Day-ahead Security Assessment under Uncertainty Relying on the Combination of Preventive and Corrective Controls to Face Worst-Case Scenarios

Florin Capitanescu University of Liège, Liège, Belgium, fcapitanescu@yahoo.com

Stéphane Fliscounakis **Patrick Panciatici** RTE - DMA, Versailles, France, stephane.fliscounakis@rte-france.com

Louis Wehenkel University of Liège, Liège, Belgium, l.wehenkel@ulg.ac.be

Abstract - This paper deals with day-ahead static security assessment with respect to a postulated set of contingencies while taking into account uncertainties about the next day system conditions. We propose a heuristic approach to check whether, given some assumptions regarding these uncertainties, the worst case with respect to each contingency is still controllable by appropriate combinations of preventive and corrective actions. This approach relies on the solution of successive optimal power flow (OPF) and security-constrained optimal power flow (SCOPF) problems of a special type. The interest of the approach is shown by illustrative examples on the Nordic32 system.

Keywords - worst-case analysis, optimal power flow, security-constrained optimal power flow, operation under uncertainty, bi-level programming

1 Introduction

1.1 Motivation and related work

Increasing levels of uncertainties (e.g. wind power, cross-border interchanges, load evolution, etc.) make the traditional deterministic day-ahead operational planning approaches targeting system security for a single forecasted system state in a given period of time of the next day insufficient. To cope with uncertainties without relying on probabilistic methods, a possible approach consists in checking whether, given some assumptions regarding uncertainties (e.g. defined as intervals on bus active/reactive power injections), the worst case with respect to each contingency is still controllable by appropriate combinations of preventive and corrective actions.

So far the worst-case operating conditions of a power system under operational uncertainty have been tackled in the literature mostly in the framework of security margins [1, 2, 3, 4]. These approaches look for computing minimum security margins under operational uncertainty with respect to either thermal overload [2, 4] or voltage instability [1, 3, 4]. These approaches yield min-max optimization problems since a security margin represents by definition the maximum value of a so-called loading parameter for a given path of system evolution.

Several works have thus been devoted to determining the minimum distance to the boundary of a feasible space. Ref. [1] uses a constrained optimization formulation to compute the closest unfeasibility to a given operating point by defining the feasible region in the power injection space as the set of all power injections for which the load flow equations have a solution. Ref. [3] proposes an iterative and a direct method to compute the locally closest saddle-node bifurcation to the current operating point in the load power parameter space, based on the euclidian distance. Ref. [4] extracts information from unstable voltage trajectories, such as the left eigenvector to the point of collapse, in order to iteratively "redirect" the computation of a worst-case uncertainty pattern.

The case where the feasible region is bounded by inequality constraints defined by branch current limits is considered in [2, 4]. Ref. [2] proposes a method to find the thermal-constrained maximum transfer capability under the worst scenario in generation-load space, by formulating a min-max optimization problem whose constrains are derived from the DC load flow equations, and by solving it with the branch and bound method. Ref. [4] computes minimal thermal security margins by using a heuristic enumerative approach which relies on the sensitivities of branch currents with respect to uncertain parameters.

Ref. [5] sets-up a broader framework of the worst case approach as a three-stage decision making process including slow preventive controls (e.g. starting up a power plant, postponing maintenance works), fast preventive controls (e.g. generation rescheduling) and corrective (or emergency) controls (e.g. generation rescheduling, network switching, phase shifter actions, etc.). The worst case with respect to a contingency is formulated as a bi-level (min-max) optimization problem which, assuming a DC load flow approximation and hence focusing on thermal overload only, can be transformed into a MILP problem for which suitable solvers are available.

1.2 Paper contributions and organization

The present paper builds upon the framework of [5]. Its main contributions are as follows:

- A new heuristic approach is proposed to compute the worst-case. This approach focuses on identifying the constraints that are violated by the worst uncertainty pattern and relies on the solution of successive OPF and SCOPF problems of a special type.
- The worst scenario is computed separately with respect to overloads and undervoltages.
- The worst-case problem is considered in its nonlinear form (i.e. using the AC network model).

Section 2 provides the general formulation of the decision making process. Section 3 presents the proposed approach. Illustrative examples to support this approach are provided in Section 4. Section 5 concludes.

2 Formulation of the problem

The problem described in [5] aims to determine strategic/slow day-ahead decisions $\mathbf{u}_p$ such that for each scenario $\mathbf{s}$ that may show up the next day there exists a combination of preventive controls $\mathbf{u}_0(\mathbf{s})$ and of corrective (post-contingency) controls $\mathbf{u}_c(\mathbf{s}, c)$ leading to secure performance for any contingency $c \in \mathcal{K}$. We reduce this problem to (and iteration over) the following two steps:

- In day-ahead operation planning, determine for each contingency $c \in \mathcal{K}$ the worst-case operating scenario, considering optimal use of preventive/corrective actions in the next day.
- Determine a strategic decision $\mathbf{u}_p$ to relieve all the constraints violated for all the worst-case scenarios for which no effective combination of next-day preventive and corrective actions was found.

2.1 Computing a worst-case scenario for a contingency

The determination of the worst-case operating scenario for a contingency requires defining a "severity" measure to quantify operating conditions. A natural choice is to express this severity in terms of the maximum total amount of post-contingency constraints violation (e.g. L_1 norm of branch overloads, or undervoltages), although the formulation can be easily adapted to any other norm (e.g. L_2 or L_∞) if this is deemed more appropriate in a particular context (e.g. using a L_∞ norm leads to focus on network weak points since it considers the worst-case by focusing on the most strongly violated constraint). However, the worst-case relative to overloads and voltage violations should be computed separately, so as to avoid mixing up quantities with different meanings.

