University of Wisconsin - Madison

MAD/PH/376
UH-511-636-87
October 1987

$B - \bar{B}$ MIXING DOES NOT EXCLUDE $m(\text{top}) < \frac{M_Z}{2}$

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ABSTRACT

We show that claims that the top is heavy (typically heavier than 55 GeV) derived from the standard model analysis of $B - \bar{B}$ mixing and the ϵ parameter of the $K - \bar{K}$ system, arise from unproven assumptions and the lack of consideration of the statistical errors on the KM matrix elements as well as on all input data. *E.g.* for the conservative case that $B_B = 1$ and $f_B = 175$ MeV, we obtain

$$m_t > 35 \text{ GeV at the } 1\,\sigma \text{ level}$$

and

$$m_t > 30 \text{ GeV at the } 1.5\sigma \text{ level}.$$

An f_B of 135 MeV only raises the 1.5σ limit to 35 GeV.

It should be stressed that this statistic ignores further theoretical ambiguities related to the calculation of ϵ, the QCD corrections to $\Delta M/\Gamma, \ldots$ Low top masses imply however a characteristic structure of the KM-matrix which we discuss in some detail.

It is straightforward to show that the ARGUS[1] observation of $B(b\bar{d}) - \bar{B}(\bar{b}d)$ mixing

$$\frac{\Delta M}{\Gamma} = 0.73 \pm 0.18 \tag{1}$$

favors a heavy top mass. In the standard model with 3 generations (1) has to be accommodated by the $\Delta B = 2$ box diagram with exchange of the top quark. In a somewhat simplified form[2]

$$\frac{\Delta M}{\Gamma} = \left[\frac{G_F^2 m_B \tau_B}{6\pi^2}\right] [f_B^2 B] \, |U_{tb}^* U_{td}|^2 m_t^2 \, . \tag{2}$$

The proportionality constant relating $\frac{\Delta M}{\Gamma}$ to m_t^2 contains several factors. The first factor contains measured quantities: the weak coupling and the B-meson mass and lifetime. The second factor describes the binding of $b\bar{d}$ quarks in B-mesons and cannot be computed perturbatively. The third factor is made up of Kobayashi-Maskawa (KM) matrix elements connecting t to b and d. As the top has not been observed they must be inferred from the unitarity of the KM matrix.

It is easy to illustrate how (1) and (2) prefer large values of m_t. Let us assume, with some authors, a symmetric form of the KM matrix with $|U_{tb}| \simeq 1$ and $|U_{td}| \simeq |U_{ub}|$. U_{ub} is constrained by the experimental result[3] that $0.07 < |U_{ub}/U_{cb}| < 0.2$, where U_{cb} is fixed by the b-lifetime. Depending on the assumed values of f_B and B we will conclude that m_t cannot be lighter than (typically) 50 GeV. This is a disastrous result for TRISTAN and SLC experiments. They do not have the beam energies to reach $t\bar{t}$ threshold for such large t-masses.

It is therefore crucial and somewhat less straightforward to ask the question what the significance is of this limit when one includes experimental errors on all input quantities and one does not introduce unproven assumptions about U_{td}. This requires using unitarity of the KM matrix only and reconsidering the determination of its matrix elements, including their errors, ab initio.

We have performed this exercise as charted in Tables 1a and 1b. In Table 1a we list all input experimental data and their errors. We fit $\Delta M/\Gamma$ and a variety of data fixing the KM matrix elements from which U_{tb} and U_{td} are derived by unitarity. We include in our fit the ϵ parameter describing CP violation in the $K - \bar{K}$ system. It also constrains the mass of the top as well as KM elements via top-exchange in the $\Delta S = 2$ box diagram relating K to $\bar{K}$. This set of data is fitted simultaneously to all parameters in the problem (listed in Table 1b) for different values of m_t. One soon realizes that including all errors (Table 1a) and varying all parameters (Table 1b) simultaneously introduces considerable freedom in the problem.

