Analytic Models for the Forward Scattering Amplitude at High Energies[a]

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We present the results of the recent comparative study by the Brown-COMPAS-Liège collaboration [1,2] on the several analytic models for the high energy forward scattering amplitude and their fits to all available data of the total cross sections and real part of the hadronic amplitudes accumulated in the PPDS data bases [3]. The stability of the parameters with respect to the energy cut-offs is used as a validity criteria [4]. It is hoped that the comparative study of several competing models would shed light on the nature of the Pomeron and where the model with a simple bare Pomeron begins to appreciably deviate from the models with a unitarized Pomeron. It is found that the data can not differentiate the model with a bare Pomeron from other models with a unitarized Pomeron. Surprisingly, the properties of factorization and quark counting rule are satisfied more or less to the same accuracy by all these competing analytic amplitude models.

I Introduction

The problem of universal description of forward scattering by hadrons and of rising total cross section have been with us unsolved over two decades[5,6]. The interest in this topic has been revived by the recent activities at HERA on deep-inelastic and diffractive scattering. The rising total cross section in energy had been discussed theoretically in connection with the rigorous proof of the Pomeranchuk theorem. The total cross section can not exceed the Froissart bound, i.e., $log^2 s$ asymptotically, s being the square of the C.M. energy. From the unitarity the total cross section is related to the imaginary part of the forward scattering amplitude and the analyticity relates the real part to the imaginary part of the scattering amplitude.

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The simplest idea for the slow rise with energy of the hadronic total cross sections is to assume a Regge trajectory with an intercept slightly larger than 1, having quantum members of vacuum, *i.e.* the Pomeron as a simple Reggeon with $\alpha_P(0) = 1+\varepsilon$, ε being a small positive number. This approach can predict the universal rise with energy of the total cross sections, the factorization of the Pomeron coupling and the quark counting based on the additive coupling of the Pomeron to the constituent quarks of hadrons. In fact our experience of several decades [9,8] has proven that the Pomeron is a simple Regge pole to a good approximation. The soft Pomeron intercept is a crucial element in HERA analyses and provides the starting point at low x and low Q^2, from which QCD evolution can be performed. It also determines the Pomeron flux factor, which enters gap cross section, and has a bearing on the extrapolation of total cross section to higher energies.

The idea to start from perturbative QCD assumes that a higher order effects would unitarize the amplitude and tame the fierce rise observed at large Q^2 to something compatible with the Froissart bound. But no one has reliably unitarized QCD amplitudes. Moreover such unitarization will involve necessarily multi-gluon exchanges between the quarks and therefore will require detailed quark structure, *i.e.,* the hadronic wave functions. In such scheme, one is likely to lose the factorization property of the Pomeron as well as the quark counting rules and the universality property [10]. Thus the simple pole model for the Pomeron remains to be the simplest and perhaps the most attractive option for near forward hadron scattering at high energies, even though this model does not offer a simple and automatic extension to the off-shell particle scattering and in particular to the deep inelastic scattering. A possible suggestion is to assume an extra "hard Pomeron" [11] that decouples at $Q^2 = 0$. Such extra trajectory is to some extent confirmed by the DGLAP evolution but the BFKL re-summation [12] of energy logarithms is generally believed to be relevant to the high energy hadron processes. It has recently been shown [13] that the next-to-leading order corrections can drastically lower the intercept of the BFKL Pomeron, leading to a very weak dependence on Q^2 result that can be consistent with the soft Pomeron behavior exhibited by the hadronic cross section. However this is achieved by a fine-tuned compensations between the leading order and next-to-leading order corrections in BFKL re-summation, which offers no simple clue for the off-shell extension.

Mainly because of the simplicity, the simple Regge pole idea has been pushed by Donnachie and Landshoff (DL) [9] to fit all soft data for scattering of on-shell particles even at non-zero t. The DL Regge model contains an exchange-degenerate meson Reggeon term in addition to the Pomeron term. However it was shown by Cudell, Kang and Kim (CKK) [4] that the assumption of an exchange-degenerate meson trajectory is not even supported by the fits to the total cross sections alone and fails to give stable parameters when fitted to the total cross sections and the real parts of the forward scattering amplitudes. Indeed, there is no reason to assume that the meson trajectories are degenerate from the data and the best fits for the meson mass spectrum. These trajectories can in fact mix together and with Pomeron, and the resulting intercepts are likely to be different. The simplest modification of the DL model suggested by CKK[4] that contains exchanges of two non-degenerate ρ/ω and f/a trajectories and is called the model RRP in our work[1].