We define the worst-case operating scenario for a given contingency c as the scenario leading to the largest total violation of post-contingency constraints (either overloads, or voltage limit violations) in the presence of the best possible combination of preventive and corrective actions. Its computation can be done by solving the following *bi-level* optimization problem:

$$\max_{\mathbf{s},\boldsymbol{\delta}} \mathbf{1}^T\boldsymbol{\delta} \tag{1}$$

$$\text{s.t. } \mathbf{s}^{\min} \leq \mathbf{s} \leq \mathbf{s}^{\max} \tag{2}$$

$$\boldsymbol{\delta} \leq \boldsymbol{\delta}_c^\star \tag{3}$$

$$\mathbf{1}^T\boldsymbol{\delta}_c^\star = \min_{\mathbf{u}_0,\mathbf{u}_c,\boldsymbol{\delta}_c} \quad \mathbf{1}^T\boldsymbol{\delta}_c \tag{4}$$

$$\text{s.t.} \quad \mathbf{g}_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{s}) = \mathbf{0} \tag{5}$$

$$\mathbf{h}_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{s}) \leq \mathbf{0} \tag{6}$$

$$\mathbf{g}_c(\mathbf{x}_c, \mathbf{u}_0, \mathbf{u}_c, \mathbf{s}) = \mathbf{0} \tag{7}$$

$$\mathbf{h}_c(\mathbf{x}_c, \mathbf{u}_0, \mathbf{u}_c, \mathbf{s}) \leq \boldsymbol{\delta}_c \tag{8}$$

$$|\mathbf{u}_0 - \bar{\mathbf{u}}_0| \leq \Delta\mathbf{u}_0 \tag{9}$$

$$|\mathbf{u}_c - \mathbf{u}_0| \leq \Delta\mathbf{u}_c \tag{10}$$

$$\boldsymbol{\delta}_c \geq \mathbf{0}, \tag{11}$$

where $\mathbf{s}$ is a vector of uncertain bus active/reactive power injections which may vary between the limits $\mathbf{s}^{\min}$ and $\mathbf{s}^{\max}$, vector $\boldsymbol{\delta}$ monitors the worst violations of the post-contingency inequality constraints, subscript 0 (resp. c) refers to the base case or pre-contingency (resp. post-contingency) state, $\mathbf{x}_0$ (resp. $\mathbf{x}_c$) is the vector of state variables (i.e. magnitude and angle of voltages) in the pre-contingency (resp. post-contingency) state, $\mathbf{u}_0$ is the vector of preventive actions (e.g. generators active power, phase shifter angle, shunt reactive power injection, transformer ratio, etc.), $\bar{\mathbf{u}}_0$ is the vector of planned optimal settings of base case controls (e.g. obtained previously by a SCOPF which satisfies all contingency constraints relative to the most likely operating scenario forecasted for the considered period of time of the next day), $\Delta\mathbf{u}_0$ (resp. $\Delta\mathbf{u}_c$) are the maximal allowed variation of preventive (resp. corrective) actions, $\mathbf{u}_c$ is the vector of corrective actions (e.g. generators active power, phase shifter angle, network switching, etc.), $\boldsymbol{\delta}_c$ is a vector of positive relaxation terms of the post-contingency inequality constraints, function $\mathbf{g}$ denotes mainly the power flow equations in a given state, function $\mathbf{h}$ denotes the operating limits (e.g. maximal branch currents, or voltage limits) in a given state, constraints (9) and (10) aim to avoid unrealistic preventive and corrective actions, constraints (10) applying only to controls that are common in both pre- and post-contingency states. In this formulation, the strategic control actions $\mathbf{u}_p$ have not been made explicit because they are frozen at this optimization stage.

The solution of this bi-level problem can be interpreted as follows. For each possible value of the operating uncertainty vector $\mathbf{s}$ lying in the domain defined by constraints (2), the slave SCOPF problem (4)-(11) which includes only one contingency, called hereafter SCOPF-1C, is solved. Let $\mathbf{1}^T\boldsymbol{\delta}_c^\star$ be its optimal solution, i.e. the minimum overall violation of constraints (8). If this value is equal to zero, it means that the uncertainty pattern does not lead to any constraint violation provided that adequate preventive and/or corrective actions are available. After considering all the values of $\mathbf{s}$ satisfying (2), the worst-case scenario, which we denote with $\mathbf{s}_c^\star$, is that leading to the largest overall violation of post-contingency constraints.

Observe that this formulation looks only for the *existence of a feasible set of preventive and corrective actions for each scenario and contingency* rather than for their optimal values. Thus, if the optimal value of this problem is strictly positive it means that strategic actions $\mathbf{u}_p$ would be required to cover the considered contingency c. Otherwise, the considered contingency is not problematic by itself. We note also that the worst-case scenario may change according to the considered contingency c, and with the range of preventive/corrective control actions that are allowed, which in turn will depend on the choice of $\mathbf{u}_p$.

2.2 Computation of a common strategic decision $\mathbf{u}_p$

If, for one or for several contingencies, the optimization problem formulated in the previous section leads to a strictly positive objective, it means that system security can not be guaranteed by the sole combination of preventive and corrective controls applied during the next day.

In this case, it will be necessary to determine an appropriate strategic decision $\mathbf{u}_p$, so as to enhance the system controllability during the next day. While we do not handle this higher level problem in this paper, we formulate below an optimization problem that could help to choose such strategic decisions, for the sake of clarity.

Let us denote with $\mathcal{C} \subset \mathcal{K}$ the subset of contingencies which require strategic preventive actions, identified in the previous step by solving the optimization problem (1)-(11) for each contingency in $\mathcal{K}$. An optimal strategic decision $\mathbf{u}_p$ could then be computed by solving the following optimization problem, focusing on the set of worst-case scenarios $\{\mathbf{s}_c^\star \mid c \in \mathcal{C}\}$ identified at the previous step:

$$\min_{\mathbf{u}_p,\mathbf{u}_0^c,\mathbf{u}_k^c} f(\mathbf{u}_p) \tag{12}$$

$$\text{s.t.} \quad \mathbf{g}_0^c(\mathbf{x}_0^c, \mathbf{u}_p, \mathbf{u}_0^c, \mathbf{s}_c^\star) = \mathbf{0} \qquad c \in \mathcal{C} \tag{13}$$

