

Using time-dependent reference profiles for an instability analysis of an evaporating binary liquid layer

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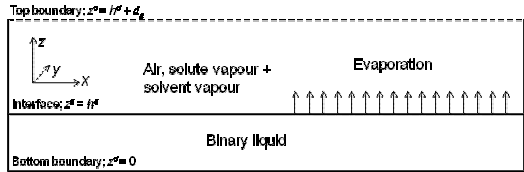
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INTRODUCTION

This work concerns a study about the evaporation of a horizontal binary-liquid layer of an aqueous solution of ethanol in contact with air. The evaporation causes concentration changes of ethanol at the liquid-gas interface accompanied with a cooling. These two phenomena can trigger both solutal and thermal Rayleigh and Marangoni instabilities in the liquid, which are to be studied in this work. For this purpose, the evaporation-induced instability is analysed taking into account horizontally uniform reference profiles. In previous work [1], the instability process, using quasi-stationary reference profiles (no time dependence of the reference profiles), has been studied thoroughly. Making comparisons between the thermal, solutal and Soret Rayleigh and Marangoni effects, it appeared that the solutal Marangoni effect is by far the most important instability mechanism. Extremely small critical liquid layer thicknesses were observed (the liquid layer thickness was used as a control parameter). This means that a liquid layer with a realistic thickness (say, 1 mm) would become unstable almost immediately after its contact with air, much before the transient diffusional boundary layers that develop from the free surface reach the bottom of the liquid. This makes considering a time-dependent reference diffusive solution of much interest for an instability analysis.

HYPOTHESES AND MODEL DESCRIPTION

- Boussinesq approximation
- Air absorption is neglected
- Local equilibrium at the interface
- Rigid bottom plate
- A transfer distance is assumed in the gas phase
- Atmospheric pressure in the gas phase
- Binary liquid: 10 wt% of ethanol in water
- External fixed temperature of 300 K



BULK EQUATIONS

$$\nabla \cdot \vec{v}_1 = 0$$

$$\frac{\partial \vec{v}_1}{\partial t} = -(\vec{v}_1 \cdot \nabla) \vec{v}_1 + P_1 \{ -\nabla p_1 - G_1 \hat{z} + \nabla^2 \vec{v}_1 + R_1 \alpha_1 \hat{z} + R_1 \beta_1 (c_1 - c_{1s}) \hat{z} \}$$

$$\frac{\partial T_1}{\partial t} = -(\vec{v}_1 \cdot \nabla) T_1 + \nabla^2 T_1$$

$$\frac{\partial c_1}{\partial t} = -(\vec{v}_1 \cdot \nabla) c_1 + L_1 \{ \nabla^2 c_1 + \psi_1 \nabla^2 T_1 \}$$

$$\frac{\partial c_2}{\partial t} = -(\vec{v}_2 \cdot \nabla) c_2 + P_2 \{ -\rho^{-1} \nabla p_2 - G_2 \hat{z} + \nu \nabla^2 \vec{v}_2 + \alpha R_2 \alpha_2 \hat{z} + \varepsilon R_2 L_2 (c_2 - c_{2s}) \hat{z} + \delta_2 \varepsilon R_2 L_2 (c_{v1} - c_{v2,0}) \hat{z} \}$$

$$\nabla \cdot \vec{v}_2 = 0$$

$$\frac{\partial T_2}{\partial t} = -(\vec{v}_2 \cdot \nabla) T_2 + \kappa \nabla^2 T_2$$

$$\frac{\partial c_{v1}}{\partial t} = -(\vec{v}_g \cdot \nabla) c_{v1} + \delta_2 D L_2 \nabla^2 c_{v1}$$

$$\frac{\partial c_{v2}}{\partial t} = -(\vec{v}_g \cdot \nabla) c_{v2} + D L_2 \nabla^2 c_{v2}$$

BOUNDARY EQUATIONS

$$\vec{v}_1 = 0 \quad T_1 = T_g \quad c_1 = c_g$$

$$\vec{v}_2 = \vec{v}_i \quad c_2 = c_{i1} \quad c_{v2} = c_{i2}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = \frac{\partial v_2}{\partial x} + \frac{\partial u_2}{\partial y} = 0 \quad -p_2 + 2\mu \frac{\partial v_2}{\partial x} = -p_2$$

$$z=h:$$

$$-\frac{\partial T_1}{\partial z} + J_1 \frac{\partial T_2}{\partial z} = J_2 + \delta_2 J_1 \quad J = J_1 + J_2$$