Then to what extent can one say that the RRP model is a unique analytic possibility to describe all hadronic soft data ? It is clear that the Froissart bound of the unitarity and positivity of the imaginary part amplitude in the Lehmann ellipse, will be violated eventually by the model RRP with the bare Pomeron intercept $\alpha_P(0) = 1 + \varepsilon$, the model RRP will not be valid at high energies where the unitarization by multi-Pomeron exchanges is needed. Thus the validity region in energy of the model RRP will necessarily have both a minimum and a maximum cut-off energies. CKK[4] proposed to use the stability of the values and errors of the parameters as a working criteria to determine the validity region, which is also adopted in our work[1,2]. The bare Pomeron term can lead to either $ln^2\ (s/s_0)$ or $ln\ (s/s_0)$ behavior for the cross section upon making an eikonal unitarization[14]. Combined with the correct analyticity and crossing symmetry of the near forward amplitude, the results of the parameterization are given by either the model $RRL2$, which saturates the Froissart bound, or the model $RRL1$, which has a $ln\ (s/s_0)$ behavior for the total cross section in our works[1] with s_0 set to 1 GeV^2. Both models, $RRL2$ and $RRL1$, can easily be derived as solutions of the derivative dispersion relations[6], or from the prescription of Block and Cahn[16]. In particular, $RRL2$ is often called as the Amaldi model[7] but has been around since 1974[6] while $RRL1$ was proposed by Block, Kang and White[17] in 1990. Another modification of the $RRL2$, with equal number of parameters as in $RRL2$, is to vary the scale parameter s_0 while setting the constant A to zero in the unitarized Pomeron term in the total cross section, which we will refer to as the model $RRL2s0$[2]. Since we may regard the model $RRL2s0$ also as arising from an isolated singularity of a triple-pole at $\alpha_P(0) = 1$ in the l - plane, this model is expected to satisfy the factorization and quark counting rules as in the case of a bare Pomeron in the RRP model, while having a maximally rising behavior allowed by the Froissart bound as in $RRL2$.
We present the results of comparative study of these four analytic models here, hoping that such comparison would differentiate the bare Pomeron from the unitarized ones and in particular the energy scale beyond which the unitariza-

tion effect sets in appreciably.

II Analytic Models for the Forward Scattering Amplitude

The analytic parametrization for the forward scattering amplitude dates back to the hey days of S-matrix theory in the 60's and gained the renewed interest[5,6] in the early 70's when the total cross section showed a spectacular rise in energy from the ISR experiments at CERN. Analytic model for the forward scattering amplitudes have an obvious advantage of being simple and easy to apply to and study the general properties from the experimental data.

We have performed the scan-fits of the four analytic amplitude models, $RRP, RRL2, RRL1$ and $RRL2s0$, to the total hadronic cross sections of $p^{\pm}p, \pi^{\pm}p, K^{\pm}p, \gamma p$, and $\gamma\gamma$ and the ρ parameters for $p^{\pm}p, \pi^{\pm}p, K^{\pm}p$. The four analytic models give the following parametrizations to the total cross sections and ρ parameters: $\boxed{RRP}$

$$\sigma_{\mp} = X \cdot s^{\varepsilon} + Y_1 \cdot s^{-\eta_1} \pm Y_2 \cdot s^{-\eta_2}$$

$$\rho_{\mp} = \frac{1}{\sigma_{\mp}} \cdot \left[-\frac{X \cdot s^{\varepsilon}}{\tan\left(\frac{(\varepsilon+1)\pi}{2}\right)} - \frac{Y_1 \cdot s^{-\eta_1}}{\tan\left(\frac{(1-\eta_1)\pi}{2}\right)} \pm \frac{Y_2 \cdot s^{-\eta_2}}{\cot\left(\frac{(1-\eta_2)\pi}{2}\right)} \right]$$

