TEMPORAL EVOLUTION AND EXTREME VALUE ANALYSIS OF PRECIPITATIONS IN BURKINA FASO

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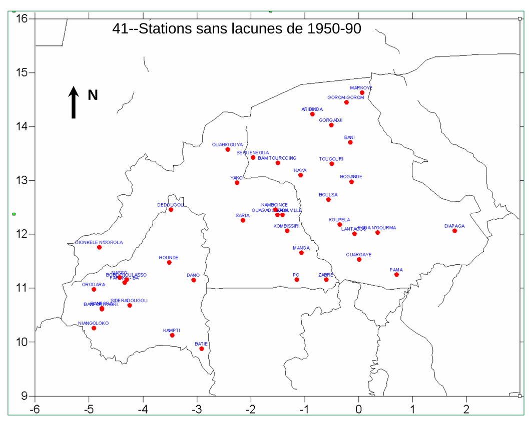
Introduction

The study presented here makes use of 41 daily rain gauges of Burkina Faso retrieved from the West Africa daily rainfall database Tapsoba (1997) from 1950 to 1990. These daily rain gauges have been chosen to maximize spatial uniformity of coverage and temporal continuity. The data were examined for continuity and missing records. A spatio temporal study shows that the decline is uniform on the region.



Ouagadougou severe event

Nevertheless in 2009 a daily rainfall event of 263mm has been recorded, causing dramatic damages in the Burkina capital city Ouagadougou. Such an event is more than twice the magnitude of previously observed maximum. A standard extreme value analysis performed on our dataset indicates that this event should be very unlikely. We investigate possible causes of such a gap.

Generalized Extreme Values

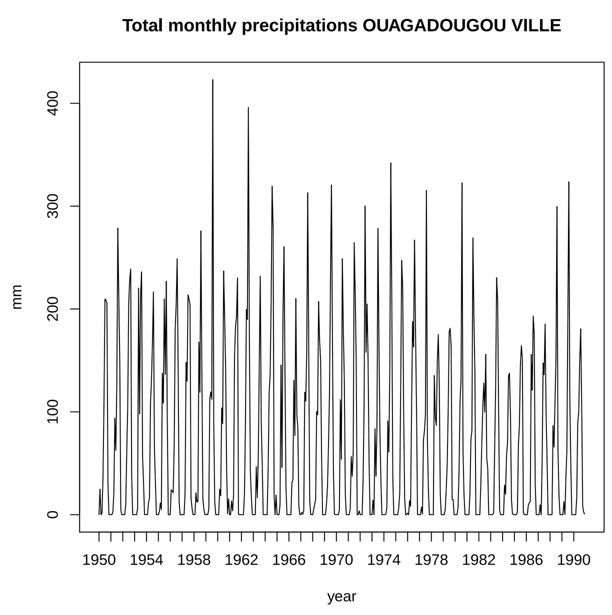
Extreme value theory

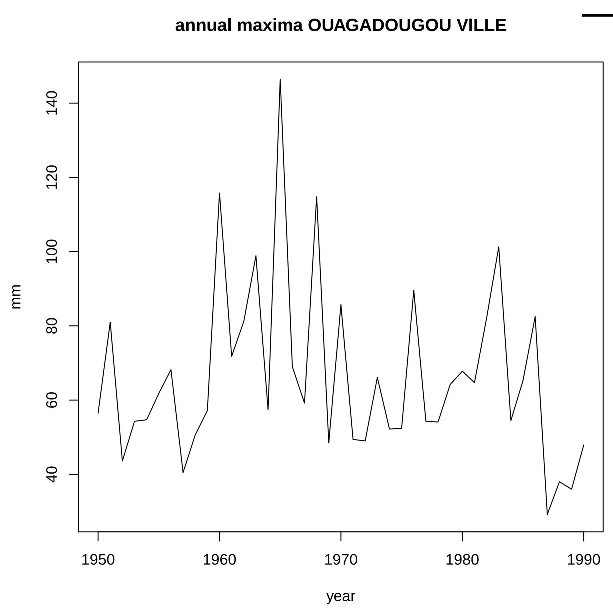
The GEV family of distribution function

$$G(z) = \begin{cases} \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]_+^{-1/\xi}\right\} & \text{if } \xi \neq 0 \\ \exp\left\{-\exp\left[-\left(\frac{z-\mu}{\sigma}\right)\right]\right\} & \text{if } \xi \to 0 \end{cases}$$

are limiting approximations to the distribution of $M_n = \max(X_1, \ldots, X_n)$ linearly rescaled as $n \to \infty$, where the X_i are i.i.d. random variables.
z_p such that $G(z_p) = 1 - p$ is the return level associated with the return period $-1/\log(1-p) \simeq 1/p$ for small p in unit of years if the GEV corresponds to the annual maximum.

