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The spin-flip amplitude
in the impact-parameter representation

O. V. Selyugin[1,a] and J.-R. Cudell[2,b]

[1] JINR, Bogoliubov Laboratory of Theoretical Physics, 141980 Dubna, Moscow Region, Russia
[2] IFPA, AGO Dept., Université de Liège , Belgium

Abstract. The impact-parameter representation of the spin-flip amplitude of had-
ron elastic scattering is examined in different unitarisation schemes, taking the
Born term of the spin-flip amplitude from the Dubna Dynamical Model (DDM).
It is shown that the basic properties of the unitarisation schemes are independent
of the functional form used for unitarisation but heavily depend on the asymptotic
value of the unitarised amplitude.

1 Introduction

There are many different models for the description of hadron elastic scattering at small angles
[1,2]. They lead to different predictions for the structure of the scattering amplitude at asymp-
totic energies, where the diffraction processes can display complicated features [3]. This concerns
especially the asymptotic unitarity bound connected with the Black Disk Limit (BDL). In this
paper, we study the impact of unitarisation on the spin properties of the elastic amplitude.

In practice, we need to sum many different waves with $l \to \infty$ and this leads to the impact
parameter representation [4] converting the summation over l into an integration over the
impact parameter b. In the impact-parameter representation, the Born term of the scattering
amplitude will be

$$\chi(s,b) \approx \int d^2q \; e^{i\mathbf{bq}} \; F_{\text{Born}}\left(s, q^2\right) , \tag{1}$$

where we have dropped the kinematical factor $1/\sqrt{s(s - 2m_p^2)}$ and a factor s in front of the
scattering amplitude.

The hadron spin-flip amplitude is expected to be connected with quark exchange between the
scattering hadrons, and at high energy and small angles it is expected to be negligible. However,
some models, which take into account non-perturbative effects, lead to a non-vanishing hadron
spin-flip amplitude [5,6] even at high energy.

After unitarisation, we get for the scattering amplitude

$$F(s,t) \approx \int e^{i\mathbf{bq}} \; \Gamma(s,b) \; d^2b, \tag{2}$$

where $t = -q^2$. The overlap function $\Gamma(s,b)$ can be a matrix, corresponding to the scattering
of different spin states. Unitarity of the S-matrix, $SS^+ = 1$, requires that $\Gamma(s,b) \leq 1$.

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[a] e-mail: `selugin@theor.jinr.ru`

[b] e-mail: `JR.Cudell@ulg.ac.be`

The unitarisation procedure can be obtained in different ways, starting from simple diagrams in the tree approximation and using the Shrödinger equation [7,8] or including the spin of particles in the relativistic case [9,10]. One must take into account many additional diagrams which include inelastic states in the s channel. This leads to different schemes, see for example [11–13] in which renormalised eikonal representations were obtained. No one really knows which are the leading diagrams and how to sum them. Therefore, all these approaches are possible phenomenological forms which can lead to different spin correlations.

There are two important constraints which any unitarisation scheme must satisfy. Firstly, in the limit of small energies, every unitarisation representation must reduce to the same scattering amplitude − the Born term − and thus must give the same result. Only at high energies, when the number of diagrams and their forms are essentially different, will the various unitarisation representations give different results [26,27].

Secondly, for any unitarisation scheme, the corresponding overlap function cannot exceed the unitarity bound. In different normalisations, this bound may equal to 1 or 2.

At LHC energies, the effects of unitarisation will be large and the experimental data will probably determine what form of unitarisation is realised.

2 Spin-dependent scattering amplitude

In the case of elastic scattering of a baryon of momentum p_1 on another baryon of momentum k_1, going to states of respective momenta p_2 and k_2, e.g. $pp \to pp$, $\bar{p}p \to \bar{p}p$, $np \to np$, $p\Lambda \to p\Lambda$, $\Lambda\Sigma \to \Lambda\Sigma$, the full representation for the scattering amplitude is