To illustrate this we start with the conservative assumption that $B = 1$ and $f_B = 175$ MeV, *i.e.* $f_B^2 B \leq 0.03$. We return later to a discussion of how varying the poorly known factor $f_B^2 B$ affects one's conclusions. The probability that a given value of m_t is consistent with all data in Table 1a (including $\Delta M/\Gamma$ given by Eq. (1)) is shown in Fig. 1. Like previous analyses we conclude that $m_t < 40$ GeV is disfavored. Notice, however, that it seems impossible to make a case for values in the $55 \sim 90$ GeV range. The more important result is, however, that the reduced probability in the $25 \sim 40$ GeV mass region is statistically hardly significant. To see this let us consider our solution for $m_t = 35$ GeV in some detail. The KM matrix results are as follows:

$$\begin{bmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{bmatrix} = \begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} 0.975 & 0.221 & 0.014 \\ 0.219 & 0.973 & 0.070 \\ 0.030 & 0.067 & 0.997 \end{bmatrix} . \tag{4}$$

Listed in (4) are the absolute values of the matrix elements. The values of the parameters are $s_1 = 0.222$, $s_2 = 0.134$, $s_3 = 0.065$, $\delta = 3.056$ rad, and $\Lambda_{\rm QCD} = 0.29$, $m_c = 1.8$ GeV, $m_b = 4.6$ GeV, $f_K = 0.172$, and $B_K = 0.88$. As previously mentioned $f_B = 0.175$ ($\simeq f_K$) and $B_B = 1$ ($\simeq B_K$). Every data point in Table 1a is individually fit to better than $1\,\sigma$. The worst fit is a $1\,\sigma$ deviation from the central value of $\Delta M/\Gamma$ given by (1). The results for $\Delta M/\Gamma$ at this and other m_t-values is shown in Fig. 2. Our overall conclusions are then

$$\begin{aligned} m_t &> 35 \text{ GeV at } 1\,\sigma \text{ level}\,, \\ &> 24 \text{ GeV at } 2\,\sigma \text{ level}\,, \end{aligned}$$

allowing any value above the TRISTAN limit of 25 GeV at the $2\,\sigma$ level. We find that even for $f_B\sqrt{B}$ as low as 135 MeV, an m_t of 35 GeV can be fitted at $1.5\,\sigma$. It should be stressed that our statistic ignores further theoretical ambiguities related to the calculation of ϵ, the QCD corrections to $\Delta M/\Gamma$, ... Our inclination is to view this exercise from a totally different perspective: the observation of $B - \bar{B}$ mixing provides information on the quantity $f_B^2 B$ once m_t will be determined experimentally. This is illustrated in Fig. 3 where we show the relation between $f_B^2 B$ and $|U_{td}|$ for $m_t = 30$ and 35 GeV. The band reflects the experimental errors on the input. For a 30 GeV top mass our solution for $|U_{td}|$ can be as large as 0.032 within $1\,\sigma$ (best fit 0.030) and therefore $f_B\sqrt{B}$ can be as small as 0.17.

Notice that for slightly larger values of $f_B\sqrt{B}$ much smaller values of $|U_{td}|$ can be accommodated for $m_t = 30$ GeV.

It is instructive to look at the somewhat unusual structure of our KM matrix. $|U_{td}|$ can be expressed as

$$|U_{td}| = |U_{ub}| \left| \left\{ -c_\delta + \sqrt{c_\delta^2 - 1 + \sin^2\theta_c |U_{cb}|^2/|U_{ub}|^2} \right\} \right| \tag{5}$$

in the approximation that all mixing angles are small. Furthermore, for $c_\delta \approx -1$,

$$|U_{td}| \approx |U_{ub}| \left(1 + \sin\theta_c |U_{cb}|/|U_{ub}|\right) \; . \tag{6}$$

For values of $|U_{ub}|/|U_{cb}|$ saturating the upper bound of ~ 0.2, we obtain

$$|U_{td}| \approx 2\,|U_{ub}| \, . \tag{7}$$

Clearly this KM matrix is not symmetric as

$$|U_{cb}| \simeq 5\,|U_{ub}| \simeq \frac{5}{2}\,|U_{td}| \, . \tag{8}$$

The fit accommodates low values of m_t by maximizing U_{td} as is obvious from inspection of Eq. (2). All of this is done, of course, within the constraints of the experimental input of Table 1a.