$$J = -\frac{L_2}{1 - c_1 - c_{v1}} \left\{ \left(\frac{\partial c_1}{\partial x} + \psi_2 \frac{\partial T_1}{\partial x} \right) + \rho \delta_2 D \frac{\partial c_{v1}}{\partial x} \right\}$$

$$J = -\frac{\rho D L_2}{1 - c_{v1} - c_{v2}} \left\{ \frac{\partial c_{v2}}{\partial z} + \delta_2 \frac{\partial c_{v1}}{\partial z} \right\}$$

$$\frac{c_{v2} \delta_2}{1 + c_{v1} (\delta_{v1} - 1) + c_{v2} (\delta_{v2} - 1)} = \frac{c_1 \delta_2}{1 + c_1 (\delta_{v1} - 1) + c_{v1} (\delta_{v2} - 1)} \frac{M_1}{M_2}$$

$$\frac{c_{v2} \delta_2}{1 + c_{v1} (\delta_{v1} - 1) + c_{v2} (\delta_{v2} - 1)} = \frac{(1 - c_1)}{1 + c_1 (\delta_{v1} - 1) + c_{v1} (\delta_{v2} - 1)} \frac{F_{v1,1}}{F_{v1,2}}$$

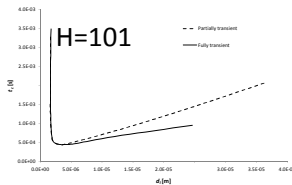
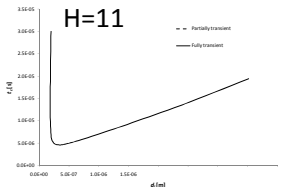
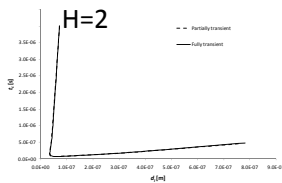
$$\frac{dh}{dt} = \frac{1}{1 - \rho} (\rho_1 - \rho_2) \quad u_1 = u_g \quad v_1 = v_g$$

$$-\kappa \left(\frac{\partial T_1}{\partial z} + \frac{\partial T_2}{\partial z} \right) + \left(\frac{\partial T_1}{\partial x} + \frac{\partial T_2}{\partial x} \right) + M_1 \frac{\partial T_1}{\partial x} + M_2 L_2 \frac{\partial c_1}{\partial x} = 0$$

$$-\kappa \left(\frac{\partial T_2}{\partial y} + \frac{\partial T_1}{\partial y} \right) + \left(\frac{\partial T_2}{\partial y} + \frac{\partial T_1}{\partial y} \right) + M_2 \frac{\partial T_2}{\partial y} + M_1 L_2 \frac{\partial c_1}{\partial y} = 0$$

OUTLINE

- Transient reference profiles are used, on one side, for the concentration profile in the liquid (partially transient) and, on the other side, for all the profiles (fully transient)
- Quasi-stationary reference profiles for the other physical quantities
- The frozen-time approach for the transient profiles
- Perturbations are added to the reference equations
- Linearization
- Decomposition in normal-mode representation allowing obtaining for each time t an eigenvalue problem for the complex growth rates as function of the wavenumber k and other parameters of the problem.
- Solving this eigenvalue problem by the Tau-Chebyshev decomposition method
- The marginal condition is then found by taking $\sigma = 0$
- The main control parameters that are kept are d_l and d_g (in an equivalent way, $H = (d_l + d_g)/d_l$)
- Parametric study is first presented at fixed values of H , obtaining a critical value of d_l below which all $\sigma < 0$ at all times $t > 0$ and for all k .
- Then, for d_l higher than this critical value, it is the purpose to calculate the instants of t at which the system passes through the marginal condition ($\sigma = 0$) at a certain k , that is the critical times.



RESULTS AND DISCUSSION

- For small H values, the fully transient model does not show any significant differences with respect to the partially transient model
- The difference becomes more pronounced as H increases
- A larger value for H (comparing $H=11$ and $H=101$, for instance) means that the gas transfer distance is larger for $H=101$. This means that the diffusion in the gas phase needs much more time to attain a quasi-stationary state
- The solutal effect being the predominant one, considering the thermal effect gains some importance (though still smaller than the solutal one), due to the larger diffusion time in the gas phase
- Considering a fully transient model (thus also a transient temperature profile next to a transient concentration profile) gives a more instable system, caused by the thermal diffusion, of which the profile dependence on time was neglected in the partially transient model (quasi-stationary profile for the temperature profiles)

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