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The random-incidence scattering coefficient of infinite periodic surfaces with rectangular and sine-shaped roughness profiles.

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Summary
The random-incidence scattering coefficient is an essential data characterizing the reflection of sound on surfaces in a room acoustics program. This research compares theoretical scattering coefficients, which have been obtained for a great number of rectangular and sine-shaped profiles by two different methods: the waveguide approach and the Holford-Urusovskii's method. The surfaces have an infinite size and, therefore, the edge effects are neglected. The comparison emphasizes the influence of the geometrical parameters of the surface's roughness profile (height and period of the corrugations) on the scattering coefficient. It is shown in this paper how the scattering coefficient varies with the height of the profile relative to the wavelength. Some similarities between both types of surfaces are demonstrated.

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1. Introduction

The random-incidence scattering coefficient is an essential data characterizing the reflection of sound on surfaces in a room acoustics program. If the total reflected power is expressed as the sum of specular and diffuse contributions, the scattering coefficient is defined as the ratio of the diffusely reflected power to the total reflected power.

Laboratory measurements of this coefficient have been defined and standardized recently [1], but only few results are presently available. Therefore, the users of room acoustics programs often have some difficulties to estimate the value of the random-incidence scattering coefficients that must be introduced in their model.

Theoretical studies like the one described in this paper can help, but they cannot solve the problem for all situations. Indeed, theoretical results are generally based on some assumptions, which of course restrict the field of possible applications. In this paper, the main assumptions are the following:

- the rough surface has an infinite size,
- its periodic roughness profile (rectangular or sine-shaped) is only developed along one dimension (1D-periodic surface),
- the surface is perfectly rigid.

These assumptions allow to study more particularly the influence of the geometrical parameters of the roughness profile on the scattering coefficient. The idea is to investigate if some similarities are observed for two different periodic profiles and if the geometrical parameters could be used as robust estimators of the scattering properties in the applications, at least if the conditions of validity of the assumptions are verified.

The finite size's effects are not taken into account in this study, since the periodic surfaces are infinite. The conclusions must therefore be restricted to applications where the wavelength is much smaller than the dimensions of the surface.

2. Theory for infinite periodic surfaces

The sound field reflected by an infinite periodic surface can be expressed by a sum of plane waves, some of which are propagating away from the surface (the *radiating* modes) while others are evanescent [2,3]. If the direction of incidence is included in a plane perpendicular to the periodic corrugations (figure 1) and the angle of incidence is ϕ_0, then the reflected pressure is:

$$p_{refl}(x,z) = \sum_{n=-\infty}^{+\infty} \overline{R_n}\, e^{jk(\alpha_n x + \beta_n z)} \qquad z > \max\{\xi(x)\} \tag{1}$$

In this expression, the terms of the series for which $|\alpha_n|<1$ are the radiating modes. In particular, the mode n=0 is the specularly reflected wave:

$$\cos\phi_n = \alpha_n = \cos\phi_0 + n\frac{\lambda}{L} \qquad \sin\phi_n = \beta_n \tag{2}$$

In formula (2), n is an integer, λ is the wavelength and L is the spatial period of the roughness profile. Each radiating mode "n" is therefore characterized by its direction (angle ϕ_n , see figure 1) and its complex amplitude R_n. These amplitudes are determined by specific methods, which are briefly described in sections 2.1 and 2.2.

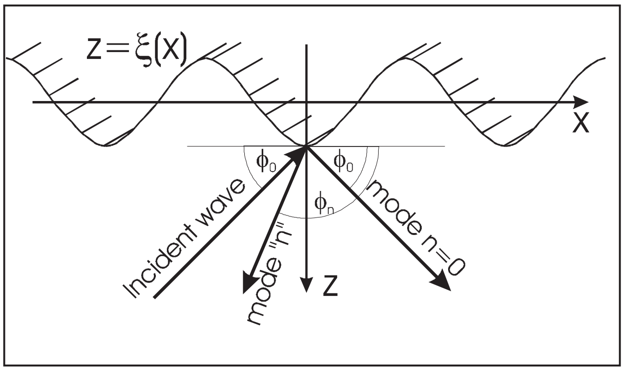

Figure 1. Sine-shaped surface profile and definition of angles ϕ_0 (incidence) and ϕ_n (scattering).