The KM matrix found by the fitting program is clearly obtained by maximizing U_{td} with respect to U_{ub} and U_{cb} which is itself fixed by the b-lifetime. A small value of $m_b = 4.6$ GeV further helps to maximize U_{cb} itself. We have checked that with the values of m_b, m_c, and Λ_{QCD}, previously listed for $m_t = 35$ GeV and characteristic for fits at other low m_t-values, we can reproduce the leptonic decay spectrum of B-mesons in the model of Altarelli *et al.*[4] This is not surprising as the model certainly contains further theoretical ambiguities (*e.g.* exponentiation of divergent graphs) and more free parameters foremost the u-quark mass and the Fermi momentum of quarks in mesons which we fit to be 0.14 and 0.7 GeV, respectively. We have also estimated bounds on $|U_{cb}|$ from the recently measured branching ratio of $7.0 \pm 1.2 \pm 1.9\%$ for the decay mode $B^0 \to D^{*-} e^+ \nu_e$ by the ARGUS collaboration.[3] We used the models of Shifman and Voloshin[5] as well of Grinstein and Wise[8] and in both cases the bound is weaker than 0.071.

The conclusion of this paper is not that the top is light, only that the case against a light top based on Eq. (1) is weak.

ACKNOWLEDGEMENTS

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Department of Energy under contracts DE-AC02-76ER00881 and DE-AN03-76SF00235.

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2. In Eq. (2) we have not shown the short distance QCD correction factor, the exact dependence on m_t/m_W and the correction due to non-zero external masses (proportional to m_B/m_t). In our actual calculations all these are taken into account. We estimate the effect of intermediate states other than $t\bar{t}$ (*e.g.* $c\bar{t}$, $c\bar{c}$, *etc.*) in the box diagram to be less than 1% and neglect them. We use results from following sources: J. Hagelin, Nucl. Phys. 193, 123 (1981); F. Franco, M. Lusignoli, and A. Pugliese, Nucl. Phys. B194, 403 (1982); T. Inami and C. S. Lim, Progr. Theor. Phys. 65, 297 (1981); M. I. Vysotsky, Yad. Fiz. 31, 1535 (1980); A. Buras, W. Slominski, and H. Steger, Nucl. Phys. B245, 369 (1984). For QCD-corrections to ϵ and τ_B we used F. Gilman and M. B. Wise, Phys. Rev. D 27, 1128 (1983) and J. L. Cortes, X. Y. Pham, and A. Tounsi, Phys. Rev. D 25, 188 (1982), respectively.

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Table 1a. Input data and experimental errors.

quantity	mean value	error
$\mid U_{ud} \mid$	0.9744	0.001
$\mid U_{us} \mid$	0.220	0.002
$\mid U_{cd} \mid$	0.207	0.024
$\mid U_{cs} \mid$	0.95	0.14
$\frac{\lvert U_{ub}\rvert}{\lvert U_{cb}\rvert}$	≤ 0.20	–
B_{sl}	0.11	0.01
τ_B (10^{-12}s)	1.16	0.14
m_B (GeV)	5.2752	0.0028
m_W (GeV)	81.8	1.5
ϵ (10^{-3})	2.275	0.021
$\frac{\Delta M}{\Gamma}$	0.73	0.18

Table 1b. Parameters of the problem and their range of variation.

parameter	minimum value	maximum value
s_1	0.21	0.23
s_2	0.	0.15
s_3	0.	0.1
δ	0.	2π
Λ_{QCD}	0.1	0.4
m_c	1.1	1.8
m_b	4.6	5.2
m_t	20	100
B_K	0.3	1
$f_B^2 \times B_B$	0.01	0.03

FIGURE CAPTIONS

Fig. 1 Maximum probability density of the fit to the data in Table 1a as a function of the top quark mass, leaving all other parameters in Table 1b free.

Fig. 2 Value of $\Delta M/\Gamma$ for the fits in Fig. 1. The data point drawn at an arbitrary value of m_t, corresponds to Eq. (1).

