1. Introduction

In studies of bedrock river incision in response to a base level change, one frequently models the upstream propagation of a wave of erosion by means of the stream power model, expressing the erosion rate E as

\[ E = K A^{m/n} \]  

(1)

where \( K \) characterizes the erosional efficiency, \( A \) is the drainage area, \( S \) the channel slope, and \( m \) and \( n \) are positive constants derived from exponents involved in various relations linking drainage area, discharge and channel width. A general expression for the knickpoint propagation celerity \( c \) has often been derived from this equation as

\[ c = (m/n)^{1/n} K \]  

(2)

where \( K = K_0 U^{m/n} \) and \( U \) is the uplift rate. The parameters appearing in these two equations can be determined by knickpoint propagation modelling. The aim is usually to minimize the stream-wise distance between observed and modelled knickpoint positions, but are the distance residual the most efficient indicator of the best fit?

2. Data set

68 streams have been selected in the Ourthe basin (2460 km², Ardennes massif, Belgium) (see Fig. 1). Mainly composed of Paleozoic slate rocks, this watershed was affected by an erosion wave which entered ~700 ka ago and caused the abandonment of the so-called ‘Younger Main Terrace’ (YMT) level (Nahas et al., 2012).

From 80 knickpoints detected in slope-area space, 18 have been attributed to the post-YMT erosion phase by geometrical correlation with the YMT profile (Fig. 1 and 3).

3. Methods

We used eq. (2) to model the knickpoint propagation in the Ourthe watershed. The parameterization was carried out by a modified stream power model with a drainage area exponent, \( m/n \), varying linearly between 0.3 and 1.5, and \( K_0 \) logarithmically between 10⁻¹² and 10⁻⁸. We tested 3 adjustment methods:

(1) Distance-based adjustment: least squares adjustment based on the stream-wise distances between observed and modelled knickpoint positions at time \( t = 0.7 \) Ma;

(2) Time-based adjustment: least square adjustment based on differences between observed and modelled knickpoint positions at the actual knickpoint location;

(3) Least rectangles adjustment: intermediate method based on the minimization of the sum of the products between distance residuals and time residuals.

4. Results

The \( m/n \), best fit 68 value significantly varies between the adjusted least squares (0.86) and the time-based one (0.75), the least rectangle best fit yielding an intermediate result of \( m/n = 0.78 \) (Fig. 4).

Residues change between methods (Table 1). They show Gaussian-like distribution, which is consistent with a good model performance. For example, time residuals for the time-based adjustment are shown as Fig. 5.

Table 1 – Results of the three types of adjustment performed on the knickpoint data set of the Ourthe catchment

<table>
<thead>
<tr>
<th>Method</th>
<th>( m/n )</th>
<th>( K ) (10⁻¹⁵ m³/s)</th>
<th>Time residual (s)</th>
<th>Distance residual (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance-based</td>
<td>0.86</td>
<td>96.3</td>
<td>66.27</td>
<td>178.61</td>
</tr>
<tr>
<td>Time-based</td>
<td>0.75</td>
<td>96.3</td>
<td>66.27</td>
<td>178.61</td>
</tr>
<tr>
<td>Least rectangles</td>
<td>0.78</td>
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<td>178.61</td>
</tr>
</tbody>
</table>

5. Discussion

Which method to choose?

The characteristics of the data set may potentially bias the estimation of \( m/n \) and \( K \), as illustrated on Fig. 6. The distribution of the data mainly either on the right side of the graph (in case of an older generation of knickpoints) or on the left side (for a more recent erosion wave) will determine the result of the adjustment. The knickpoint positions appearing on the steeper part of the \( x = f(t) \) curve (Fig. 6) are located in the steeper part of the \( x = f(t) \) curve where the adjustment on distances is much more sensitive than that on time, and the reverse is true for a set of older knickpoints (red point) appearing in the flattened part of the curve. Consequently, if a data set contains knickpoints distributed over a large range of drainage areas, best fit based on distance or time will adjust preferentially on the knickpoints respectively located in the larger or smaller drainage areas.

As there is no physical reason to privilege distance over time, or conversely, in the adjustment, the intermediate least rectangles approach may appear the most appropriate. In the case of the Ardennes, a time adjustment seems to be more appropriate because most of the knickpoints are located close to the headwaters.

Fig. 1 – Study area and knickpoint spatial distribution

Fig. 2 – Knickpoint detection

Fig. 3 – Knickpoint validation

Fig. 4 – Least square fit. Contours represent time model

Fig. 5 – Time residuals for the time adjustment

Fig. 6 – Schematic representation of the influence of the knickpoint spatial distribution on the parameterization.

Comparison between modelled and field derived \( m/n \) ratio values

If we now recall that \( m/n = (c-1)/b \) (Whipple & Tucker, 1999), where \( c \) and \( b \) are the exponents of the power law relations respectively linking distance to drainage area and channel width to discharge, we can compare the calculated \( m/n \) value to the field derived one. Field derived \( m/n \) value is usually to minimize the stream-wise distance between observed and modelled knickpoint positions.

Fig. 7 – Schematic representation of the time adjustment works mainly toward lower m/n

Fig. 8 – Schematic representation of the distance adjustment works mainly toward larger m/n

References

Nahas G. et al., 2011. Quaternary river incision in NE Ardennes (Belgium) - insights from 10Be/26Al dating of river terraces. Quat. Int. 271-272, 127-39


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Modelling the migration of a mid-Pleistocene erosion wave in the Ardennes (western Europe) drainage network: approach and first implications