Once determined, the amplitude R_0 of the specular mode is simply used in the definition of the *directional* scattering coefficient. This coefficient of course depends on the frequency or equivalently on the wave number k:

$$s(\phi_0, k) = 1 - \left|\overline{R_0}\right|^2 \tag{3}$$

If the direction of incidence is not in a plane perpendicular to the corrugations as in figure 1, it can be shown [3] that the scattering coefficient is simply derived from (3) by shifting the frequency:

$$s(\phi_0, \theta_0, k) = s(\phi_0, \theta_0 = 0, k\cos\theta_0) \tag{4}$$

In this expression, θ_0 is the angle between the vertical plane containing the direction of incidence and the vertical plane (x,z), while the angle ϕ_0 is the angle between the projection of the direction of incidence in the plane (x,z) and the plane $z = 0$.

Finally, the random-incidence scattering coefficient is found by integrating (4) for all possible directions of incidence (θ_0 , ϕ_0) [3].

2.1. Sine-shaped surfaces

If we refer to figure 1, sine-shaped surfaces are defined by the following equation:

$$z = \xi(x) = H\cos(2\pi x / L) \tag{5}$$

in which H is the height of the profile (half the peak-to-peak depth) and L is the spatial period.

The reflection coefficients R_n of these surfaces have been determined by the Holford-Urusovskii's method, described in [2,3]. Some results of random-incidence scattering coefficients calculations have already been presented in [4]. These results are recalled in figure 2.

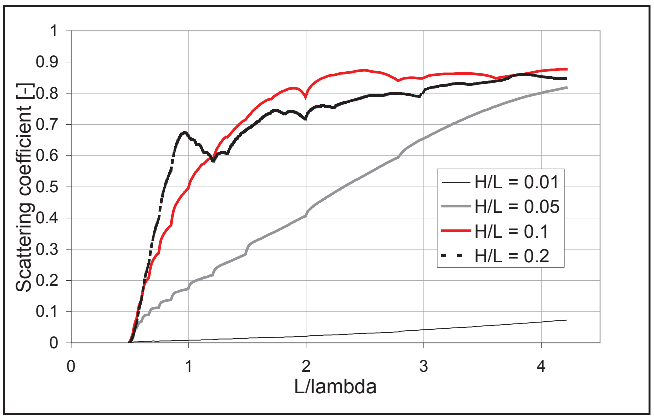

Figure 2. Random-incidence scattering coefficient of some sine-shaped surfaces, as a function of the ratio of the spatial period L to the wavelength.

This figure shows that scattering begins at L/λ=0.5 for all surfaces. Indeed, at smaller values of this ratio, only the specular mode is propagated,

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following (2). The scattering coefficient then increases with a rate depending on the relative height *H/L*. More precisely, the increase rate is greater for greater values of the ratio *H/L*.

These theoretical results have been compared to laboratory measurements' values obtained for a specific surface sample and a very good correspondence was demonstrated [3].

2.2. Surfaces with a periodic rectangular profile

Parallel grooves with a rectangular section define this periodic rough surface. If we refer again to figure 1, the profile if defined by:

$$z = \xi(x) = -h \quad if \ L(m-0.25) < x < L(m+0.25)$$

$$z = \xi(x) = 0 \qquad L(m-0.75) < x < L(m-0.25) \tag{6}$$

In this definition, *m* is an integer, *L* and *h* are respectively the spatial period and the depth of the profile.

Another method has been applied to find the reflection coefficients R_n of these infinite periodic surfaces. This method is a *waveguide* approach: it has been recently described by Ducourneau et al [5] and much earlier by De Bruijn [6]. Briefly, it consists in solving the equation of propagation to find the pressure inside the rectangular (perfectly rigid) grooves ($-h<z<0$). Then, boundary conditions are expressed in $z=0$:

- continuity of the pressure (with eq. (1)) and its gradient at the exit of a groove,
- rigid boundary elsewhere.

