ROBUSTNESS OF BUILDING FRAMES FURTHER TO A COLUMN LOSS
Substructure approach with account of dynamic effects

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INTRODUCTION

Nowadays, it is a concern to mitigate the risk of progressive collapse of a whole structure further to exceptional events such as explosions, impacts, fire... Different general approaches are proposed in the Eurocodes and some other national design codes to ensure the robustness of structures, although very few practical guidelines are provided. In the present study, the so-called “alternative load path method” is followed and the conventional scenario “loss of a column” is considered.
Investigations were conducted at the University of Liège in the last few years regarding the static behaviour of two-dimensional building frames suffering the loss of a column further to an unspecified accidental event. They resulted in the development of simplified analytical methods for the prediction of the structural response assuming a static behaviour ([3] and [4]). In particular, it was shown that a substructure composed of the double-beam overhanging directly the lost column and its beam-to-column joints (*Fig. 1* – “CAD” in *Fig. 2*) is able to reproduce the response of a 2D frame when membrane forces develop in the beams after the formation of the global beam plastic mechanism induced by the column loss (*Fig. 2*).

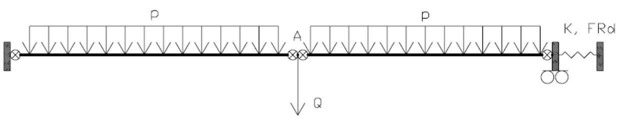

Fig. 1. Double-beam simplified substructure

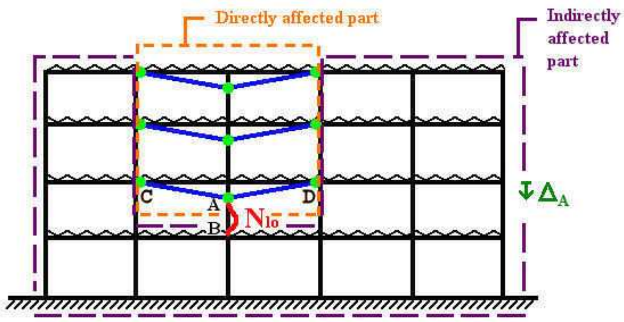

Fig. 2. Plastic mechanism caused by the loss of column “AB”

The dynamic behaviour of such a substructure was more recently investigated [2]. As a result, a simplified approach was developed for the prediction of the maximal displacement of the system and subsequently the internal forces developing in the substructure, permitting the definition of ductility and resistance requirements for robustness.
In the present paper, the whole analytical procedure is applied to a particular substructure. First, the static response is determined. Then the proposed model is used to predict the dynamic behaviour of the substructure as a function of the load initially supported by the failing column and the duration of its removal. In this paper, analytical approaches developed at the University of Liège are applied to predict the response of the considered substructure and the results are compared to numerical simulations; but the development of these methods and the observed phenomena are not detailed. For more information, the interested reader can mainly refer to [1].

1 DEFINITION OF THE INVESTIGATED SUBSTRUCTURE

1.1 Geometrical and mechanical characteristics

Each beam of the studied substructure has an initial length l_0 = *6,5 m*. They are made of a S235 steel profile IPE400. The material behaviour law is considered to be elastic-perfectly plastic with an infinite ductility and is admitted to be independent of the strain rate. The lateral spring characterising the influence of the rest of the structure is elastic with a stiffness *K = 10000 kN/m* which is considered to remain constant in case of dynamic loadings and a resistance F_{Rd} which is assumed to be infinite. The joints are perfectly rigid and fully resistant.

1.2 Static and dynamic loading conditions

As justified in [1] on the basis of [3] and [2], the substructure is studied under a simplified loading consisting of a single concentrated force P applied at mid-length of the system (*Fig. 3*). For dynamic analyses, the force is considered to linearly increase during a time t_r and then to remain constant (*Fig. 4*). Static loading conditions correspond to t_r tending to infinity.