$\boxed{RRL2}$

$$\sigma_{\mp} = \lambda \left(A + B \cdot \ln^2 s\right) + Y_1 \cdot s^{-\eta_1} \pm Y_2 \cdot s^{-\eta_2}$$

$$\rho_{\mp} = \frac{1}{\sigma_{\mp}} \cdot \left[\lambda \cdot B \cdot \pi \cdot \ln s - \frac{Y_1 \cdot s^{-\eta_1}}{\tan\left(\frac{(1-\eta_1)\pi}{2}\right)} \pm \frac{Y_2 \cdot s^{-\eta_2}}{\cot\left(\frac{(1-\eta_2)\pi}{2}\right)} \right]$$

$\boxed{RRL1}$

$$\sigma_{\mp} = \lambda \cdot \left(A + B \cdot \ln s\right) + Y_1 \cdot s^{-\eta_1} \pm Y_2 \cdot s^{-\eta_2}$$

$$\rho_{\mp} = \frac{1}{\sigma_{\mp}} \cdot \left[\frac{\lambda \cdot B \cdot \pi}{2} - \frac{Y_1 \cdot s^{-\eta_1}}{\tan\left(\frac{(1-\eta_1)\pi}{2}\right)} \pm \frac{Y_2 \cdot s^{-\eta_2}}{\cot\left(\frac{(1-\eta_2)\pi}{2}\right)} \right]$$

$\boxed{RRL2s0}$

$$\sigma_{\mp} = \left(B \cdot \ln^2(s/s_0)\right) + Y_1 \cdot (s/s_0)^{-\eta_1} \pm Y_2 \cdot (s/s_0)^{-\eta_2}$$

$$\rho_{\mp} = \frac{1}{\sigma_{\mp}} \cdot \left[B \cdot \pi \cdot \ln(s/s_0) - \frac{Y_1 \cdot (s/s_0)^{-\eta_1}}{\tan\left(\frac{(1-\eta_1)\pi}{2}\right)} \pm \frac{Y_2 \cdot (s/s_0)^{-\eta_2}}{\cot\left(\frac{(1-\eta_2)\pi}{2}\right)} \right]$$

Note that these models have a universal rising of the cross sections and satisfy the Pomeranchuk theorem as well as the universality of the effective secondary meson Regge-pole contributions. They have the same total number of adjustable parameters, namely 16 parameters, to fit 8 kind of the total cross sections and 6 kind of ρ data samples. There are three Regge terms coming from the Pomeron and $C = \pm 1$ meson trajectories in RRP, while a unitarized

Pomeron and $C = \pm 1$ meson Reggeon terms in $RRL1$ and $RRL2$, for a given $h\ p$ scattering process where $h = p^{\pm}$, $\pi^{\pm}$ and $K^{\pm}$. In addition, $\gamma\ p$ and $\gamma\ \gamma$ cross section are described by two terms representing contributions from the bare or unitarized Pomeron and $C = +1$ (a/f) Reggeon. The results presented here supplement Ref.[1,2] and also the earlier reports [18,19]

III Dataset and Statistical Fits of the Analytic Models

We used the experimental data extracted from the Particle Physics Data System (PPDS)[3], the largest data set prepared by collaborative efforts of COMPAS (IHEP), BPDG (LBNL), PDG (Durham,UK), ITEP (Moscow), and JINR (Dubna), and maintained by the COMPAS Group at IHEP for the cross sections and ρ parameters. They contain 711 (178) data points for total cross sections (ρ parameters) for $\sqrt{s}\ \geq 3$ GeV, while 492 (141) and 271 (112) data points for total cross sections (ρ parameters) for $\sqrt{s}\ \geq 5$ and 9 GeV respectively. As the dataset is large enough and contains no substantial inconsistencies, we have used the conventional definition of χ^2.

Each model is fitted to the data in the intervals from E^{CM}_{min} to the maximal available cm energies for each collisions and E^{CM}_{min} is varied starting from 3 -GeV in 1 -GeV steps up to $E^{CM}_{min} = 13$ -GeV, to be able to establish the area of applicability of each model by demanding $\chi^2/dof \leq 1.$ and that the model parameters and their errors are stable with respect to E^{CM}_{min}. This criterion implies that the Pomeron parameters are stable with respect to $\sqrt{s}_{min}$ and the low energy data becomes of no primary importance. The stability pattern of the parameters and their errors are presented in Ref. [1,2] and here we give in Fig. 1 only the χ^2/dof comparison of the four different models to see their areas of applicability.