$$
\begin{aligned}
\Phi(s,t) = \ & \Phi_1(s,t)\bar{u}(p_2)u(p_1)\bar{u}(k_2)u(k_1) + \Phi_2(s,t)\bar{u}(p_2)\gamma K u(p_1)\bar{u}(k_2)\gamma P u(k_1) \\
& + \Phi_3(s,t)\bar{u}(p_2)\gamma_5(\gamma K)u(p_1)\bar{u}(k_2)\gamma_5(\gamma P)u(k_1) + \Phi_4(s,t)\bar{u}(p_2)\gamma_5 u(p_1)\bar{u}(k_2)\gamma_5 u(k_1) \\
& + \Phi_5(s,t)\left[\bar{u}(p_2)\gamma K u(p_1)\bar{u}(k_2)u(k_1) + \bar{u}(p_2)u(p_1)\gamma P u(k_1)\bar{u}(k_2)\right] \\
& + \Phi_6(s,t)\left[\bar{u}(p_2)\gamma K u(p_1)\bar{u}(k_2)u(k_1) - \bar{u}(p_2)u(p_1)\gamma P u(k_1)\bar{u}(k_2)\right] \\
& + \Phi_7(s,t)\bar{u}(p_2)\gamma_5 u(p_1)\bar{u}(k_2)\gamma_5(\gamma P)u(k_1) \\
& + \Phi_8(s,t)\left[\bar{u}(p_2)\gamma_5(\gamma K)u(p_1)\bar{u}(k_2)\gamma_5 u(k_1)\right] .
\end{aligned}
\tag{3}
$$

Here

$$
K = \frac{1}{2}(k_1 + k_2); \quad P = \frac{1}{2}(p_1 + p_2); \quad Q = (k_2 - k_1) = (p_2 - p_1).
\tag{4}
$$

The last two terms of (3) do not satisfy charge and time invariance and must be zero. If all four particles are identical, the amplitude does not change under the exchange $p_1, p_2 \rightleftharpoons k_1, k_2$ and $P \rightleftharpoons K$. Hence, for proton-proton scattering there are only five helicity amplitudes [15]:

$$
\begin{aligned}
\Phi_1^B(s,t) = \ & <++|++>; \quad \Phi_2^B(s,t) = <++|-->; \\
\Phi_3^B(s,t) = \ & <+-|+->; \quad \Phi_4^B(s,t) = <+-|-+>; \quad \Phi_5^B(s,t) = <++|+-> .
\end{aligned}
\tag{5}
$$

In the Regge limit, t fixed and $s \to \infty$, one can write the Regge-pole contributions to the helicity amplitudes in the s-channel as [14]

$$
\Phi_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}^B(s,t) \approx \sum_i g_{\lambda_1,\lambda_2}^i(t)g_{\lambda_3,\lambda_4}^i(t)[\sqrt{|t|}]^{|\lambda_1-\lambda_2|+|\lambda_3-\lambda_4|}\left(\frac{s}{s_0}\right)^{\alpha_i}\left(1 \pm e^{-i\pi\alpha_i}\right).
\tag{6}
$$

The differential cross sections is then given by

$$
\frac{d\sigma}{dt} = \frac{2\pi}{s^2}\left(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2\right) .
\tag{7}
$$

The total helicity amplitudes can be written as $\Phi_i(s,t) = \Phi_i^h(s,t) + \Phi_i^{\rm em}(s,t)e^{\varphi(s,t)}$, where $\Phi_i^h(s,t)$ comes from the strong interactions, $\Phi_i^{\rm em}(s,t)$ from the electromagnetic interactions and

$\varphi(s,t)$ is the interference phase factor between the electromagnetic and strong interactions [16, 17].

The spin correlation parameters, the analysing power - A_N and the and double-spin parameter A_{NN}, can be extracted from experimental measurements:

$$A_N = \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)} = \frac{\Delta\sigma^s}{\sigma_0} , \tag{8}$$

$$A_{NN} = \frac{\sigma(\uparrow\uparrow) - \sigma(\uparrow\downarrow)}{\sigma(\uparrow\uparrow) + \sigma(\uparrow\downarrow)} = \frac{\Delta\sigma^d}{\sigma_0} , \tag{9}$$

where $\Delta\sigma^s$ and $\Delta\sigma^d$ refer to the difference of single- and double-spin-flip cross sections. The expressions for these parameters will be

$$A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} \left[Im(\Phi_1 + \Phi_2 + \Phi_3 - \Phi_4)\Phi_5^* \right] ; \tag{10}$$

$$A_{NN} \frac{d\sigma}{dt} = \frac{4\pi}{s^2} \left[Re(\Phi_1\Phi_2^* - \Phi_3\Phi_4^*) + |\Phi_5|^2 \right] . \tag{11}$$

Regge factorisation together with the experimental information about the spin-correlation effects at high energy and small momentum transfer in proton-proton elastic scattering, suggest that the double helicity flip is a second-order effect and consequently that one can neglect the amplitudes $\Phi_{++--}^B(s,t)$ and $\Phi_{+--+}^B(s,t)$. Furthermore, when the exchanged Regge trajectories have natural parity, we have for spin-non-flip amplitudes [18] $\Phi_{++++}^B(s,t) = \Phi_{+-+-}^B(s,t)$.