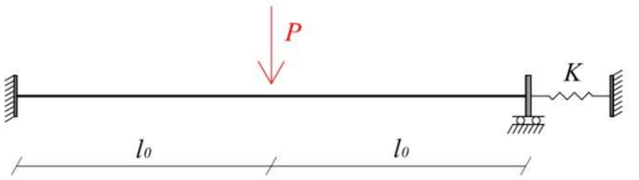

Fig. 3. Considered system with simplified loading

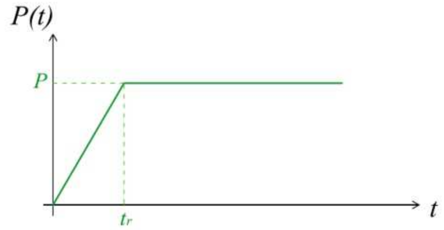

Fig. 4. Time evolution of the applied load $P(t)$

1.3 Mass and damping properties of the system

To investigate the dynamic behaviour of the substructure, the mass of the system has to be defined. A uniformly distributed mass $m = 3000\ kg/m$ is considered along the beams. The mass mainly influences the natural period of the system. It is demonstrated in [2] that the dynamic response of a given structure is governed by two parameters: P and t_r/T, where T is the period of the principal eigenmode in the elastic domain ($T = 0{,}31\ sec$ for the considered system).
No damping is introduced in the system, which constitutes a conservative approach. Besides, it is shown in [2] that damping does not induce a significant decrease of the maximum displacement.

2 STATIC RESPONSE OF THE SUBSTRUCTURE

In this section, the curve representing the static evolution of the vertical displacement at mid-length of the double-beam, u, when the force P increases is established using analytical methods. It is then compared to the numerical curve obtained using the homemade finite element software Finelg [6].

2.1 First order elastic behaviour

For low values of the applied load, the response of the substructure is elastic and the second order effects are negligible. The relation between the concentrated force P applied in the middle of the system and the corresponding displacement u is thus simply given by *Eq. (1).*

$$u = \frac{P \cdot L^3}{192 \cdot E \cdot I} = \frac{P \cdot l_0^3}{24 \cdot E \cdot I} \tag{1}$$

Where $L = 2 \cdot l_0$ is the initial length of the double-beam, E is the material Young modulus and I is the inertia of the profile. This equation gives the first linear part of the curve represented in *Fig. 5*.

2.2 Second order rigid-plastic behaviour

The three plastic hinges appear for a force $P_{pl} = 4 \cdot M_{pl} / l = 246\ kN$, where M_{pl} is the plastic resistant moment of the beam cross-section. After the formation of the plastic mechanism, the displacement rapidly increases and the second order effects become important. In particular, significant tension forces develop in the beams implying the increase of the system stiffness and the decrease of the deformation rate.
The second part of the analytical curve is established using the developments presented in [3]. They are based on a second-order rigid-plastic analysis and lead in the present case to the following relations between the tension load in the beams N_{Rd}, the applied force P, and the displacement u:

$$N_{Rd} = \frac{K_N \cdot \left[\sec(\theta) \cdot \left(2 \cdot K \cdot L + P \cdot \mathrm{tg}(\theta)\right) - 2 \cdot K \cdot L\right]}{8 \cdot K + 2 \cdot K_N \cdot \sec^2(\theta)} \tag{2}$$

$$P = \cos^2(\theta)\cdot\Big[N_{Rd}\cdot\mathrm{cosec}(\theta) - K\cdot L\cdot\mathrm{cotg}(\theta)$$
$$+\frac{\mathrm{cotg}(\theta)\cdot\sqrt{\sec^4(\theta)\cdot\left(2\cdot K^2\cdot L^2 + N_{Rd}^2 - 4\cdot K\cdot L\cdot N_{Rd}\cdot\cos(\theta) + N_{Rd}^2\cdot\cos(2\theta) + 16\cdot K\cdot M_{Rd}\cdot\sin(2\theta)\right)}}{\sqrt{2}} \tag{3}$$
$$-K\cdot L\cdot\mathrm{tg}(\theta) + 2\cdot N_{Rd}\cdot\sec(\theta)\cdot\mathrm{tg}(\theta)\Big]$$

$$u = \left(\frac{L}{2} + 2\cdot\delta_N\right)\cdot\sin(\theta) = \left(\frac{L}{2} + 2\cdot\frac{N_{Rd}}{K_N}\right)\cdot\sin(\theta) \tag{4}$$

In these expressions, δ_N is the elasto-plastic elongation of half a beam and K_N is the extensional stiffness associated to half a beam when yielding extends along the beams due to the increase of the tension load after the formation of the plastic mechanism. No analytical method exists to determine the value of this parameter yet; it was numerically estimated at K_N = *56200 kN/m*. The resistant moment of the plastic hinges M_{Rd} correlates with the internal tension force N_{Rd} to fulfil the profile cross-section plastic resistance under M-N interaction.

