difficulties to define the bounds and non accurate control on the design.

- Stress constraints have been included in the optimization process due to the identification of a critical time.
Perspectives

- Mixed formulations including global (average) constraints and some time step constraints

- Dynamic stress constraints when a critical time does not exist.

- Improve algorithms for dynamic problems!
  - Structural approximations: local / global: trust regions?
  - Reliability and robustness when starting from unfeasible design points

- Other design criteria for time domain analysis of dynamic systems.
THANK YOU VERY MUCH FOR YOUR ATTENTION
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