Fig. 3a Relation (with 1.5σ errors) between $\sqrt{B}f_B$ and $|U_{td}|$ implied by the constraint of Eqs. (1) and (2). The horizontal line is our maximum value of $\sqrt{B}f_B$ in previous fits. Vertical lines represent the maximum value of $|U_{td}|$ for 3 generations and for any number of generations.

Fig. 3b Same as Fig. 3a for $m_t = 35$ GeV. Error is 1σ.

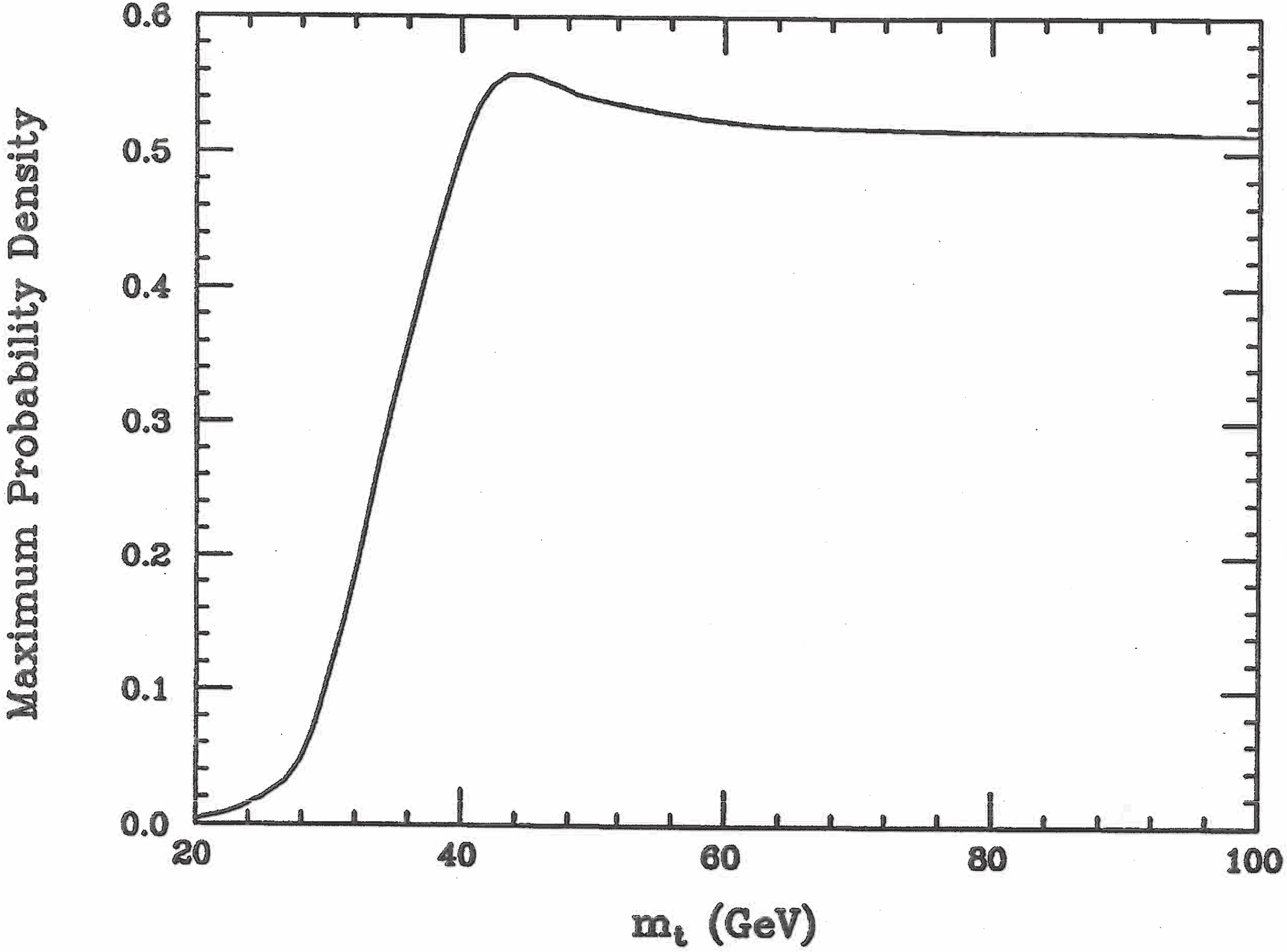
Maximum Probability Density
0.6
0.5
0.4
0.3
0.2
0.1
0.0
20
40
60
80
100
m_t (GeV)