2.3 Full membrane behaviour

When the tension load in the beams reaches the value of the plastic resistance N_{pl}, the system resists through catenary actions only. The beams plastically elongate under a constant tension force N_{pl} and the bending moments are negligible. In [2], the equation of the limit curve defining the last part of the static response is established using force equilibrium and geometrical considerations:

$$u = P\cdot\left(\frac{l_0}{2\cdot N_{pl}\cdot\sqrt{1-\left(\frac{P}{2\cdot N_{pl}}\right)^2}} - \frac{1}{4\cdot K}\right) \tag{5}$$

2.4 Global analytical static curve and comparison to numerical results

The complete curve representing the static relation between the vertical displacement at mid-length of the system and the applied force is analytically established from the combination of the three successive behaviours described in 2.1, 2.2 and 2.3. It is depicted in *Fig. 5* where it is also compared to the numerical response of the substructure.

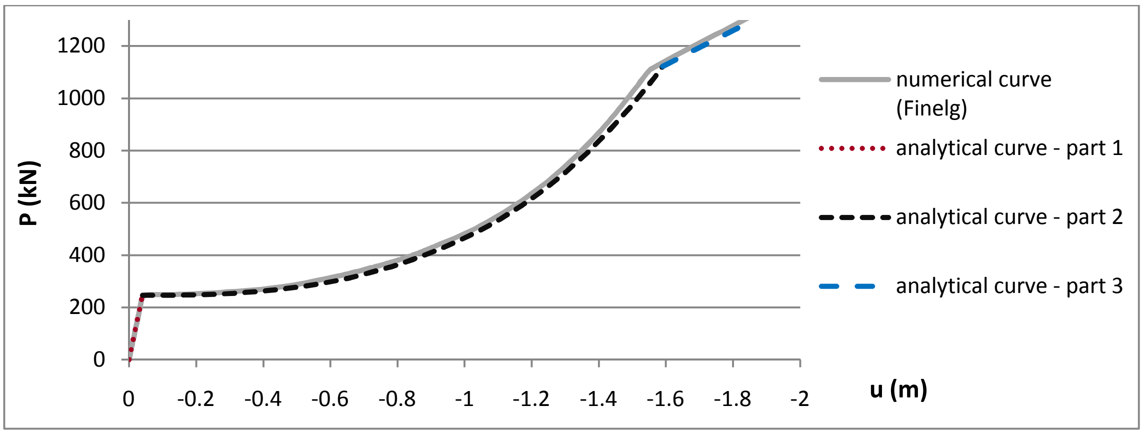

Fig. 5. Static curve (P,u): comparison of the analytical and numerical results

3 DYNAMIC RESPONSE OF THE SUBSTRUCTURE

3.1 Dynamic maximum displacement caused by an instantaneous load

The curve providing the maximum displacement reached if the force P is applied instantaneously ($t_r = 0$), called pseudo-static curve, can easily be established using the nonlinear static curve, following a procedure developed at Imperial College London [5]. This method is based on energetic considerations. When the maximum displacement is reached, the external work W is equal to the internal strain energy U that can be estimated as the area under the static curve until the value of the maximum displacement u_{max}. Thus, for any value of u_{max}, the level of the instantaneous load P causing this maximum displacement can be deduced from *Eq. (6)* and eventually the pseudo-static curve can be entirely derived from the static one (*Fig. 6*).

$$W = U \quad \Leftrightarrow \quad P \cdot u_{max} = \int_0^{u_{\max}} P(u)_{stat} \cdot du \tag{6}$$

3.2 Dynamic behaviour according to the loading conditions

Several dynamic analyses were numerically achieved (using Finelg [6]) considering the substructure subjected to different loading conditions (P, t_r). The maximum displacement u_{max} was recorded for each of them in order to establish curves providing u_{max} as a function of the applied force P for different values of t_r (constant along one curve). These curves are given in *Fig. 6*. The static and pseudo-static curves are also represented on this graph; they form the upper (static curve, $t_r \to \infty$) and lower (pseudo-static curve, $t_r = 0$) boundaries between which the other curves lie. A simplified analytical procedure is proposed in 3.4 to approach such curves (P, u_{max}) corresponding to given values of t_r. This method is based on the model presented in 3.3.