Again, the random-incidence scattering coefficients have been computed for a great number of combinations of *L/λ* and *h/L*. Some results are shown in figure 3.

The curves included in this figure have been chosen to allow a first comparison with figure 2. First, the geometrical parameter on the horizontal axis is the same in both figures, i.e. the ratio of the spatial period of the surface to the wavelength. Secondly, the relative height is defined in both figures as the ratio of half the peak-to-peak depth to the spatial period.

As can be seen, the scattering coefficients of periodic rectangular profiles are characterized by resonant modes and this results in quite different evolutions of the curves illustrated in figures 2 and 3. Except for a general increase of the scattering coefficient as a function of *L/λ*, it is difficult to find some resemblance between both figures.

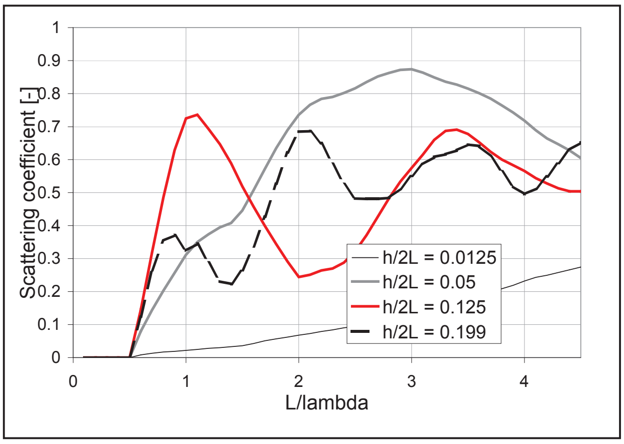

Figure 3. Random-incidence scattering coefficient of some surfaces with a periodic rectangular profile, as a function of the ratio of the spatial period *L* to the wavelength.

The resonance can be explained as follows: first consider a plane wave incident on the surface with perpendicular incidence (ϕ_0 = 90°). If the frequency (or the wavelength) is such that *h/λ* is a multiple of 1/2, then the plane wave propagating into the groove in the direction of the *z*-axis imposes a null admittance as boundary condition, at the exit of the groove (in $z=0$). This condition is similar to the rigid boundary condition imposed on the flat parts of the profile in $z=0$. Therefore, the specular mode alone (with R_0 =1) satisfies all boundary conditions and it is the solution to the problem of scattering. As a consequence of (3), the scattering coefficient s = 0. On the contrary, if the depth *h* is an odd multiple of *λ/4*, then the boundary condition at the exit of the groove becomes a null impedance (zero pressure). As the reflecting plane in $z=0$ is composed of a juxtaposition of very different acoustic impedances, scattering is enhanced and a maximum of the scattering coefficient is observed.

Now, if the direction of incidence is not perpendicular to the horizontal plane $z=0$, this resonance is modified and attenuated in some way, due to the fact that the specular modes and the plane waves propagating into the groove in the *z*-direction are no longer sufficient to fulfill the boundary conditions. The maximum values of the scattering coefficient are shifted to somewhat different *h/λ*, depending on the direction of incidence. As a consequence, the maximum scattering coefficients for *random* incidence are not exactly situated at odd multiple of $h=\lambda/4$ (see figure 4).

This figure 4 shows a remarkable "synchronicity" between all the curves corresponding to different values of the ratio *h/L*. With only few exceptions, the random-incidence scattering coefficient is practically independent of *h/L*, if this ratio remains less than 0.4.