$$\mathbf{h}_0^c(\mathbf{x}_0^c, \mathbf{u}_p, \mathbf{u}_0^c, \mathbf{s}_c^\star) \leq \mathbf{0} \qquad c \in \mathcal{C} \tag{14}$$

$$\mathbf{g}_k^c(\mathbf{x}_k^c, \mathbf{u}_p, \mathbf{u}_0^c, \mathbf{u}_k^c, \mathbf{s}_c^\star) = \mathbf{0} \qquad c \in \mathcal{C}, k \in \mathcal{K} \tag{15}$$

$$\mathbf{h}_k^c(\mathbf{x}_k^c, \mathbf{u}_p, \mathbf{u}_0^c, \mathbf{u}_k^c, \mathbf{s}_c^\star) \leq \mathbf{0} \qquad c \in \mathcal{C}, k \in \mathcal{K} \tag{16}$$

$$\mathbf{u}_p \in \mathcal{U}_p \tag{17}$$

$$|\mathbf{u}_0^c - \bar{\mathbf{u}}_0| \leq \Delta\mathbf{u}_0 \qquad c \in \mathcal{C} \tag{18}$$

$$|\mathbf{u}_k^c - \mathbf{u}_0^c| \leq \Delta\mathbf{u}_k \qquad c \in \mathcal{C}, k \in \mathcal{K} \tag{19}$$

where $f(\mathbf{u}_p)$ is a cost function of strategic preventive actions, $\mathcal{U}_p$ is the set of strategic preventive actions, $\mathbf{s}_c^\star$ is the worst uncertainty pattern of contingency c, $\mathbf{u}_0^c$ is the vector of preventive actions corresponding to the worst case of contingency c, $\mathbf{x}_0^c$ is the vector of state variables corresponding to the worst case of contingency c, $\mathbf{u}_k^c$ is the vector of corrective actions in post-contingency state k corresponding to the worst case of contingency c, and $\mathbf{x}_k^c$ is the vector of state variables in post-contingency state k corresponding to the worst case of contingency c.

Notice that to simplify the problem formulation the constraints relative to the most likely state (i.e. obtained by using $\mathbf{s}_c^\star = \mathbf{0}$ in the above formulation) have not been explicitly highlighted. Observe also that for each contingency c the preventive actions $\mathbf{u}_0^c$ must not only satisfy the constraints relative to the worst case relative to this contingency but for all postulated contingencies $k \in \mathcal{K}$, given the available corrective actions $\mathbf{u}_k^c, k \in \mathcal{K}$.

The size of this SCOPF-like problem might be very large, i.e. $|\mathcal{C}|$ times larger than the size of a classical SCOPF. Appropriate techniques aiming to decompose the problem (e.g. by identifying the binding constraints at the optimum) would thus be required in practical conditions in order to reduce the problem size [6]. Furthermore, once common strategic actions $\mathbf{u}_p$ have been computed, the worst-cases with respect to the new system state must be re-computed by solving again the problem (1)-(11) for each contingency $k \in \mathcal{K}$. If, subsequently to this computation some worst-cases are found that still require strategic actions, their constraints must be added to above optimization problem and new iterations must be performed.

Clearly, due to the infinite number of possible uncertainty patterns $\mathbf{s}$, this approach can not guarantee that common strategic actions will be found after a finite number of iterations. Nevertheless, at each iteration the strategic control actions determined lead to a more secure strategy than at the previous iteration (e.g. starting up a power plant generally enhances security by providing an additional degree of freedom), thus yielding an anytime optimization framework for day-ahead risk management.

3 The proposed approach for computing the worst uncertainty pattern for a single contingency

3.1 *Principle and assumptions*

Nowadays there is no theoretically or practically sound algorithm able to solve in a generic way the bi-level programming problem (1)-(11), given its features: non-convex, non-linear, and large scale [7]. Consequently, in the power systems area, only linear approximations of nonlinear bi-level optimization problems have been reported [2, 5, 8]. Furthermore, although the formulation (1)-(11) fits into a Monte-Carlo simulation framework (i.e. that solves the classical SCOPF (4)-(11) for any uncertain scenario), such approach is computationally intractable unless one considers only a limited number of uncertain scenarios $\mathbf{s}$ which strongly limits the aim of our approach.

In this paper we propose a practical heuristic approach aiming to provide an acceptable solution of the original bi-level programming problem by decomposing it into a number of OPF- or SCOPF-like problems.

Furthermore, in order to provide useful information for the TSO, our approach distinguishes between four classes of contingencies, according to the type of control actions required by a contingency to meet the worst-case constraints:

- contingencies that do not require any control action;
- contingencies that require only corrective actions ($\mathbf{u}_c$);
- contingencies that require both preventive and corrective actions ($\mathbf{u}_0, \mathbf{u}_c$);
- contingencies that require strategic, preventive, and corrective actions ($\mathbf{u}_p, \mathbf{u}_0, \mathbf{u}_c$).

To ease the approach comprehension let us assume that neither preventive nor corrective actions are allowed to satisfy post-contingency constraints. In this particular case the general bi-level problem (1)-(11) becomes:

$$\max_{\mathbf{s},\boldsymbol{\delta}} \mathbf{1}^T\boldsymbol{\delta} \tag{20}$$

$$\text{s.t.} \ \mathbf{s}^{\min} \leq \mathbf{s} \leq \mathbf{s}^{\max} \tag{21}$$

$$\boldsymbol{\delta} \leq \boldsymbol{\delta}_c^\star \tag{22}$$

$$\mathbf{1}^T\boldsymbol{\delta}_c^\star = \min_{\boldsymbol{\delta}_c} \mathbf{1}^T\boldsymbol{\delta}_c \tag{23}$$

$$\text{s.t.} \ \mathbf{g}_0(\mathbf{x}_0, \mathbf{s}) = \mathbf{0} \tag{24}$$

$$\mathbf{h}_0(\mathbf{x}_0, \mathbf{s}) \leq \mathbf{0} \tag{25}$$

$$\mathbf{g}_c(\mathbf{x}_c, \mathbf{s}) = \mathbf{0} \tag{26}$$

$$\mathbf{h}_c(\mathbf{x}_c, \mathbf{s}) \leq \boldsymbol{\delta}_c \tag{27}$$

$$\boldsymbol{\delta}_c \geq \mathbf{0}. \tag{28}$$

We denote with $\mathcal{VC}$ the set of constraints that have been relaxed at the solution of the optimization problem (20)-

(28), and hence for which $\boldsymbol{\delta}_c^\star > 0$, or equivalently the set of original constraints $\mathbf{h}_c(\mathbf{x}_c, \mathbf{s}) \leq \mathbf{0}$ violated by the worst uncertainty pattern.