Fig. 1

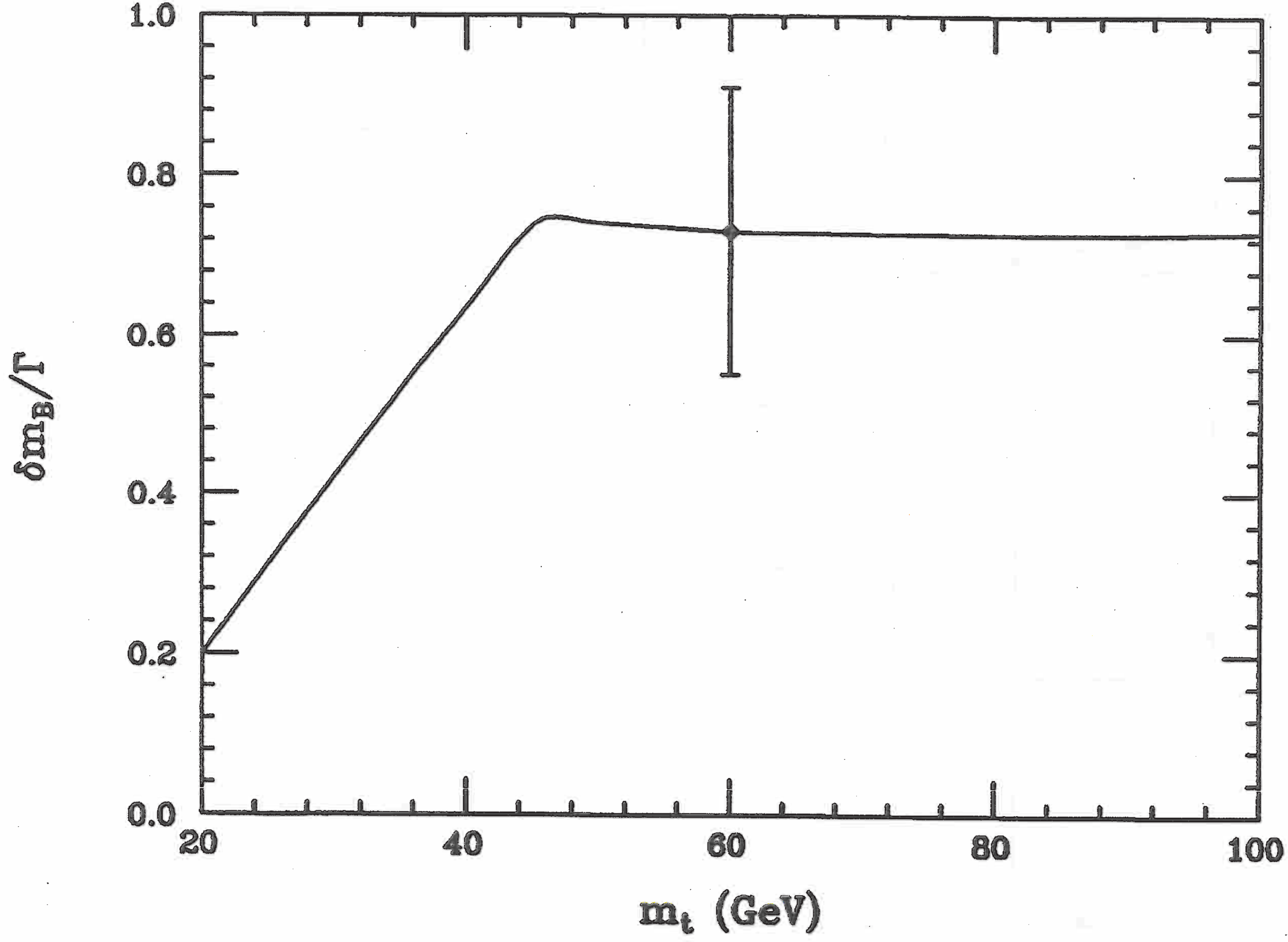