The $\chi^2/d.o.f.$ shown in Fig.1 clearly indicates that the fit is bad for small energies in all cases and in particular that all four analytic models reproduce the data to a practically same accuracy starting from $E^{CM}_{min}\ \geq 9$ - GeV, with slight preference of ranking in the order of $RRL1, RRL2so, RRL2$ and RRP. In particular, the stability of the parameters set in already at $E^{CM}_{min}\ \geq 5$ - GeV for the models $RRL1$ and $RRL2s0$. Thus we see that the χ^2 value alone can not differentiate the RRP model from the other models with a unitarized Pomeron, though the latter have slightly better χ^2/dof in general. In particular, the scan-fit of $RRL2s0$ is slightly better than that of $RRL2$ and RRP already at $\sqrt{s}_{min}\ = 5$ GeV and is quite similar to that of $RRL1$[2]. Note however that the most interesting parameter ε in RRP model is sensitive to the highest energy region where the data are scarce. One may thus consider a scheme giving more weight to the high energy data. Also one may remember that the total cross

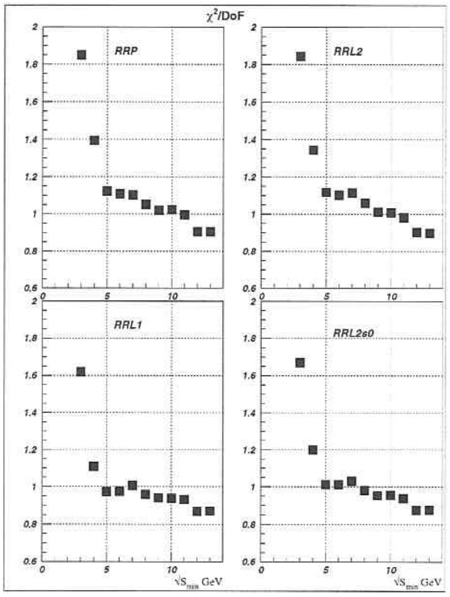

Figure 1: χ^2/dof dependence of the minimum energy $\sqrt{s}_{min}$.

sections are better measured than the ρ parameters because of lingering doubt on the Coulomb - nuclear interference calculations. One further point is that these analytic models can not describe the resonance region and they can be trusted only above a certain minimum energy $\sqrt{s}_{min}$ well above the threshold of resonance production. Thus the minimum cut-off energy $\sqrt{s}_{min}$ could be in principle process dependent. In spite of these worries we see no reason to use non-conventional definition of χ^2 and to invent different weighting schemes for different energy data points but using the inverse squares of their total errors as weights.

As in the previous work by CKK [4], for the RRP model the proposed criteria fails to meet for the exchange-degenerate meson trajectories because $\chi^2/d.o.f.$ is large and of order 1.3 and the parameter values and their errors are unstable when fitted to the total cross sections and ρ parameters. In fact the results confirm the conclusion reached by CKK [4] that the assumption of exchange-degeneracy for $C = \pm 1$ meson trajectories is not supported even by the fits to the total cross sections alone. We need $C = \pm 1$ meson trajectories that are non-degenerate, particularly when we want to fit ρ parameters in addition to the total cross sections. Since the minimum cut-off energy $\sqrt{s}_{min}$ turns out to be as high as 9 GeV, the Pomeron parameters are reasonably well determined while those of the lower trajectories are determined less accurate. The parameters of the RRP model determined for $\sqrt{s}_{min} = 9$ GeV are given in Table 1. We note that the RRP and $RRL2$ fits for the total cross sections and ρ parameters for $\sqrt{s}_{min} = 9$ GeV, extrapolated down to $\sqrt{s} \leq$

ε	η_1	η_2	$\chi^2/d.o.f.$	Total number of data points	
0.0933±0.0024	0.357±0.015	0.560±0.017	1.02	383	
	pp	πp	Kp	$\gamma p \times 10^{-2}$	$\gamma\gamma \times 10^{-4}$
X_1 (mb)	18.79±0.51	12.08±0.29	10.76±0.23	5.98±0.17	1.55±0.14
Y_1 (mb)	63.0±2.3	26.2±0.74	14.08±0.57	11.64±0.88	3.9±2.0
Y_2 (mb)	36.2±3.2	7.63±0.72	14.7±1.3		

Table 1: The values of the parameters of the hadronic amplitude in modelRRP corresponding to a cut off $\sqrt{s} \geq 9$ GeV.