The proposed approach relies on the observation that if the set $\mathcal{VC}$ was known *beforehand*, then the worst uncertainty pattern and its corresponding maximum degree of constraints violation could be computed by solving the following SCOPF-1C problem (a detailed formulation of this problem is provided in the Appendix):

$$\mathbf{s}_c^\star = \arg\max_{\mathbf{s}} \quad \sum_{j \in \mathcal{VC}} h_{cj}(\mathbf{x}_c, \mathbf{s}) \tag{29}$$

$$\text{s.t.} \quad \mathbf{s}^{\min} \leq \mathbf{s} \leq \mathbf{s}^{\max} \tag{30}$$

$$\mathbf{g}_0(\mathbf{x}_0, \mathbf{s}) = \mathbf{0} \tag{31}$$

$$\mathbf{h}_0(\mathbf{x}_0, \mathbf{s}) \leq \mathbf{0} \tag{32}$$

$$\mathbf{g}_c(\mathbf{x}_c, \mathbf{s}) = \mathbf{0}, \tag{33}$$

where $h_{cj}(\mathbf{x}_c, \mathbf{s})$ is the value of j-th component of the vector $\mathbf{h}_c(\mathbf{x}_c, \mathbf{s})$. For instance when focusing on the worst case with respect to overloads, the constraint $h_{cj}(\mathbf{x}_c, \mathbf{s}) \leq 0$ corresponds to a branch thermal limit of type $I_{cj} - I_{cj}^{\max} \leq 0$. Here, the inequality constraints which do not belong to the set $\mathcal{VC}$ have been removed beforehand from the SCOPF-1C problem because they are supposed to be known a priori as being anyway satisfied (by definition of the set $\mathcal{VC}$, i.e. $h_{cj}(\mathbf{x}_c, \mathbf{s}) \leq 0, \forall j \notin \mathcal{VC}$).

The aim of the proposed approach is therefore to compute the worst uncertainty pattern by identifying in a *combinatorial fashion* the set $\mathcal{VC}$. To this end we identify the set $\mathcal{APC}$ of *all possible sets of problematic constraints*, where a set $\mathcal{PC}$ of *problematic constraints* comprises post-contingency constraints for which there exists an uncertainty pattern leading to their *simultaneous violation* in the *absence* of any preventive/corrective action. Each set $\mathcal{PC}$ has associated a worst uncertainty pattern, i.e. a pattern that leads to the largest total violation of all the constraints of this set, which we call *problematic pattern*. We denote with $\mathcal{PP}$ the set of problematic patterns corresponding to all possible sets of problematic constraints $\mathcal{APC}$.

The proposed approach comprises three main steps that are described hereafter in sections 3.2, 3.3, and 3.4.

3.2 *Determination of the set of problematic patterns without any preventive/corrective action*

The proposed algorithm is as follows:

0. Initialization: $\mathcal{APC} = \emptyset$, and $\mathcal{PP} = \emptyset$.

1. For each inequality constraint $j = 1, \ldots, n_h$, where n_h is the size of vector $\mathbf{h}_c$, compute its corresponding worst uncertainty pattern (i.e. that maximizes the violation of post-contingency constraint j) by solving the following SCOPF-1C problem:

$$\mathbf{s}_{cj}^\star = \arg\max_{\mathbf{s}} \quad h_{cj}(\mathbf{x}_c, \mathbf{s}) \tag{34}$$

$$\text{s.t.} \quad \mathbf{s}^{\min} \leq \mathbf{s} \leq \mathbf{s}^{\max} \tag{35}$$

$$\mathbf{g}_0(\mathbf{x}_0, \mathbf{s}) = \mathbf{0} \tag{36}$$

$$\mathbf{h}_0(\mathbf{x}_0, \mathbf{s}) \leq \mathbf{0} \tag{37}$$

$$\mathbf{g}_c(\mathbf{x}_c, \mathbf{s}) = \mathbf{0}. \tag{38}$$

   If the objective of this optimization problem is less or equal to zero, it means that, whatever the uncertainty pattern, the constraint j is always satisfied. Consequently this constraint may be *omitted* in the subsequent steps of the algorithm.

   Because for the computation of the maximum violation of constraint j the other post-contingency inequality constraints $h_{ci}(\mathbf{x}_c, \mathbf{s}) \leq 0, i \neq j$ have been removed from the optimization problem some of them may be violated at the optimum.

   If only constraint j is violated at this SCOPF-1C solution, we augment the set of problematic patterns $\mathcal{PP} \leftarrow \mathcal{PP} \cup \{\mathbf{s}_{cj}^\star\}$. Otherwise, augment the set $\mathcal{APC} \leftarrow \mathcal{APC} \cup \mathcal{PC}_j$, where the set of problematic constraints $\mathcal{PC}_j$ is composed by all violated constraints at the SCOPF-1C (34)-(38) solution.

2. Compute the worst uncertainty pattern of each set of problematic constraints $\mathcal{PC}_j \in \mathcal{APC}$ by solving the SCOPF-1C (29)-(33), with set $\mathcal{PC}_j$ replacing set $\mathcal{VC}$. Let $\mathbf{s}_{c\mathcal{PC}_j}^\star$ denote the worst uncertainty pattern derived from this problem (note that this step is skipped for sets $\mathcal{PC}_j$ that contain a single constraint, since this computation has been already performed in the previous step). Augment the set of problematic uncertainty patterns $\mathcal{PP} \leftarrow \mathcal{PP} \cup \{\mathbf{s}_{c\mathcal{PC}_j}^\star\}$.

