Two-level Mixtures of Markov Trees

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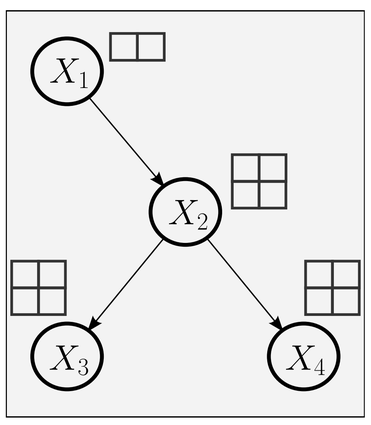
Bayesian Networks efficiently encode a probability distribution on a large set of variables but their **poor scaling** in terms of the number of variables may make them unfit to tackle learning and inference problems of increasing size. **Mixtures of Markov trees** however scale well by design. We investigate whether the two approaches to **learn such mixtures from data** (maximum likelihood and variance reduction) can be combined together by building a **two level Mixture** of Markov Trees. Our experiments on synthetic data show the interest of this model.

Markov Tree T :

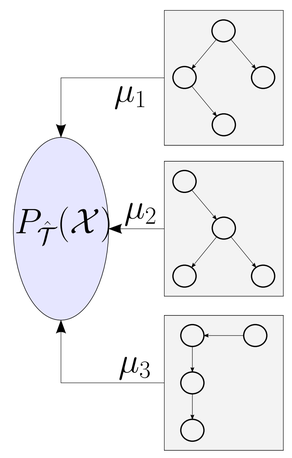


- A class of Bayesian Network.
- No cycle, each variable has only one parent.
- Encodes a joint probability distribution over n variables $\mathcal{X}$:

$$P_T(\mathcal{X}) = \prod_{i=1}^{n} P(X_i | Pa_{\mathcal{G}}(X_i)) \ .$$

- Learning from a data set is $\mathcal{O}(n^2 \log(n))$ (Chow-Liu algorithm).
- Inference is $\mathcal{O}(n)$.

Mixtures of Markov trees :



- Composed of a set $\hat{\mathcal{T}} = \{T_1, \ldots, T_m\}$ of m elementary Markov Tree densities and a set $\{\mu_i\}_{i=1}^m$ of weights.
- Convex combination of tree predictions :

$$P_{\hat{\mathcal{T}}}(\mathcal{X}) = \sum_{k=1}^{m} \mu_k P_{T_k}(\mathcal{X}) \ .$$

Key points:

- Trees → efficient algorithms.
- Mixture → improved modeling power.

There are 2 approaches to learn such mixtures from data :

Maximum-likelihood [1]

- Learning the mixture is viewed as a global optimization problem aiming at maximizing the data likelihood.
- Can be solved (for a fixed m) using the EM algorithm by iteratively :
  - optimizing μ based on $\mathcal{T}$,
  - optimizing a soft partition the data set based on $\mathcal{T}$ and μ,
  - optimizing each $T \in \mathcal{T}$ on a part by the Chow-Liu algorithm.

Variance reduction [2]

- This approach can be viewed as an approximation of Bayesian learning in the space of Markov Tree structures.
- A sequence of trees is generated by a randomized Chow-Liu algorithm :
  - pure random structure,
  - edge subsampling,
  - bootstrapping...

We attempt to combine both.

Motivation:

- Variance reduction methods are good on low samples sets.
- Maximum-likelihood methods partition the data set.

→ It seems natural to combine them.