3 U_e-matrix unitarisation

The form of the unitarisation of the scattering amplitude in the impact parameter representation depends on the non-linear processes which lead to the saturation of the gluon density. There are many approaches to the equations describing such processes [19]. The most popular one, the dipole-dipole interaction model [20], describes the process of saturation as a function of the dipole size. Its inclusion into a real hadron-hadron interaction requires a phenomenological model.

Here we shall consider only non-linear equations which lead to the known unitarisation schemes. One of the simplest such equation is the well-known logistic equation [3,21], used for long time in many different branches of physics:

$$\frac{dN}{dy} = \Delta N \left[1 - N \right], \tag{12}$$

where $y = \log(s/s_0)$ and $\Delta = 1 - \alpha(0)$, ($\alpha(0)$ being the intercept of the leading pole).

Its solution has the form

$$N = \frac{\chi(s,b)}{1 + \chi(s,b)} , \tag{13}$$

where $\chi(s,b) \approx s^\Delta$ is connected with the Born term of the scattering amplitude.

Then the scattering amplitude is

$$\Phi^h(s,t) = \frac{i}{2\pi} \int d^2b \, e^{i\mathbf{bq}} \frac{\chi(s,b)}{1 + \chi(s,b)} . \tag{14}$$

This unitarisation scheme gives results similar to those of the eikonal representation, and we will refer to it as U_e-unitarisation.

The phase $\chi(s,b)$ is connected to the interaction quasi-potential which can have real and imaginary parts and, in the case of a spin-dependent potential, a matrix structure:

$$\chi(s,b) = F_{\mathrm{Born}}(s,b) \approx \frac{1}{k} \int \hat{V} \left(\sqrt{b^2 + z^2} \right) dz. \tag{15}$$

If the quasi-potential contains a non-spin-flip part and, for example, spin-orbital and spin-spin interactions, the phase will be

$$\chi(s,b) = \chi_0(s,b) - i\,\mathbf{n}\cdot(\sigma_1+\sigma_2)\chi_{\mathrm{LS}}(s,b) - i(\sigma_1\cdot\sigma_2)\,\chi_{\mathrm{SS}}(s,b). \tag{16}$$

If we take into account only the spin-flip and spin-non-flip parts and neglect the second order on the spin-flip amplitude, the overlap function will be

$$\Gamma(s,b) = \frac{\chi_0(s,b) + \sigma\chi_{sf}(s,b)}{1+\chi_0(s,b)+\sigma\chi_{sf}(s,b)} = 1 - \frac{(1+\chi_0(s,b)) - \sigma\chi_{sf}(s,b)}{(1+\chi_0(s,b))^2 - (\sigma\chi_{sf}(s,b))^2}\,. \tag{17}$$

Using the representation for the Bessel functions

$$J_0(x) \;=\; \frac{1}{2\pi}\int_0^{2\pi} e^{ix\cos\phi}\,d\phi \qquad J_1(x) \;=\; -\frac{1}{2\pi}\int_0^{2\pi} e^{ix\cos\phi}\,\sin\phi\,d\phi\,, \tag{18}$$

the representation of spin-non-flip and spin flip amplitude is

$$\Phi_1^h(s,t) \;=\; i\int_0^\infty bJ_0(bq)\frac{\chi_0(s,b)}{1+\chi_0(s,b)}db\,; \tag{19}$$

$$\Phi_5^h(s,t) \;=\; i\int_0^\infty b^2 J_1(bq)\frac{\chi_{sf}(s,b)}{(1+\chi_0(s,b))^2}db\,. \tag{20}$$

4 U_T-matrix unitarisation

If Eq. (12) has an additional coefficient n

$$\frac{dN}{dy} = \Delta N\,[\,1 - N/n\,]. \tag{21}$$

we obtain, for $n=2$, the unitarisation in the standard U-matrix form intensively explored in [22].

In the impact parameter representation, the properties of the U-matrix are explored in [23]. In this scheme, the hadronic amplitude is given by

$$\Phi^h(s,t) \;=\; \frac{i}{2\pi}\int d^2b\,e^{i\mathbf{bq}}\frac{\chi(s,b)}{1+\chi(s,b)/2}\,, \tag{22}$$

where $\chi(s,b)$ is the same Born amplitude as before.