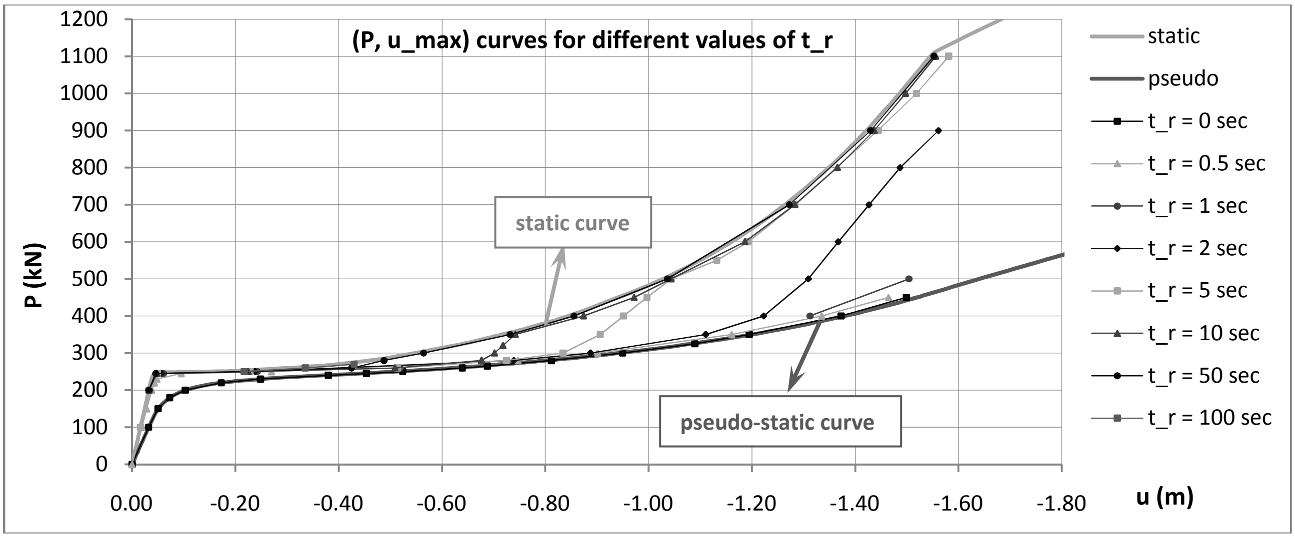

Fig. 6. Maximal dynamic displacement according to the value of the load and its rise time

The different kinds of behaviour that can be observed according to the loading conditions (P, t_r) are summarised in [1]. In particular, the time evolution of the displacement, yielding and internal forces are briefly described and explained. More details can be found in [2].

3.3 Simplified model predicting the time evolution of the displacement

Fig. 7 shows examples of dynamic response of the considered system for t_r equal to 10 seconds and different values of P higher than the static plastic load P_{pl}. For each of them, the dynamic curve, providing the time evolution of the displacement $u_{dyn}(t)$, is compared to the static curve $u_{stat}(t)$, representing the evolution of the displacement if the dynamic amplification is neglected. Accordingly, $u_{dyn}(t^*)$ is the static displacement associated with the value of the applied load $P(t^*)$ at the time t^*.

It is observed that, after the formation of the plastic mechanism, $u_{dyn}(t)$ rapidly increases before it stabilises. Two types of response can be highlighted according that the displacement corresponding to this plateau ($u_{plateau}$) is greater or smaller than the static displacement associated to the final load P. In the first case (type 1), the plateau is infinite. In the second case (type 2), it continues until the dynamic curve meets the static one; then the dynamic displacement starts to increase again, oscillating around the static curve.

A model was developed in [2] to estimate the evolution of the dynamic displacement until the first maximum of the curve is reached after the development of the plastic mechanism. The rest of the dynamic curve can be approached by an infinite horizontal line for a response of type 1 and by a horizontal plateau followed by the last part of the static curve for a response of type 2. Such approximated curves obtained with this procedure are also represented in *Fig. 7*.