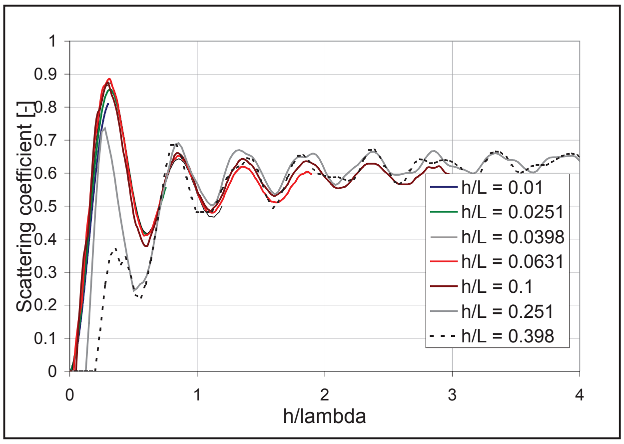

Figure 4. Random-incidence scattering coefficient of some surfaces with a periodic rectangular profile, as a function of the ratio of the depth of the grooves *h* to the wavelength.

3. Comparison of sine-shaped and periodic rectangular profiles

This comparison is inspired by the choice of variables used in figure 4: the random-incidence scattering coefficient will be expressed for both periodic surfaces as a function of the ratio of the depth of the grooves to the wavelength. This choice is also motivated by the study described in [4], in which some similarities were observed between *gaussian* and sine-shaped profiles (*gaussian* surfaces are characterized by a random roughness profile obeying gaussian probability density distributions).

Now, there still remains a problem as *H* and *h* don't have the same signification in both periodic profiles: for sine-shaped profiles, *H* is half the peak-to-peak depth while, for periodic rectangular profiles, *h* is the peak-to-peak depth. Again inspired by the previous study on gaussian surfaces [4], it is decided to choose the *rms* height of the profile as the typical variable on the horizontal axis: for sine-shaped surface, the *rms* height is $0.707*H$, while for periodic rectangular profiles it is *h/2*. Figure 5 shows the results of the comparison.

In this figure, the parameter identifying the individual curves is the ratio of the peak-to-peak amplitude of the profile to the spatial period *L*.

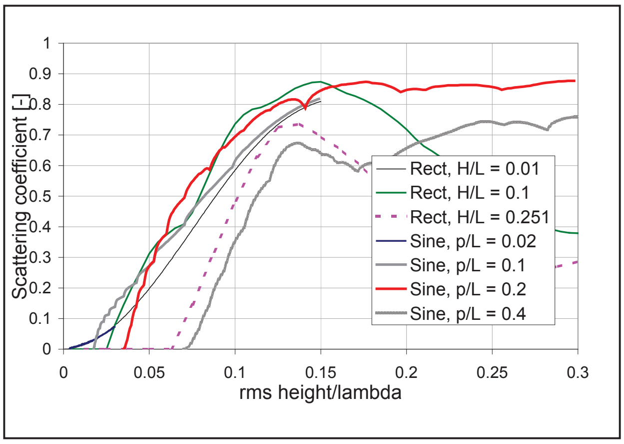

Figure 5. Random-incidence scattering coefficient of some surfaces with sine-shaped and periodic rectangular profiles, as a function of the ratio of the *rms* height of the profile to the wavelength. The parameter identifying each individual curve is the ratio of the peak-to-peak amplitude of the profile to the spatial period *L* ($p=2H$ for sine-shaped surfaces).

Figure 5 shows similar trends for both periodic surfaces, as long as their *rms* height is less than 0.15λ and the ratio (*H/L* or *p/L*) less than *0.2*. The *rms* height seems to be the key parameter for weakly rough surfaces, since the scattering coefficients values seem to coincide for sine-shaped and periodic rectangular profiles. This was also found earlier for random rough surfaces [4,7].

If the *rms* height exceeds 0.15λ, the scattering coefficient tends to an asymptotic value for the sine-shaped surfaces, while it decreases for the periodic rectangular surfaces (due to the first resonance mode as shown in figure 4). This last figure also shows that an asymptotic value is also reached for periodic rectangular profiles, but at much greater values of the relative *rms* height.

If the ratio (*H/L* or *p/L*) exceeds 0.2, then very different evolutions are observed and no generalization is possible.

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In the previous study on gaussian surfaces [4], we excluded the profiles for which the correlation length was less than the wavelength. For periodic surfaces, the parameter most similar to the correlation length is the spatial period *L*. Therefore, figure 5 has been reproduced in figure 6, except that we have arbitrarily excluded the profiles for which $L<\lambda$. The correspondence is somewhat better between both surfaces.