Comparing Eq. (22) with Eq. (14), we see that both have the same rational form but differ by the additional coefficient in the denominator. This additional coefficient leads to different analytic properties: the upper bound at which the overlapping function saturates will be in twice as large as in the eikonal or U_e representations, and the inelastic overlap function at $b=0$ will be tend to zero at high energies. This leads to the new relation [23] $\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}}\to 1$.

For the helicity amplitudes of pp scattering, the corresponding solution of the unitarity equations [24]:

$$\Phi_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(\mathbf{p},\mathbf{q}) = U_{\lambda_3,\lambda_4,\lambda_1,\lambda_2}(\mathbf{p},\mathbf{q}) + \tag{23}$$
$$i\frac{\pi}{8}\sum_{\lambda',\lambda''}\int d\Omega_{\hat{\mathbf{k}}}U_{\lambda_3,\lambda_4,\lambda',\lambda''}(\mathbf{p},\mathbf{k})\Phi_{\lambda',\lambda'',\lambda_1,\lambda_2}(\mathbf{k},\mathbf{q})\,,$$

In the impact parameter representation, one obtains the following equations relating the unitarised helicity amplitudes $\Phi_i(s,b)$ to the Born amplitudes $u_i(s,b)$ [24]:

$$\Phi_1 = \frac{(u_1 + u_1^2 - u_2^2)(1 + u_3 + u_4) - 2(1 + 2u_1 - 2u_2)u_5^2}{(1 + u_1 - u_2)\left[(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2\right]},$$

$$\Phi_2 = \frac{u_2(1 + u_3 + u_4) - 2u_5^2}{(1 + u_1 - u_2)\left[(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2\right]},$$

$$\Phi_3 = \frac{(u_3 + u_3^2 - u_4^2)(1 + u_1 + u_2) - 2(1 + 2u_3 - 2u_4)u_5^2}{(1 + u_3 - u_4)\left[(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2\right]},$$

$$\Phi_4 = \frac{u_4(1 + u_1 + u_2) - 2u_5^2}{(1 + u_3 - u_4)\left[(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2\right]},$$

$$\Phi_5 = \frac{u_5}{(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2}, \tag{24}$$

where for simplicity we omitted the arguments in the functions $\Phi_i(s,b)$ and $u_i(s,b)$. If we take $\chi_c = u_1 + u_3$ and $\chi_{sf} = u_5$ in the same approximation as in the U_e case, the spin-non-flip and spin-flip amplitude will be

$$\Phi_1^h(s,t) = i \int_0^\infty bJ_0(bq)\frac{\chi_c(s,b)}{1 + \chi_c(s,b)/2}db; \tag{25}$$

$$\Phi_5^h(s,t) = i \int_0^\infty b^2 J_1(bq)\frac{\chi_{sf}(s,b)}{\left(1 + \chi_c(s,b)/2\right)^2}db. \tag{26}$$

It is clear that these forms can also be obtained by the same procedure as we used in the case of U_e unitarisation.

5 Eikonal unitarisation scheme

To obtain the standard eikonal representation of the elastic scattering amplitude in the impact parameter representation one must take the non-linear equation in the form [3]

$$\frac{dN_e}{dy} = -\Delta \log(1 - N_e)[1 - N_e]. \tag{27}$$

where $y = \log(s/s_0)$ and where the subscript "e" implies that the solution N_e has exactly the standard eikonal form as shown in ref. [3], *i.e.*

$$N_e = \Gamma(s,b) = [1 - e^{-\chi(s,b)}]. \tag{28}$$

The eikonal representation is then

$$\Phi^h(s,t) = \frac{i}{2\pi} \int e^{i\mathbf{bq}} \left[1 - e^{-\chi(s,b)}\right] d^2b, \tag{29}$$

where the eikonal phase in the case of spin-dependent potential has a matrix structure and the quasi-potential $V(s,r)$ contains the non-spin-flip part and , for example, spin-orbital and spin-spin interaction:

$$\chi(s,b) = \chi_0(s,b) - i\, \mathbf{n} \cdot (\sigma_1 + \sigma_2)\chi_{LS}(s,b) - i(\sigma_1 \cdot \sigma_2)\, \chi_{SS}(s,b). \tag{30}$$