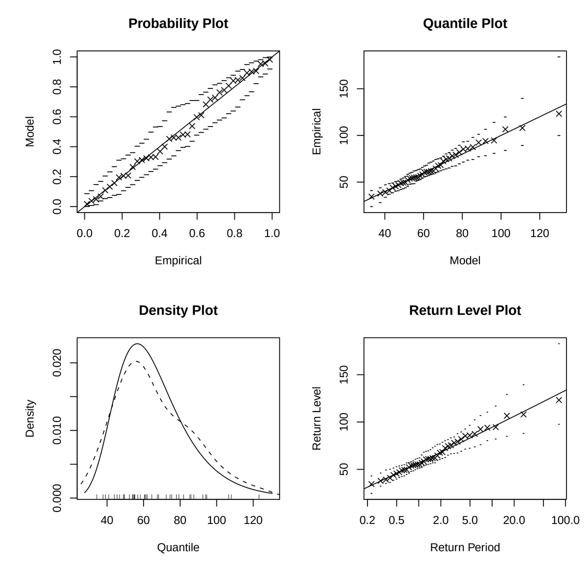
Ouagadougou series





Parameter estimation

	loc μ	scale σ	shape ξ
Estimates	56.77594	16.11875	0.01380
Standard Errors	2.9253	2.1866	0.1430



The return level $z_p = 260mm$ corresponds to a return period of 100 000 years!

Spatial extremes

Spatial max-stable process

A stochastic process $Z(\cdot)$ with continuous pathes is called max-stable if

$$\max_i \frac{Z_i(\cdot) - a_n(\cdot)}{b_n(\cdot)} \stackrel{d}{=} Z(\cdot)$$

$Z_i(\cdot)$ independent replications of $Z(\cdot)$, $a_n(\cdot)$, $b_n(\cdot)$ continuous functions.

Extremal coefficient

For Z max-stable process, Fréchet margins, the extremal coefficient θ_N is

$$P(Z(s_1) < z, \ldots, Z(s_N) < z) = \exp\left(-\frac{\theta_N}{z}\right)$$

$\theta_N = N$, if and only if margins are independent.
The extremal coefficient function for a stationary max stable process $\exp\theta(h) = P(Z(s+h) < z, Z(s) < z)$ characterizes the asymptotic spatial dependence of the process.

Spatial max stable models

Brown Resnick process (Kabluchko *et al.*)

$$Z(s) = \max_i \xi_i \exp\left(W_i(s) - \sigma^2(s)\right) \qquad s \in \mathbb{R}^2$$

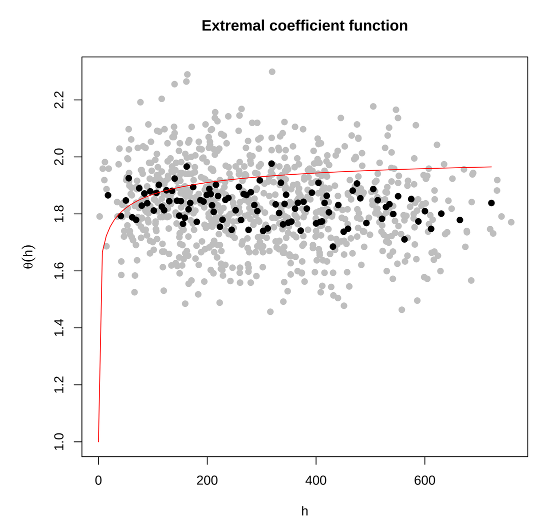
ξ_i Poisson point process of intensity $\xi^{-2}d\xi$ on $\mathbb{R}^+$, W_i independent copies of a zero mean Gaussian process with stationary increment and $\mathrm{Var}(W(s)) = \sigma^2(s)$.
Extremal coefficient function