Fig. 2

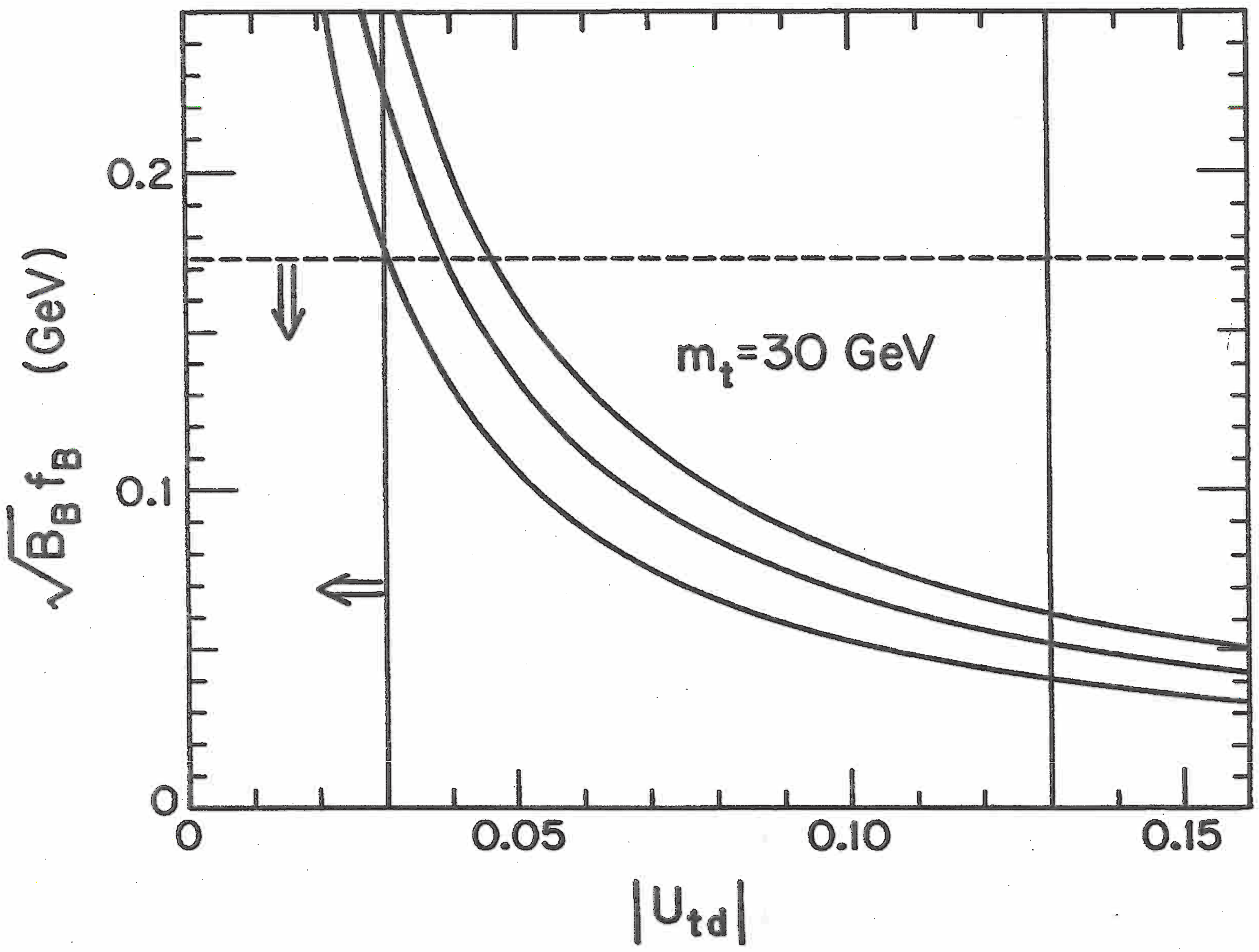

Fig. 3a