3. Notice that if, for each and every constraint $j = 1, \ldots, n_h$, the objective of the SCOPF-1C (34)-(38) is less or equal to zero, then the worst uncertainty pattern for the contingency c does not lead to any post-contingency constraint violation and the overall computation terminates.

3.3 *Checking whether corrective actions alone suffice to face the identified problematic patterns*

For each problematic scenario $\mathbf{s} \in \mathcal{PP}$ indentified in the previous step, we check whether corrective actions alone would suffice to remove the violated constraints, by solving the following OPF problem:

$$\min_{\mathbf{u}_c, \boldsymbol{\delta}_c} \mathbf{1}^T \boldsymbol{\delta}_c \tag{39}$$

$$\text{s.t.} \ \mathbf{g}_c(\mathbf{x}_c, \mathbf{u}_c, \mathbf{s}) = \mathbf{0} \tag{40}$$

$$\mathbf{h}_c(\mathbf{x}_c, \mathbf{u}_c, \mathbf{s}) \leq \boldsymbol{\delta}_c \tag{41}$$

$$|\mathbf{u}_c - \bar{\mathbf{u}}_0| \leq \Delta \mathbf{u}_c \tag{42}$$

$$\boldsymbol{\delta}_c \geq \mathbf{0}, \tag{43}$$

where $\bar{\mathbf{u}}_0$ are the optimal settings of base case controls computed by the classical SCOPF for the most likely operation state.

Observe that this problem does not include base case constraints of type (36)-(37) since any stress pattern $\mathbf{s}$ computed from the SCOPF-1C (34)-(38) must indeed satisfy these constraints.

If the objective (39) is equal to zero the TSO may want to compute what is the minimum amount of corrective actions to remove constraint violations. This can then be achieved by using $\min\ |\mathbf{u}_c - \bar{\mathbf{u}}_0|$ as objective function together with $\boldsymbol{\delta}_c = \mathbf{0}$ in constraints (41).

3.4 *Checking whether both preventive/corrective actions suffice to face the identified problematic patterns*

For each problematic scenario $\mathbf{s} \in \mathcal{PP}$ for which corrective actions alone do not suffice to solve the problem, we check whether a suitable combination of preventive and corrective actions would be able to meet post-contingency constraints, by solving the following SCOPF-1C problem:

$$\min_{\mathbf{u}_0, \mathbf{u}_c, \boldsymbol{\delta}_c} \ \mathbf{1}^T \boldsymbol{\delta}_c \tag{44}$$

$$\text{s.t.} \ \ \mathbf{g}_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{s}) = \mathbf{0} \tag{45}$$

$$\mathbf{h}_0(\mathbf{x}_0, \mathbf{u}_0, \mathbf{s}) \leq \mathbf{0} \tag{46}$$

$$\mathbf{g}_c(\mathbf{x}_c, \mathbf{u}_0, \mathbf{u}_c, \mathbf{s}) = \mathbf{0} \tag{47}$$

$$\mathbf{h}_c(\mathbf{x}_c, \mathbf{u}_0, \mathbf{u}_c, \mathbf{s}) \leq \boldsymbol{\delta}_c \tag{48}$$

$$|\mathbf{u}_0 - \bar{\mathbf{u}}_0| \leq \Delta \mathbf{u}_0 \tag{49}$$

$$|\mathbf{u}_c - \mathbf{u}_0| \leq \Delta \mathbf{u}_c \tag{50}$$

$$\boldsymbol{\delta}_c \geq \mathbf{0}. \tag{51}$$

The worst uncertainty pattern identified for the contingency c thus corresponds to the problematic pattern leading to the largest value of the objective (44). As in the previous case, if the objective (44) is zero the TSO may want to compute what is the minimum cost or amount of preventive actions (e.g. $\min \ \mathbf{c}^T|\mathbf{u}_0 - \bar{\mathbf{u}}_0|$) while using $\boldsymbol{\delta}_c = \mathbf{0}$ in constraints (48).

Otherwise, if for at least one uncertainty pattern in $\mathcal{PP}$ the objective (44) is positive then the best combination of preventive and corrective actions is not able to meet post-contingency constraints and hence strategic preventive actions will be required.

3.5 *Remarks*

A drawback of the proposed algorithm is that, since the worst uncertainty pattern computed depends on the type of control actions allowed (e.g. ranging from no action allowed to both preventive and corrective actions allowed), the algorithm may not provide the same solution as the original bi-level optimization problem (4)-(11). Nevertheless, the way of problem decomposition makes sense from an engineering point of view.

On the other hand, since all constraints are enumerated in step 1, the algorithm can identify tricky situations where patterns leading to smaller constraint violations than the worst pattern turn out to be more dangerous than the worst pattern because no efficient control actions are available.

3.6 *Computational issues*

The proposed algorithm is computationally intensive and depends on the total number of inequality constraints n_h, the size of the set $\mathcal{PP}$, and the number of postulated contingencies (the size of set $\mathcal{K}$). To reduce its computational time three solutions can be envisaged:

- use parallel computations for the various SCOPF-1C problems;
- the solution of OPF (39)-(43) can be skipped, since it is performed for the sake of distinguishing between cases where corrective actions alone suffice or not to satisfy worst-case constraints, and replaced with the solution of SCOPF (44)-(51);
- not all inequalities $\mathbf{h}_c(\mathbf{x}_c, \mathbf{s}) \leq \mathbf{0}$ should be treated but only those that are closer to their limits and hence prone to be violated (i.e. the weak-points). TSO expertise can be very useful to filter-out harmless constraints and reduce the set of postulated contingencies $\mathcal{K}$.

4 Numerical results

4.1 *Description of the test system*

We consider a variant of the "Nordic 32" system [9], shown in Fig. 1. The system contains 60 buses, 23 generators, 57 lines, 22 loads, 14 shunts, 27 transformers with fix rations, and 4 transformers with variable ratio.