Taking into account the Eqs. (18) and (31), we have for the spin non-flip

$$\Phi_1^h(s,t) = i \int_0^\infty bJ_0(bq)\left[1 - e^{\chi_0(s,b)}\right]\left[1 - b^2\chi_{\mathrm{LS}}^2(s,b) - 3/2\chi_{\mathrm{SS}}^2(s,b)\right]\, db, \tag{31}$$

and for the spin-flip

$$\Phi_5^h(s,t) \;=\; i \int_0^\infty J_1(bq)\chi_1 e^{\chi_0(s,b)} \; b \; [\chi_{\mathrm{LS}}(s,b) + i \; \chi_{\mathrm{LS}}(s,b) \; \chi_{\mathrm{SS}}(s,b)] \; db, \tag{32}$$

where

$$\chi(s,b)_0 \approx \int_{-\infty}^\infty V_0(s,b,z)dz\,; \tag{33}$$

$$\chi(s,b)_1 \approx \frac{b}{2} \int_{-\infty}^\infty V_1(s,b,z)dz\,. \tag{34}$$

If, for example, the potentials V_0 and V_1 are assumed to be Gaussian

$$V(s,b)_{0,1} \;\approx\; \int_{-\infty}^\infty e^{-Br^2} \; dz = \frac{\sqrt{\pi}}{\sqrt{B}} e^{-Bb^2} \,, \tag{35}$$

in the first Born approximation, Φ_0^h and Φ_1^h will have the forms

$$\Phi_1^h(s,t) \;\approx\; \int_0^\infty bJ_0(bq)e^{-Bb^2}db = e^{-Bq^2}\,; \tag{36}$$

$$\Phi_5^h(s,t) \;\approx\; \int_0^\infty b^2 J_1(bq)e^{-Bb^2}db = qBe^{-Bq^2}\,. \tag{37}$$

6 The analysing power in the different unitarisation schemes

Now let us compare the spin correlation parameter A_N in the different unitarisation schemes. For that we use the Born terms of the spin-non-flip and spin-flip Born terms of the proton-proton elastic scattering calculated in the framework of the Dubna Dynamical Model (DDM) [2,25].
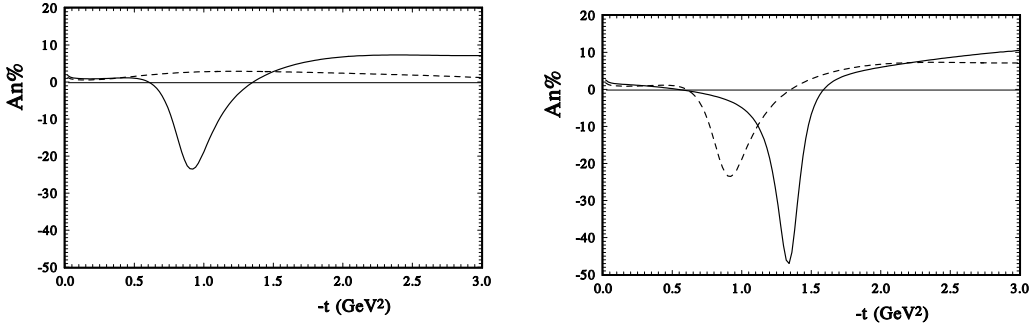

Fig. 1. a[left] $A_N(s,t)$ for the Born amplitudes calculated in the DDM [6] at $\sqrt{s} = 50\,\mathrm{GeV}$ (full line) and at $\sqrt{s} = 500\,\mathrm{GeV}$ (dashed line); **b**[right] $A_N(s,t)$ in the case of the eikonal unitarisation with the Born amplitudes calculated in the DDM [6] at $\sqrt{s} = 50\,\mathrm{GeV}$ (full line) and at $\sqrt{s} = 500\,\mathrm{GeV}$ (dashed line).

This model, which takes into account the interactions at large distances, predicts non-vanishing spin effects at high energies [6]. The values of $A_N(s,t)$ corresponding to the Born terms are shown in Fig. 1a. We can see that whereas the analysing power is not small at $\sqrt{s} = 50\,\mathrm{GeV}$, it is negligible at $\sqrt{s} = 500\,\mathrm{GeV}$. Now we can use the eikonal form of the unitarisation. Our results are shown in Fig. 1b. We see that in this case the size of A_N grows and now it is a measurable effect up to $\sqrt{s} = 500\,\mathrm{GeV}$. This result is linked to the fact that the diffractive structure of proton-proton scattering does not disappear at this energy.