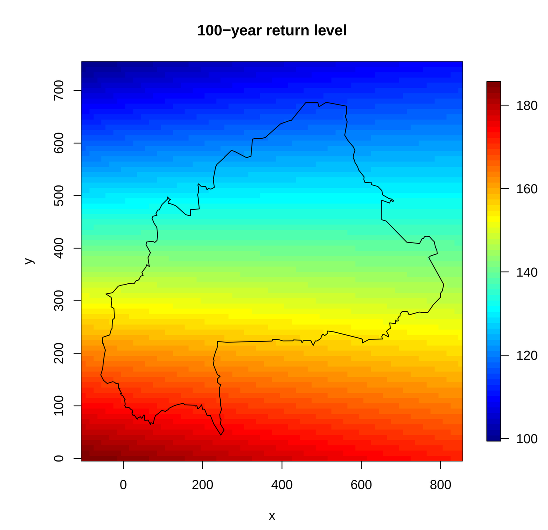
$$\theta(h) = 2\Phi\left(\sqrt{\frac{\gamma(h)}{2}}\right)$$

Φ is the standard normal cumulative distribution function, γ is the W process semi-variogram function.

Modeling the Burkina data

Linear model on the coordinates for the GEV parameters
Variogram $\gamma(h) = \left(\frac{\|h\|}{\lambda}\right)^\alpha$
Maximum composite likelihood estimator.

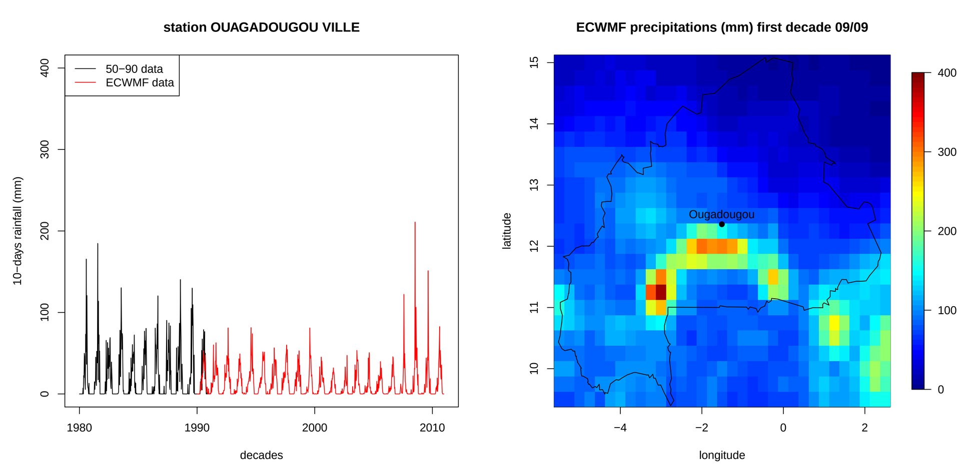




The return level $z_p = 260mm$ corresponds to a return period of 29 000 years!

ECWMF data

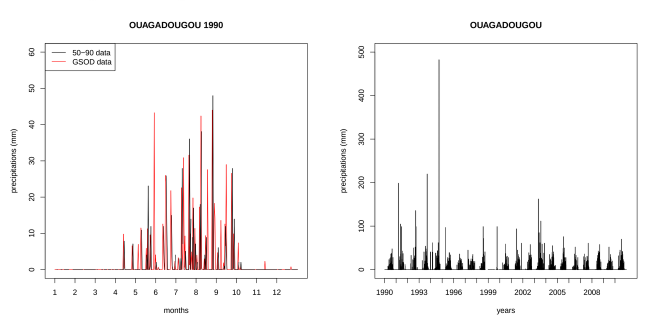
The ECMWF (European Centre for Medium-Range Weather Forecast) provides 10-daily and monthly outputs of global circulation model. They are reanalysis time series.



These data are consistent with our 1950-1990 database. They show no severe event in Ouagadougou for that period but a spot 250km away from Ouagadougou.

GSOD data

The National Climatic data Center provides Global Surface summary Of Day data (GSOD) over 9000 Worldwide Stations. Daily precipitations are available for 10 stations from Burkina, including Ouagadagou.



The GSOD data coincides with our 1950-1990 database for year 1990. This data show no flood event in September 2009 in Ouagadougou, neither in the 9 other stations. But there is one severe event in 2004.

A GEV study on the maxima series gathered from the two database give this time a return period corresponding to the return level 260mm equal to 72 years.

Concluding remarks

This study shows that assessing future levels of extreme rainfall from collected data is a very difficult task. An important issue is the reliability of past data: a severe flood as the september 2009 in Ouagadougou one, does not figure in databases. Models of extreme have to take into account sufficiently large periods and all information available as spatial asymptotic dependence.

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