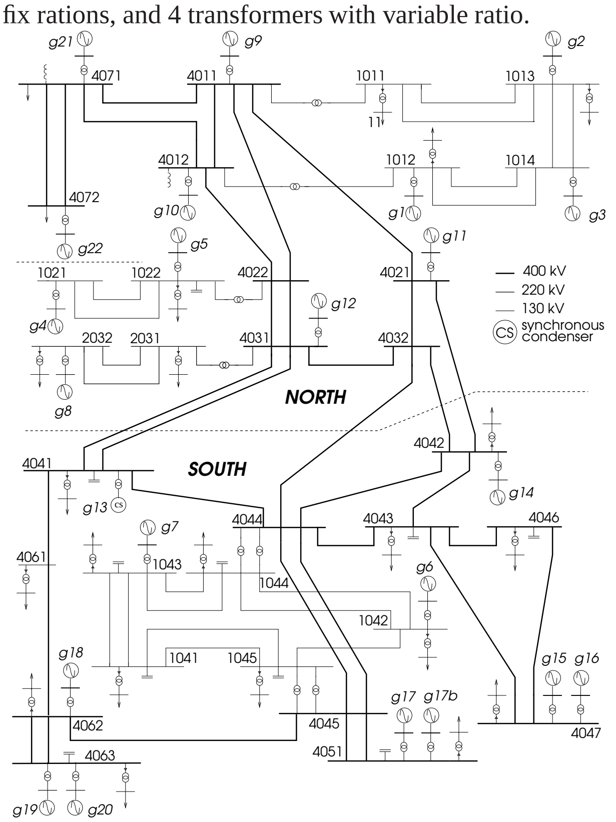

Figure 1: The modified Nordic32 test system.

4.2 *Simulation assumptions*

Uncertainty consists in variable active and reactive power injections at any load bus, modeled by constraints (54)-(55), in the range of -5% to +5% of the nominal active/reactive load. Furthermore, the total variation of uncertain active (resp. reactive) power injections, modeled by constraints (56)-(57), is trimmed to the range +/- 100 MW (resp. MVar).

We consider a list of 33 N-1 contingencies.

The following simulation cases are considered:

- **case 0**: the contingency is simulated at the classical SCOPF solution by a power flow program (hence without considering any corrective action);
- **case WP**: the worst uncertainty pattern (WP) corresponding to the contingency, computed by solving the SCOPF-1C (29)-(33);

- **case WP+CA**: the worst uncertainty pattern corresponding to the contingency considering corrective actions (CA), computed by solving the OPF (39)-(43);
- **case WP+PA+CA**: the worst uncertainty pattern corresponding to the contingency considering both preventive actions (PA) and corrective actions, computed by solving the SCOPF-1C (44)-(51).

We consider separately the problems of thermal overload constraints and voltage magnitude constraints.

4.3 Worst-case with respect to thermal overload

4.3.1 Type of allowed preventive/corrective actions

To satisfy worst cases constraints we consider that both preventive and corrective actions are only of type generation rescheduling. Table 1 shows the range of allowed preventive actions (PA), as up/down deviations with respect to the classical SCOPF settings, and corrective actions (CA), as up/down deviations with respect to the precontingency state. The overall amount of preventive (resp. corrective) actions is of 584.6 (resp. 120) MW.

Table 1: Range of generation rescheduling (MW) as preventive and corrective actions

generator	g1	g2	g3	g4	g5	g6
PA	21.6	16.2	18.9	16.2	14.2	16.8
CA	20			10		
generator	g7	g8	g9	g10	g11	g12
PA	11.4	28.9	27.0	21.6	17.1	17.2
CA	10					
generator	g13	g14	g15	g16	g17	g17b
PA		18.9	84	16.2	18.9	18.9
CA			40		10	
generator	g18	g19	g20	g21	g22	
PA	16.2	16.2	16.2	16.8	135	
CA			10		20	

4.3.2 Computation of the worst uncertainty pattern for a contingency

We first compute a reference schedule for the nominal scenario by minimizing generation cost with a SCOPF formulation [6] including the 33 contingencies and relying on the preventive/corrective actions provided in Table 1. At this SCOPF optimum we compute the worst uncertainty pattern for each contingency.

We illustrate the search procedure of the worst uncertainty pattern, described in sections 3.2, 3.3, and 3.4, for the loss of line 4011-4021.

At step 1 of the algorithm we notice that, only for 3 lines (4031-4032, 4012-4022, and 4022-4031) considered separately, there exists uncertain patterns leading to overload. In particular the worst pattern with respect to any of these 3 lines also overloads the two other lines (see Table 2). Then we build up all sets of problematic constraints as all possible combinations among these 3 lines.

At step 2 we compute the worst pattern for each set of problematic constraints. Table 2 provides the lines overloaded for the 6 sets of problematic constraints $\mathcal{PC}$. Due to the simplicity of the test network and the small number of lines overloaded, only two problematic patterns (set $\mathcal{PP}$) have been found. For instance the worst pattern for the overload of line 4031-4032 coincides with the worst pattern for any set of problematic constraints $\mathcal{PC}$ which includes line 4031-4032. Also, the worst patterns for the overload of lines 4012-4022 and 4022-4031 coincide, as expected, given the location of these lines (see Fig. 1).

Table 2: Lines overloaded (%) and overall overload (%) for all sets of problematic constraints $\mathcal{PC}$

all sets $\mathcal{PC}$	lines overloaded 4031-4032	4012-4022	4022-4031	overall overload
4031-4032	19.7	7.0	2.5	29.2
4012-4022	13.3	7.2	2.7	23.2
4022-4031	13.3	7.2	2.7	23.2
4031-4032, 4012-4022	19.7	7.0	2.5	29.2
4031-4032, 4022-4031	19.7	7.0	2.5	29.2
4012-4022, 4022-4031	13.3	7.2	2.7	23.2
4031-4032, 4012-4022, 4022-4031	19.7	7.0	2.5	29.2

Next we check for the two problematic patterns whether the preventive/corrective actions suffice, and provide in Table 3 the loading of critical lines in various cases.