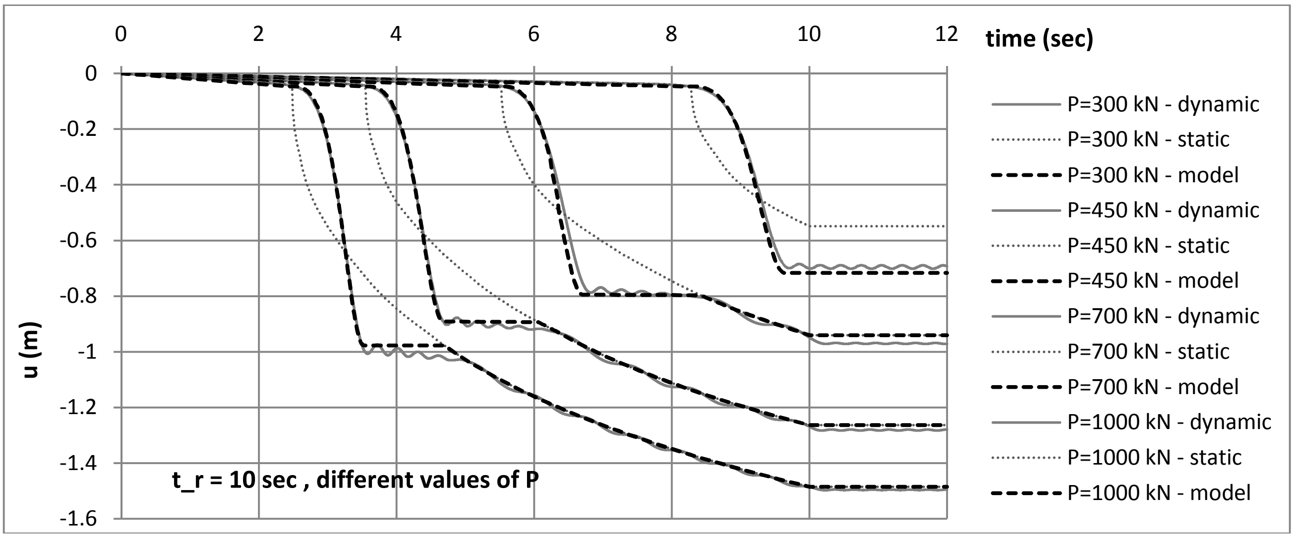

Fig. 7. Time evolution of the displacement: comparison to static response and simplified model

The model considers a rigid-plastic behaviour and is based on an energy equation expressing that the work done by the external force *P(t)* is equal to the sum of the kinetic energy, the work related to the plastic hinges rotation and the energy included in the lateral spring. Developing this equation and making a few conservative hypotheses [2], it leads to *Eq. (7)*, which is only valid until the first maximum of the curve *u(t)* is reached. In order to take account of the elastic deformations that are neglected in the rigid-plastic model, the displacement reached before the formation of the plastic mechanism is added to the curve deduced from *Eq. (7)*. The value of the first maximum of the obtained curve is $u_{plateau}$.

$$M_g \cdot \ddot{u}(t) + 4 \cdot \frac{M_{pl}}{l(u)} \cdot f\left(N\left(u, l_0\right)\right) + \frac{2 \cdot K}{l(u)^2} \cdot u^3(t) = P(t) \tag{7}$$

where:

- $M_g = 1/3 \cdot m \cdot 2 \cdot l_0 = 1/3 \cdot M_{tot}$ is the generalised mass of the system
- the plastic resistance of the profile section for M-N interaction is: $M_{pl,N}(N) = M_{pl} \cdot f\left(N\right)$
- $N(u) = \dfrac{K \cdot \delta_N}{\cos(\theta)} \approx \dfrac{K \cdot u^2 / l(u)}{1 - u^2 / \left(2 \cdot l(u)^2\right)} \approx K \cdot \dfrac{u^2}{l(u)}$ and $N\left(u, l_0\right) \approx K \cdot \dfrac{u^2}{l_0}$
- $l(u) \approx l_0 + 2 \cdot N\left(u, l_0\right) / K_N$