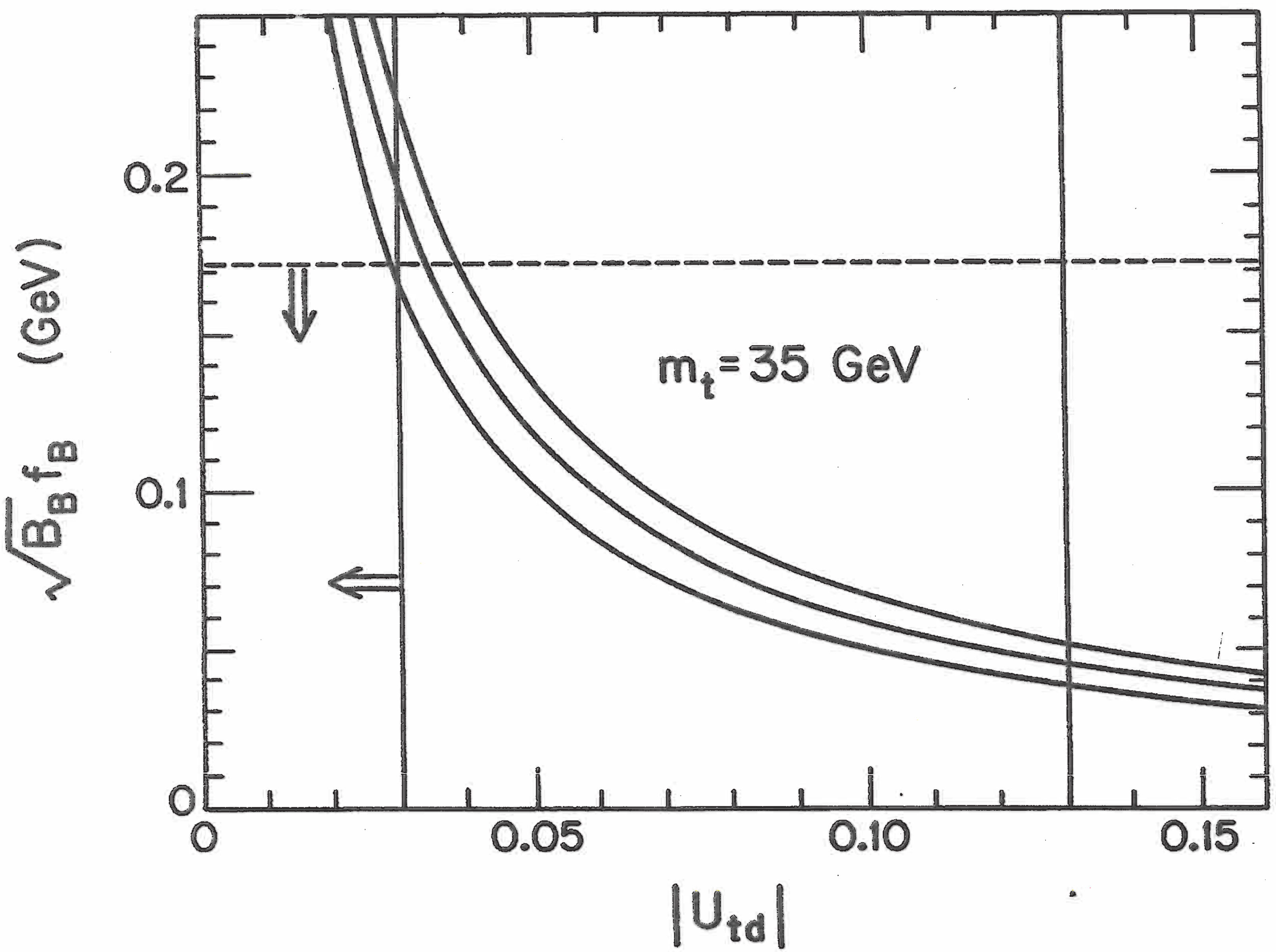

Fig. 3b