Table 3: Loading (%) of critical lines in various cases for the two problematic patterns

line	0	WP	WP+CA	WP+CA+PA
first problematic pattern				
4031-4032	**102.4**	**119.7**	**116.3**	**109.3**
4012-4022	95.3	**107.0**	**103.7**	99.9
4022-4031	90.1	**102.5**	99.4	91.1
second problematic pattern				
4031-4032	**102.4**	**113.3**	**107.2**	**100.5**
4012-4022	95.3	**107.2**	**103.9**	100.0
4022-4031	90.1	**102.7**	99.8	92.0

We conclude that the first problematic pattern is the worst pattern for this contingency as it leads to the largest overall overload in the case WP+CA+PA.

4.3.3 Contingencies not requiring any control action

The loss of line 4011-4012 belongs to this class because in the case WP the most loaded line is 4012-4022 with a loading of 77.4 % but no line is overloaded. Consequently, for this contingency no branch is overloaded whatever the uncertainty pattern in the assumed range.

4.3.4 Contingencies requiring only corrective actions

We have not identified any contingency in this class because, on the one hand, the range of corrective actions is much smaller than the assumed uncertain injections and, on the other hand, no contingency that satisfies all constraints in case 0 violates any constraint in case WP (due to most loaded lines are significantly below their limit).

4.3.5 *Contingencies requiring both preventive and corrective actions*

The loss of line 4042-4044 belongs to this class. Table 4 provides the loading of line 4042-4043 in various cases.

Table 4: Loading of line 4042-4043 (%) in various cases

line	0	WP	WP+CA	WP+CA+PA
4042-4043	**100.6**	**113.3**	**111.1**	100.0

4.3.6 *Contingencies requiring strategic decisions*

Table 3 shows that both preventive and corrective actions do not suffice to remove the overload for the worst case of contingency 4011-4021 and therefore strategic preventive actions are required.

4.3.7 *Comparison with the DC approximation*

We compute the worst uncertainty pattern (case WP) by the proposed approach and by the approach of [5] for the contingency 4042-4044 (see section 4.3.5). To enable a fair comparison we consider that the overall variation of uncertain active/reactive power injections, modeled by constraints (56)-(57), is zero. In this case both approaches provide the same worst pattern (although obviously different overloads e.g. 115.7 % vs. 125.3 %). This result is due to: the normal load level of this operating point, the low impact of reactive power injections (indeed from the overload of 113.3-100.6=12.7 %, see Table 4, uncertain reactive injections count for only 0.4 % of the overload while active powers count for the remaining 12.3 %), the rather small number and range of uncertain injections.

4.4 *Worst-case with respect to undervoltage limits*

We first perform a classical corrective SCOPF which minimizes the active power losses and considers the 33 postulated contingencies [6]. Preventive and corrective actions are shunt reactive power and transformer ratio with the ranges provided in Table 5. Voltage limits are chosen as 0.95 pu (resp. 0.92 pu) and 1.05 pu in base case (resp. contingency) state.

4.4.1 *Range and type of preventive/corrective actions*

Table 5 provides the range and type of preventive/corrective actions as up/down deviations with respect to the classical SCOPF settings.

Table 5: Range and type of preventive and corrective actions

shunts	all (MVar)	*transformers ratio*	all (pu)
PA	+/- 80	PA	+/- 0.05
CA	+/- 40	CA	+/- 0.02

4.4.2 *Contingencies not requiring any control action*

For the loss of line 4041-4061 no voltage limit is violated in case 0, the lowest voltage being of 0.943 pu (it drops with 0.071 pu due to the contingency) at bus 4061. In the case WP the lowest voltage is again at bus 4061 with a value of 0.921 pu but still slightly above the limit. Consequently, for this contingency no voltage limit is violated whatever the uncertainty pattern in the assumed range.

4.4.3 *Contingencies requiring only corrective actions*

Table 6 yields the voltage at the most affected buses in various cases for the loss of line 4043-4047. Observe that in the WP case two voltages violate the minimum post-contingency voltage limit of 0.92 pu. However, both voltages are brought back within their limits thanks to corrective actions only.

Table 6: Voltage (pu) for most affected buses in various cases

bus	0	WP	WP+CA
4046	0.935	**0.884**	> 0.92
4047	0.949	**0.915**	> 0.92

4.4.4 *Contingencies requiring strategic decisions*

Table 7 provides the voltage at the most affected bus in various cases for the loss of line 4061-4062. Observe that the voltage at bus 4061 drops severly under the minimal limit due to contingency (case 0) and further falls significantly for the worst uncertainty pattern (case WP). Since for the best combination of preventive/corrective actions (case WP+CA+PA) this voltage is still lower than the minimal limit strategic preventive actions are required. Note that despite the reasonably large amount of preventive actions their full use is limited by the risk of over-voltages in the base case.

Table 7: Voltage (pu) for most affected bus in various cases

bus	0	WP	WP+CA	WP+CA+PA
4061	**0.875**	**0.823**	**0.871**	**0.895**

5 Conclusion and future works

This paper has proposed a heuristic approach to compute the worst-case under operation uncertainty for a contingency with respect to static constraints (e.g. overloads and under-voltages), and to check whether there exists appropriate combinations of preventive and corrective actions to face this worst-case.

The untractable benchmark bi-level worst-case optimization problem is decomposed into more tractable OPF-like and SCOPF-like optimization problems which are solved sequentially. However, although its assumptions make engineering sense, this heuristic approach does not guarantee to provide the same solution as the benchmark worst-case problem. Unfortunately, no method exists yet to check this assumption.

Future research will be devoted to the problem of finding strategic decisions when the best combination of preventive/corrective actions do not suffice to satisfy the constraints relative to the worst-case of a contingency. Another extension of this work concerns the handling of challenging discrete corrective actions (e.g. topology changes) which increases considerably the difficulty of the worst-case problem.

Acknowledgments

This paper presents research results of the European FP7 project PEGASE.

Louis Wehenkel and Florin Capitanescu also acknowledge the support of the Belgian Network DYSCO (Dynamical Systems, Control, and Optimization), funded by the Interuniversity Attraction Poles Programme, initiated by the Belgian State, Science Policy Office. The scientific responsibility rests with the authors.