A (mb)	B (mb)	s_0	η_1	η_2	χ^2/d.o.f.
25.29 ± 0.98	0.2271 ± 0.0071	1 (fixed)	0.341 ± 0.024	0.558 ± 0.017	1.01
	pp	πp	Kp	$\gamma p \times 10^{-2}$	$\gamma\gamma \times 10^{-4}$
λ	1	0.6459 ± 0.0043	0.5772 ± 0.0065	0.3201 ± 0.0055	0.083 ± 0.076
Y_1 (mb)	52.6 ± 2.2	20.17 ± 0.62	9.00 ± 0.75	8.65 ± 0.87	(3.0 ± 2.0)
Y_2 (mb)	36.0 ± 3.2	7.50 ± 0.71	14.6 ± 1.3		

Table 2: The values of the parameters of the hadronic amplitude in model $RRL2$ corresponding to a cut off $\sqrt{s} \geq 9$ GeV.

5 GeV, look deceptively good, though the value of $\chi^2/d.o.f$ goes above 2 so that statistically unacceptable [1,2]. This is why we need a careful statistical analysis of the fits with physically sound criteria imposed on. For completeness. we give the parameters of the other three models in Table 2 - 4.

IV Discussion on the Results

Universality: All these models give more or less indistinguishable fits and χ^2 values for $\sqrt{s_{min}} \geq 9$ GeV, though the models $RRL1$ and $RRL2s0$ are slightly better and stable already at 5 GeV. Thus the only discrimination that

A (mb)	B (mb)	s_0	η_1	η_2	χ^2/d.o.f.
-30.8 ± 3.6	6.74 ± 0.22	1 (fixed)	0.2078 ± 0.0079	0.545 ± 0.0063	0.97
	pp	πp	Kp	$\gamma p \times 10^{-2}$	$\gamma\gamma \times 10^{-4}$
λ	1	0.6839 ± 0.0045	0.6439 ± 0.0073	0.3566 ± 0.0048	0.0845 ± 0.0061
Y_1 (mb)	106.3 ± 2.9	61.2 ± 2.4	49.7 ± 2.5	29.4 ± 1.3	8.1 ± 3.5
Y_2 (mb)	33.36 ± 0.96	5.78 ± 0.16	13.42 ± 0.38		

Table 3: The values of the parameters of the hadronic amplitude in model $RRL1$ corresponding to a cut off $\sqrt{s} \geq 5$ GeV.

		$\sqrt{s_0}$ (MeV)	η_1	η_2	χ^2/d.o.f.
		20.17 ± 5.70	0.288 ± 0.010	0.555 ± 0.009	1.01
	pp	πp	Kp	$\gamma p \times 10^{-2}$	$\gamma\gamma \times 10^{-4}$
B (mb)	0.145 ± 0.008	0.095 ± 0.006	0.086 ± 0.056	0.048 ± 0.003	0.012 ± 0.001
Y_1 (mb)	70.38 ± 0.77	34.29 ± 0.30	22.63 ± 0.40	14.86 ± 0.35	4.90 ± 0.76
Y_2 (mb)	34.97 ± 1.46	5.98 ± 0.24	13.93 ± 0.57		

Table 4: The values of the parameters of the hadronic amplitude in model $RRL2s0$ corresponding to a cut off $\sqrt{s} \geq 5$ GeV.