Now we can consider the unitarisation procedure in the form of the U_e-matrix, Eqs. (19, 20). The result of the calculation is shown in Fig. 2. The size of $A_N(s,t)$ is above that in the case of the eikonal unitarisation. It remains large at $\sqrt{s} = 500 \, \text{GeV}$. The size of A_N is positive and large after the diffraction minimum, and reaches $20\,\%$ at $|t| = 2 \, \text{GeV}^2$. However, a comparison with the eikonal unitarisation (see Fig. 1b) shows that we have practically the same form of the analysing power in both cases. So, the difference is only quantitative but not qualitative. Despite an essential change in the functional form of the unitarisation procedure, we obtain very similar results.
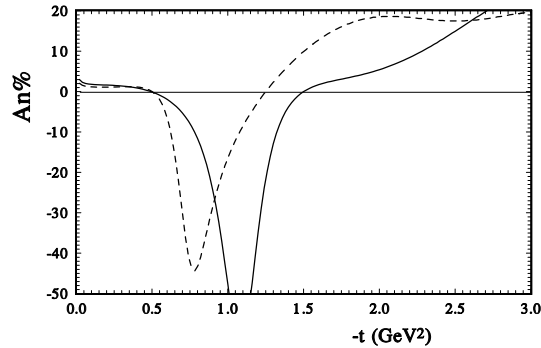

Fig. 2. $A_N(s,t)$ in the case of the U_e-matrix unitarisation with the Born amplitudes calculated in the DDM [6] at $\sqrt{s} = 50 \, \text{GeV}$ (full line) and at $\sqrt{s} = 500 \, \text{GeV}$ (dashed line).

A very different picture, presented in Fig. 3a, is obtained if we use U_T-matrix unitarisation. In this case, $A_N(s,t)$ has a very different form, coming mainly from the form of the spin-non-flip amplitude, which for the Born term of the DDM does not reproduce elastic proton-proton scattering.
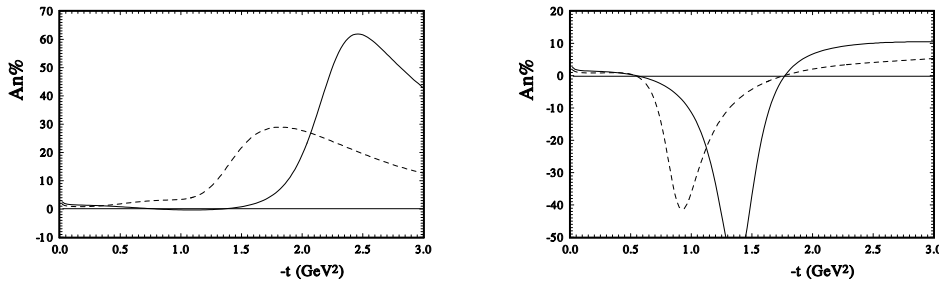

Fig. 3. a[left] $A_N(s,t)$ in the case of the U_T-matrix unitarisation with the Born amplitudes calculated in the DDM [6] at $\sqrt{s} = 50 \, \text{GeV}$ (full line) and at $\sqrt{s} = 500 \, \text{GeV}$ (dashed line); **b**)[left] $A_N(s,t)$ in the case of the U_T-matrix unitarisation with the new Born amplitudes for U_T calculated by new fit at $\sqrt{s} = 50 \, \text{GeV}$ (full line) and at $\sqrt{s} = 500 \, \text{GeV}$ (dashed line)

In order to fix this problem, we made a new fit of the scattering amplitude to obtain the correct description the differential cross sections at high energy in the framework of the U_T-matrix unitarisation. The new Born leads to the analysing power shown in Fig. 3b. The resulting Born term is rather different from that in the eikonal or U_e cases, but we see that all the unitarisation schemes lead to similar curves for the analysing power.

7 Conclusion

From the above analysis, we are led to the conclusion that the unusual properties of the U_T matrix unitarisation are not connected with its functional form. Other rational forms, such as the U_e matrix, have the same properties as the eikonal.

The spin-non-flip and spin-flip amplitudes also have very similar functional forms in the U_T and U_e schemes, differing only by an additional coefficient in the denominator. The comparison of the polarisation effects calculated for different unitarisation shows that the eikonal and U_e matrix qualitatively give the same results for the same Born term, but that a very different Born term needs to be used in the U_T-matrix case, suggesting that the underlying dynamical picture of the scattering must be quite different.

Acknowledgements

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