Concept:

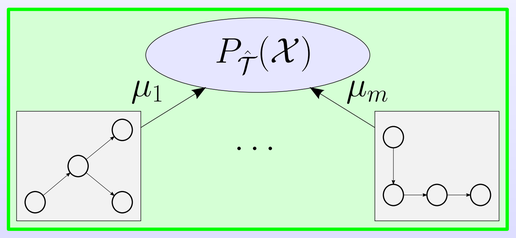
Replacing in the ML mixture the Chow-Liu algorithm by a variance reduction mixture learning algorithm:

$$P_{\hat{\mathcal{T}}}(\mathcal{X}) = \sum_{k=1}^{m} \mu_k P_{\hat{\mathcal{T}}_k}(\mathcal{X}) \ ,$$

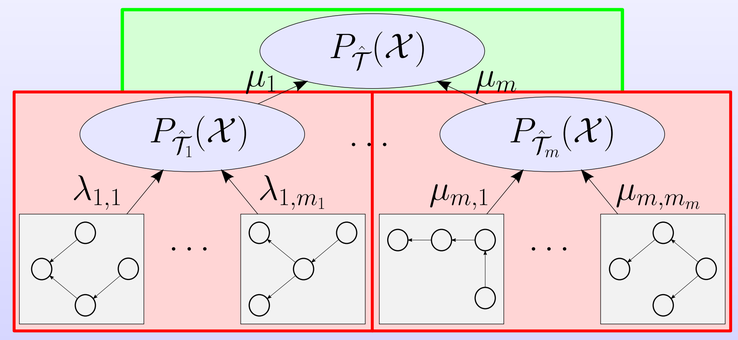
$$P_{\hat{\mathcal{T}}_k}(\mathcal{X}) = \sum_{j=1}^{m_k} \lambda_{k,j} P_{T_{k,j}}(\mathcal{X}) \ \ \forall k \ .$$

Algorithm tested:

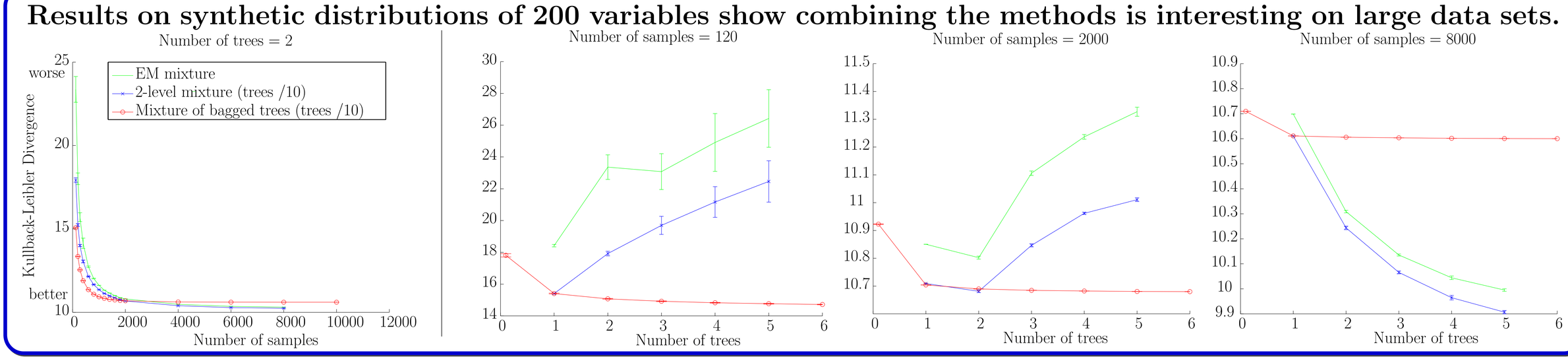
1 : build an EM mixture



2 : replace each tree by a mixture of 10 bagged trees



Results on synthetic distributions of 200 variables show combining the methods is interesting on large data sets.



References

[1] Meila, M., Jordan, M.: Learning with mixtures of trees. JMLR 1, 1–48 (2001)

[2] Ammar, S., Leray, P., Schnitzler, F., Wehenkel, L.: Sub-quadratic Markov tree mixture learning based on randomizations of the Chow-Liu algorithm. In: The Fifth European Workshop on Probabilistic Graphical Models. pp. 17–24 (2010)

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