the soft data can bring in favor of RRP must come from some other evidence, such as in the confirmation of the properties that follow from that the Pomeron is a simple Regge pole coupled to constituent quarks, *i.e.*, factorization, quark counting and universality, though such property is also expected from the model $RRL2s0$ and to some extent from $RRL2$ and $RRL1$ models too. The Pomeron and meson Reggeon parameters in each case are given in Table 1 - 4. As the l-plane singularities in the Regge theory are universal, the energy dependence of the data has to be a combination of points which rise or fall with energy in a process-independent manner. Withinn the context of perturbative QCD, one could *a priori* expect a small deviation from universality [10], because the hadronic wave functions must come into calculation of the various terms. The universality of the Pomeron and meson trajectory intercepts is closely linked to the structure function F_2 at HERA [20]. There the effective Pomeron intercept, extracted from the power behavior in $1/x$ from $F_2(x)/x$, seems to depend on Q^2, the negative mass squared of the virtual photons. In order to check for a target mass M-dependence of the Pomeron intercept on the other side of $M^2 = 0$, we made a partial fit, fixing $C + \pm 1$ meson intercepts, at $\sqrt{s}_{min} = 9$ GeV with the errors in the intercept corresponding to a change of 1 unit in the $\chi^2/d.o.f.$ The result shows that the soft Pomeron intercept may be universal and independent of the target mass.

Factorization and Quark Counting Rule: The Reggeon couplings are expected to factorize into a product of two couplings, one for each interacting hadron. Also the Pomeron couples to single quarks is like a $C = +1$ photon in Regge theory. The Pomeron being an extended object, this will be a viable scenario only for constituent quarks if the Pomeron is a simple pole. The result of Table 1 gives

$$\frac{(X_{pp}/X_{\pi p})}{3/2} = 1.04 \pm 0.11$$

$$X_{Kp}/X_{\pi p} = 0.89 \pm 0.05$$

$$X_{\gamma p}/\left\{ g_{em}^2 \left[\frac{1}{f_p^2} + \frac{1}{f_\omega^2} + \frac{1}{f_\rho^2}\right] (1+\delta) X_{\pi p} \right\} \approx \frac{213.9 X_{\gamma p}}{X_{\pi p}} = 1.06 \pm 0.04$$

$$X_{pp} X_{\gamma\gamma}/X_{\gamma p}^2 = 0.78 \pm 0.15$$

The first and second relations reflect the quark counting rule, the third is the factorization combined with generalized vector dominance [21] where the contribution from off-diagonal terms δ is expected to be about 15%, and the forth is an example of factorization. From these, we see that the properties of factorization and quark counting, expected from the Pomeron as a simple Regge pole, seem to hold within 10 %. Note however that quark counting fails to be present for meson Reggeons, as they have to probe multi-quark configuration unlike the Pomeron which seems to couple to single quarks.

The quark counting rule and factorization property are also expected to be satisfied by the Pomeron term in $RRL2s0$ model, as it can be interpreted as an isolated triple-pole in the l -plane. In fact, one can see from B parameters in Table 4 that the corresponding quantities as those of RRP model above are:

1.02 ± 0.03; 0.91 ± 0.06; 1.09 ± 0.07; 0.74 ± 0.15.

Thus the properties of the quark counting rule and factorization are also satisfied to the same accuracy by $RRL2s0$ model and we need further tuning of the models in order to differentiate them, for example, by distinquishing the strange quark coupling from the non-starnage quark couplings since the universality between strange and light quarks is satisfied only to within 10 % in both RRP and $RRL2s0$ models. It would then mean at least a 17 - parameter model fits. What is more interesting, however, is that the unitarized Pomeron terms in both $RRL2$ and $RRL1$ models also exhibit similar proteries. Namely, the four corresponding quantities are given by [1.03, 0.89, 1.06, 0.81] and [0.97, 0.94, 1.11, 0.66] with similar errors as those of RRP in $RRL2$ and $RRL1$ respectively. Thus it appears that such approximate factorization relation is satisfied by any high energy models that are constrained by the conditions of unitarity, analyticity and acceptable fits to the cross section datas of $pp, \gamma p$ and $\gamma\gamma$ [22]. This is because the Pomeron term, whether a simple bare Pomeron or otherwise, dominates the amplitudes and the meson Reggeon contributions are practically negiligible in the energy region above the ISR energies.

Other Possible Trajectories: The sharp rise observed at the HERA might be due to the presence of another singularity of a simple pole[11], hither to undetected, called the hard Pomeron, which can fit the deep inelastic scattering data. We tested its contribution with the hard Pomeron intercept 0.4 and concluded to the 2σ level that it must decouple at $Q^2 \leq 0$ if a simple pole.