6 Appendix

The compact SCOPF-1C formulation (29)-(33) can be detailed as follows:

$$\max_{\mathbf{P}_u,\mathbf{Q}_u} \sum_{ij\in\mathcal{VC}} I^c_{ij}(V^c_i, V^c_j, \theta^c_i, \theta^c_j) \quad \text{or} \tag{52}$$

$$\min_{\mathbf{P}_u,\mathbf{Q}_u} \sum_{i\in\mathcal{VC}} V^c_i \tag{53}$$

$$\text{s.t. } P^{\min}_{ui} \le P_{ui} \le P^{\max}_{ui}, \quad \forall i \in \mathcal{N} \tag{54}$$

$$Q^{\min}_{ui} \le Q_{ui} \le Q^{\max}_{ui}, \quad \forall i \in \mathcal{N} \tag{55}$$

$$P^{\min}_{u} \le \sum_{i\in\mathcal{N}} c_{Pi} P_{ui} \le P^{\max}_{u} \tag{56}$$

$$Q^{\min}_{u} \le \sum_{i\in\mathcal{N}} c_{Qi} Q_{ui} \le Q^{\max}_{u} \tag{57}$$

$$P^0_{gi} - P_{li} + c_{Pi}P_{ui} - \sum_{j\in\mathcal{B}^0_i} P^0_{ij}(V^0_i, V^0_j, \theta^0_i, \theta^0_j) = 0, \quad \forall i \in \mathcal{N} \tag{58}$$

$$Q^0_{gi} - Q_{li} + c_{Qi}Q_{ui} - \sum_{j\in\mathcal{B}^0_i} Q^0_{ij}(V^0_i, V^0_j, \theta^0_i, \theta^0_j) = 0, \quad \forall i \in \mathcal{N} \tag{59}$$

$$Q^{\min}_{gi} \le Q^0_{gi} \le Q^{\max}_{gi}, \quad \forall i \in \mathcal{G} \tag{60}$$

$$I^0_{ij}(V^0_i, V^0_j, \theta^0_i, \theta^0_j) \le I^{\max 0}_{ij}, \quad \forall i,j \in \mathcal{N} \tag{61}$$

$$V^{\min 0}_i \le V^0_i \le V^{\max 0}_i, \quad \forall i \in \mathcal{N} \tag{62}$$

$$P^c_{gi} - P_{li} + c_{Pi}P_{ui} - \sum_{j\in\mathcal{B}^c_i} P^c_{ij}(V^c_i, V^c_j, \theta^c_i, \theta^c_j) = 0, \quad \forall i \in \mathcal{N} \tag{63}$$

$$Q^c_{gi} - Q_{li} + c_{Qi}Q_{ui} - \sum_{j\in\mathcal{B}^c_i} Q^c_{ij}(V^c_i, V^c_j, \theta^c_i, \theta^c_j) = 0, \quad \forall i \in \mathcal{N} \tag{64}$$

$$Q^{\min}_{gi} \le Q^c_{gi} \le Q^{\max}_{gi}, \quad \forall i \in \mathcal{G} \tag{65}$$

where, superscript 0 (resp. c) refers to the base case (resp. contingency c state), objectives (52) and (53) refer respectively to overloads and undervoltages, P_{ui} (resp. Q_{ui}) denotes uncertain active (resp. reactive) power injection at bus i, $c_{Pi}, c_{Qi} \in \{0, 1\}$ are coefficients indicating buses where power injections are uncertain (i.e. $c_{Pi} = 1$ or $c_{Qi} = 1$), $\mathcal{N}$ is the set of buses, $\mathcal{G}$ is the set of generators, $\mathcal{B}_i$ is the set of branches connected to bus i, the other notations being self-explanatory. A slack generator, not shown explicitly in this formulation, is chosen to clear the mismatch due to uncertain injections. Uncertain injections are limited at each individual bus by constraints (54) and (55) as well as overall by constraints (56) and (57).

Note that since the base case constraints (58)-(62) are generally less restrictive than contingency constraints, they are also satisfied for the worst contingency pattern, which allows further simplification of this formulation.

REFERENCES

[1] J. Jarjis, F.D. Galiana, "Quantitative analysis of steady state stability in power networks", IEEE Trans. on Power Apparatus and Systems, vol. PAS-100, no. 1, 1981, pp. 318-326

[2] D. Gan, X. Luo, D.V. Bourcier, R.J. Thomas, "Min-Max Transfer Capability of Transmission Interfaces", International Journal of Electrical Power and Energy Systems, vol.25, no. 5, 2003, pp. 347-353

[3] I. Dobson, L. Lu, "New methods for computing a closest saddle node bifurcation and worst case load power margin for voltage collapse", IEEE Trans. on Power Systems, vol. 8, no. 2, 1993, pp. 905-911

[4] F. Capitanescu, T. Van Cutsem, "Evaluating bounds on voltage and thermal security margins under power transfer uncertainty", PSCC Conference, Seville (Spain), June 2002

[5] P. Panciatici, Y. Hassaine, S. Fliscounakis, L. Platbrood, M. Ortega-Vazquez, J.L. Martinez-Ramos, L. Wehenkel, "Security management under uncertainty: from day-ahead planning to intraday operation", IREP Symposium, Buzios (Brazil), 2010

[6] F. Capitanescu, L. Wehenkel, "A new iterative approach to the corrective security-constrained optimal power flow problem" IEEE Trans. on Power Systems, vol. 23, no.4, November 2008, pp. 1342-1351

[7] B. Colson, P. Marcotte, G. Savard, "An overview of bilevel optimization", Annals of Operations Research, vol. 1, 2007, pp. 235-256

[8] J. M. Arroyo, F. D. Galiana, "On the Solution of the Bilevel Programming Formulation of the Terrorist Threat Problem", IEEE Trans. on Power Systems, vol. 20, no. 2, May 2005, pp. 789-797

[9] Nordic32 system, CIGRE Task Force 38.02.08, "Long-Term Dynamics, Phase II", 1995.