3.4 Analytical method to approach a curve (P,u_{max})

The final objective is not to determine the whole evolution of the dynamic displacement, but to evaluate the maximum displacement in order to predict the required deformation capacity and tension resistance of the structural members. In view of the shape of the (P,u_{max}) curves (*Fig. 6*), the concept is to approximate them, beyond the plastic plateau, using segmented curves (*Fig. 8*) established as follows [2]: segment 1 ≡ horizontal line at the level of P_{pl}; segment 2 ≡ pseudo-static curve; segment 3 ≡ vertical line between the pseudo-static and the static curve, at the abscissa named u_{trans} at which the actual (P,u_{max}) curve joins the static curve; segment 4 ≡ static curve.
To be able to draw such an approximate curve, the value of u_{trans} has to be determined. The point (P_{trans}, u_{trans}) at which the dynamic curve (P,u_{max}) associated with a given value of t_r joins the static curve corresponds to a transition between the two types of response previously described. Indeed, $u_{max} > u_{stat}$ for $P < P_{trans}$ (type 1) and $u_{max} \approx u_{stat}$ for $P > P_{trans}$ (type 2). As explained before, the behaviour type is governed by the value of $u_{plateau}$: type 1 corresponds to $u_{plateau} > u_{stat}(P)$ while type 2 is associated with $u_{plateau} < u_{stat}(P)$.

Consequently, the approximate dynamic curve $(P,u_{max})_{appr}$ corresponding to a fixed value of t_r can be established following this procedure: (i) evaluation of the displacement $u_{plateau}$ for different values of P and comparison with the static displacement $u_{stat}(P)$; (ii) identification of the force for which $u_{plateau}$ equals $u_{stat}(P)$: this value of the load is P_{trans}; (iii) deduction of $u_{trans} = u_{stat}(P_{trans})$ from the static curve; (iv) determination of the complete curve $(P,u_{max})_{appr}$. The model presented in 3.3 can be used to estimate $u_{plateau}$. As the final objective is the evaluation of P_{trans}, only responses relatively close to the intermediate situation between the two behaviour types are interesting. In such cases, the plateau always starts at a time $t_{plateau} < t_r$, which means $P(t) = P \cdot t / t_r$ can be used in *Eq. (7)*. Besides, it was shown that the proposed model gives very good results for the evaluation of $u_{plateau}$ in these situations (*Fig. 7*). The approximate curve obtained through this method for $t_r = 10$ *sec* is represented in *Fig. 8*, where it is compared to the curve determined using numerical simulations. The maximum unsafe error is about 3%.

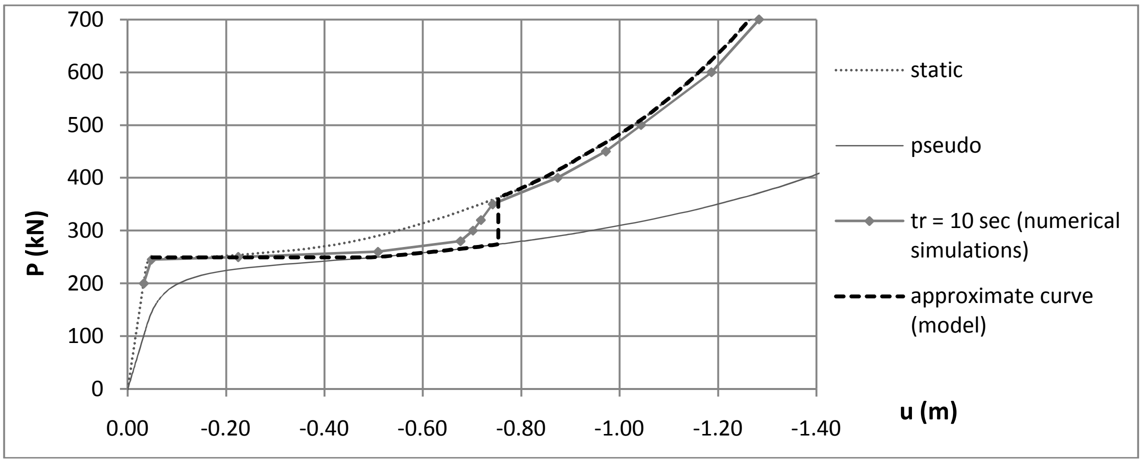

Fig. 8. Approximate dynamic curve $(P, u_{max})_{appr}$ established using the proposed model

Unfortunately, *Eq. (7)* has no analytical solution and numerical calculation has to be performed to determine *u(t)*. To obtain a full analytical procedure, an approximate solution giving the first maximum of the curve *u(t)* fulfilling *Eq. (7)* via an analytical expression would be needed.

4 CONCLUSIONS

In this paper, different analytical approaches mainly developed at the University of Liège were applied to predict the static and dynamic behaviour of a substructure composed of a double-beam further to the loss of its central support; and compared to numerical results. Detailed information about the development and validation of these analytical methods is available in [2] and [3].

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