The exchange of a $C = -1$ trajectory [23] with intercept close to 1 is needed within the Donnachie- Landshoff model to reproduce the large $-t$ dip in elastic scattering. Assuming its intercept to be as low as 1, we concluded again to the 2σ level that such object did not seem to be present at $t = 0$.

Further Remarks: Both RRP and $RRL2$ models produce very good fits to all data for $\sqrt{s}_{min} \geq 9$ GeV, with $\chi^2/d.o.f \leq 1$ with stable parameters, and in particular reproduce the $UA4/2$ and CDF experimental results for $p\bar{p}$. The RRP model gives the Pomeron intercept, $\alpha_P(0) = 1.093 \pm 0.003$, in agreement with the conclusion of CKK [4]. The $C = \pm 1$ meson trajectories are non-degenerate and have intercepts as given in Table 1. The interplay between the Pomeron and $C = +1$ meson Reggeon contribution makes the determination of the couplings of the latter Reggeon problematic, as there seems to be characteristic $\sqrt{s}_{min}$ dependence. Further stabilization of these couplings is needed. The two models are hardly distinquishable up to the LHC energy, *i.e.,* $\sigma_{tot} = 117.77 \pm 2.33$ (114.86 ± 1.72) and $\rho = 0.155 \pm 0.004$ (0.131 ± 0.002) for both pp and $p\bar{p}$ from the model RRP ($RRL2$), which mean that the unitarity effect is in no sight in the forward scattering data well beyond the available Tevatron energy.

Furthermore we have indicated that the $t = 0$ data, based on the χ^2 consideration alone, can not rule out other analytic models, such as $RRL1$ and $RRL2s0$, which are in some respects better than RRP and $RRL2$ models because the validity region starts already at $\sqrt{s}_{min} \geq 5$ GeV. In paricular, both $RRL1$ and $RRL2s0$ models reproduce the $UA4/2$ results but $RRL1$ prediction for the total cross section is about 76 mb closer to the upper limit of the E710 or E811 results while $RRL2s0$ gives 78 mb closer to the lower limit of the CDF cross section at the Tevaron energy for $p\bar{p}$, and 101 mb and 104 mb respectively at the LHC energy for pp scattering. The LHC cross section may therefore differentiate the $RRL1$ model from RRP and $RRL2$ models which predict 118 mb and 115 mb respectively , it will be difficult to discriminate $RRL1$ from $RRL2s0$ model. The unitarity effect in the form of $RRL1$ will thus be easier than that of $RRL2$ to test at the LHC energy but a Pomeron of a triple-pole singularity in $RRL2s0$ appears to be slightly more favorable from the $\chi^2/d.o.f.$ values alone. For the latter type of Pomeron, one needs no unitarization as it respects the Froissart bound already. What is more intriguing is that the $RRL2s0$ model, a special case of $RRL2$ in which the constant term $A = 0$ with varing scale s_0, gives similar $\chi^2/d.o.f$ values as in the $RRL1$

model but with a low $\sqrt{s_0}$ about 20 MeV. Thus we seem to have two options, based on this comparative data fittings, for the Pomeron singularity in the l-plane. In view of the phenomenological success of the Pomeron as a simple bare Regge pole with an intercept slightly larger than 1, one may choose to favor the RRL1 model as a unitarized version of such bare Pomeron. On the other hand, the special RRL2s0 model correspond to the l-plane singularity of a triple pole with the intercept exactly at 1. It will be important to be able to differentiate the two options for the Pomeron.

It is intriguing that the factorization and quark counting properties are well respected by all these models within 10 to 15 %, though such properties are more naturally expected from a Pomeron as a simple Regge pole.