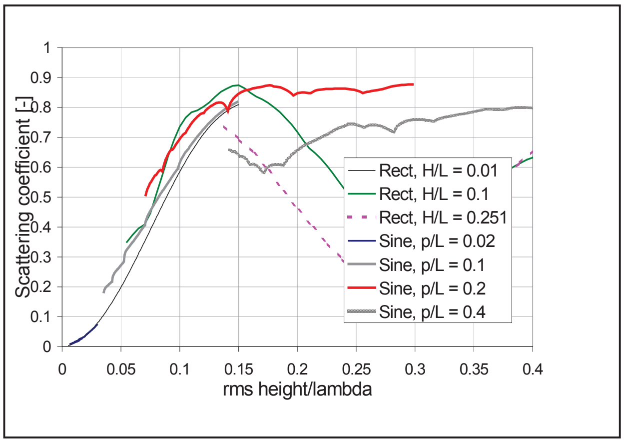

Figure 6. Same as figure 5, with $L>\lambda$.

4. Conclusions

The random-incidence scattering coefficient has been computed for infinite, perfectly rigid, periodic surfaces. More particularly, sine-shaped and periodic rectangular profiles were investigated.

For both types of profiles, the ratio of the *rms* height to the wavelength seems to be a good estimator of the scattering properties of these surface **as long as**:

- the random-incidence scattering coefficient is concerned,
- the *rms* height does not exceed 0.15λ,
- the ratio of the peak-to-peak amplitude of the profile to the spatial period is less than 0.2.

This means that, in the absence of more precise information about the scattering coefficient of a given rough surface, the evaluation of the *rms* height of its profile can be used to estimate this coefficient approximately, especially for applications in room acoustics simulations (provided that the conditions listed previously are verified, of course).

It is recalled that the finite size's effects are not taken into account in this study, and particularly in figures 5 and 6. These effects can be significant at low frequencies, depending on the size and the type of surface.

It has been also shown in this study that it is generally not allowed to assume that the random-incidence scattering coefficient keeps a high and quasi-constant value beyond 0.15λ. This was suggested in a previous study [4], based on computations carried out on gaussian and sine-shaped surfaces. It has been shown in this paper that for periodic rectangular profiles, an asymptotic behaviour is reached for much greater *rms* heights than 0.15λ (see figure 4).

Finally, more laboratory measurements are still necessary to precise the conditions of validity of these simulations, and also to set up databases for the users of room acoustics programs.

References

[1] ISO 17497-1: Acoustics – Measurement of the sound scattering properties of surfaces – Part 1: Measurement of the random-incidence scattering coefficient in a reverberation room, 2004.

[2] R.L. Holford: Scattering of sound waves at a periodic, pressure release surface: An exact solution. Jnl. Acoust. Soc. Amer. 70(4) (1981) 1116-1128.

[3] J.J. Embrechts, L. De Geetere, G. Vermeir, M. Vorländer, T. Sakuma: Calculation of the random-incidence scattering coefficients of a sine-shaped surface. Acta Acustica united with Acustica 92 (2006) 593-603.

[4] J.J. Embrechts: Random-incidence scattering coefficients of infinite sine-shaped surfaces. Proc. 2005 Forum Acusticum in Budapest, 2151-2154.

[5] J. Ducourneau, L. Bos, V. Planeau, A. Faiz, S.S. Lami, A. Nejade: Prediction of the acoustic pressure above priodically uneven facings in industrial workplaces. Jnl. Sound and Vib. 329 (2010) 2276-2290.

[6] A. De Bruijn: The sound absorption of an absorbing periodically uneven surface of rectangular profile. Acustica 18 (1967) 123-131.

[7] J.J. Embrechts, D. Archambeau, G.B. Stan: Determination of the scattering coefficient of random rough diffusing surfaces for room acoustics applications. Acta Acustica united with Acustica 87 (2001) 482-494.