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References

1. J. R. Cudell, V. Ezhela, K. Kang, S. Lugovsky and N. Tkachenko, Phys. Rev. **D 61** (2000) 034019, hep-ph/9908218, and K. Kang, Proc. VIIIth Blois Workshop, BROWN-HET-1193.
2. V. V. Ezhela, Proc. VIIIth Blois Workshop, BROWN-HET-1194.
3. Data used were extracted from the COMPAS Group PPDS accessible at http://wwwppds.ihep.su:8001/ppds.html. Computer readable data files are also available at http://pdg.lbl.gov.
4. J.R. Cudell, K. Kang and S.K. Kim, Phys. Lett. **B395** (1997) 311, hep-ph/9601336. See also R.J.M.Covolan, J. Montanha and K. Goulianos, Phys. Lett. **B 389** (1996) 176.
5. C Bourelly and J.Fisher, Nucl. Phys. B61 (1973) 513; H. Lipkin, Phys. Rev. D11 (1975) 1827; P. D. B. Collins, F. D. Gault, and A. Martin, Nucl. Phys. B85 (1975) 141.
6. K. Kang and B. Nicolescu, Phys. Rev. D11 (1975) 2461.
7. U. Amaldi et al., Phys. Lett. 66B (1977) 390.
8. A. Donnachie and P.V. Landshoff, Phys.Lett. **B296** (1992) 227.
9. A. Donnachie and P.V. Landshoff, Nucl. Phys. **B244** (1984) 322; A. Donnachie and P.V. Landshoff, Nucl. Phys. **B267** (1986) 690; P.D.B. Collins, F.D. Gault and A. Martin (Ref. 1).
10. J.R. Cudell and B.U. Nguyen, Nucl. Phys. **B420** (1994) 669, hep-ph/9310298.
11. A. Donnachie and P.V. Landshoff, Phys. Lett. **B437** (1998) 408, hep-

ph/9806344; J.R. Cudell, A. Donnachie and P.V. Landshoff, Phys. Lett. **B448** (1999) 281, hep-ph/9901222.

12. L.N.Lipatov, Sov. J. Nucl. Phys. **23** (1976) 642; V.S.Fadin, E.A.Kuraev and L.N.Lipatov, Phys. Lett. **B60** (1975) 50; E.A.Kuraev, L.N.Lipatov and V.S.Fadin, Sov.Phys.JETP **44** (1976) 45, **45** (1977) 199; I.I.Balitsky and L.N.Lipatov, Sov.J.Nucl.Phys. **28** (1978) 822.
13. V. Kim, Proc. VIIIth Blois Workshop, hep-ph/9911228.
14. J. Finkelstein, H. M. Fried, K. Kang and C.-I Tan, Phys. Lett. **B232** (1989) 257.
15. K. Kang and B. Nicolescu (Ref. [6]) and also K. Kang, Nucl. Phys. **B** (Proc. Suppl.) **12** (1990) 64.
16. M. M. Block and R. N. Cahn, Rev. Mod. Phys. **57** (1985) 563.
17. M. M. Block, K. Kang and A. R. White, J. Mod. Phys. **A7** (1992) 4449. See also K. Kang, P. Valin, and A.R. White, Nuovo Cim. **107A** (1994) 2103.
18. A short preliminary summary presented by three of us (V.E., S.L., and N.T.) in C. Caso *et al.*, Eur. Phys. J. **C3** (1998) 1, http://pdg.lbl.gov.
19. K. Kang *et al.*, hep-ph/9812429, presented at 4th Workshop on Quantum Chromodynamics, Paris, France, 1-6 June 1998 and presented at 4th Workshop on Small x and Diffractive Physics, Batavia, IL, 17-20 Sept. 1998, In Batavia 1998, Small-x and diffractive physics, pp. 203-211.
20. C. Adloff *et al.* [H1 Collaboration], Nucl. Phys. **B497** (1997) 3, hep-ex/9703012; J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. **C7** (1998) 609, hep-ex/9809005.
21. G. Shaw, Phys. Rev. **D47** (1993) 3676 and Phys. Lett. **B228** (1989) 129; P. Ditsas and G. Shaw, Nucl. Phys. **113** (1976) 246; J.J. Sakurai and D. Schildknecht, Phys. Lett. **B40** (1972) 121.
22. M. M. Block, E. M. Gregores, F. Halzen and G. Pancheri, Phys. Rev. **D 60** (1999) 054021.
23. L. Lukaszuk and B. Nicolescu, *Lett. Nuovo Cim.* **8** (1973) 405; G. Bialkowski, K. Kang and B. Nicolescu, Lett. Nuovo Cim. **13** (1975) 401;and also K. Kang and B. Nicolescu (Ref. [6]).