Statistical derivation of the evolution equation of liquid water path fluctuations in clouds

K. Ivanova
Department of Meteorology, Pennsylvania State University, University Park, Pennsylvania, USA

M. Ausloos
SUPRAS and GRASP, B5, Sart Tilman Campus, Liège, Belgium

Received 5 March 2002; revised 5 March 2002; accepted 17 June 2002; published 11 December 2002.

[1] How to distinguish and quantify deterministic and random influences on the statistics of turbulence data in meteorology cases is discussed from first principles. Liquid water path (LWP) changes in clouds, as retrieved from radio signals, upon different delay times, can be regarded as a stochastic Markov process. A detrended fluctuation analysis method indicates the existence of long range time correlations. The Fokker-Planck equation which models very precisely the LWP fluctuation empirical probability distributions, in particular, their non-Gaussian heavy tails is explicitly derived and written in terms of a drift and a diffusion coefficient. Furthermore, Kramers-Moyal coefficients, as estimated from the empirical data, are found to be in good agreement with their first principle derivation. Finally, the equivalent Langevin equation is written for the LWP increments themselves. Thus rather than the existence of hierarchical structures, like an energy cascade process, strong correlations on different timescales, from small to large ones, are considered to be proven as intrinsic ingredients of such cloud evolutions. *INDEX TERMS:* 3220 Mathematical Geophysics: Nonlinear dynamics; 3230 Mathematical Geophysics: Numerical solutions; 3250 Mathematical Geophysics: Fractals and multifractals; 3399 Meteorology and Atmospheric Dynamics: General or miscellaneous; *KEYWORDS:* Liquid water path, Fokker-Planck equation, statistical derivation, fluctuations

Citation: Ivanova, K., and M. Ausloos, Statistical derivation of the evolution equation of liquid water path fluctuations in clouds, *J. Geophys. Res.*, *107*(D23), 4708, doi:10.1029/2002JD002266, 2002.

1. Introduction

[2] In order to establish a sound understanding for any scientific phenomenon, one has from numerical data to obtain laws which can be next derived theoretically. Some difficulty arises in particular in nonlinear dynamical systems because of the problem to sort out noise from both chaos and deterministic components [*Bhattacharya and Kanjilal*, 2000; *Provenzale et al.*, 1992]. Moreover, algorithms for solving the inverse problem for nonlinear systems with underlying unknown dynamical processes are notoriously hardly reliable [*Theiler et al.*, 1992]. In fact, to extract dynamical equations for chaotic-like data is an enormous challenge [*Rowlands and Sprott*, 1992]. Practically one is often led to empirical relationships. This is the case of the meteorology field where there is a widely mixed set of various (sometimes) unknown influences, over different time and space scales. Works of interest for sorting out deterministic ingredients in chaotic systems, as in [*Hilborn*, 1994; *Ott*, 1993; *Schreiber*, 1999; *Cellucci et al.*, 1997; *Davis et al.*, 1996a], can be mentioned for general purpose though this is not an exhaustive list of references.

[3] It is known that two equivalent master equations govern the dynamics of a system, i.e. the Fokker-Planck equation and the Langevin equation, the former for the probability distribution function of time and space signal increments, the latter for the increments themselves [*Reichl*, 1980; *Ernst et al.*, 1969; *Risken*, 1984; *Hänggi and Thomas*, 1982; *Gardiner*, 1983]. They are both condensates of the huge set of (6N) Hamilton equations which should in practice describe the whole dynamics of the system of N particles by giving the time evolution of both coordinates and momenta of each individual particle. This is a highly unrealistic scheme of work, and therefore conservation laws are used in order to derive the Navier-Stokes equations, from the averaging of basic quantities weighted by the above probability functions [*Reichl*, 1980; *Huang*, 1967]. Such a "micro-turned-macroscopic description" can be by passed when describing the evolution of the quasi-equilibrium probability distribution function of such particles, i.e. in writing a Boltzmann equation [*Huang*, 1967]. However this corresponds to a mean field description of the Fokker-Planck equation (FPE) [*Reichl*, 1980; *Ernst et al.*, 1969; *Risken*, 1984; *Hänggi and Thomas*, 1982; *Gardiner*, 1983], an approximation which might be too rough for this sort of nonlinear system dynamics.

[4] Recently much advance has been made in describing nonlinear phenomena in meteorology, after the canonical

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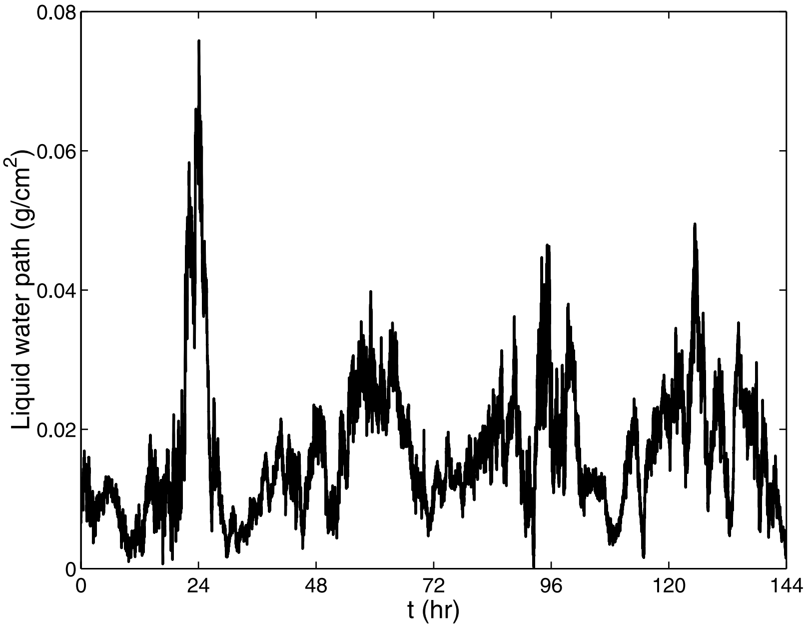

Figure 1. Time dependence of a stratus cloud liquid water path (LWP) from January 9 to 14, 1998, obtained at the ARM Southern Great Plains site with time resolution of 20 s.

Lorenz approximation [*Lorenz*, 1963; *Schuster*, 1984] of the Navier-Stokes equations for sorting out some deterministic and stochastic components of turbulent fluid motion in the atmosphere. In fact, turbulence [*Frisch*, 1995; *Parisi and Frisch*, 1985; *Hunt*, 1999] seems now to be part of more general class of problems similar to those found when there are long range fluctuations, as in financial data [*Ghashghaie et al.*, 1996; *Mantegna and Stanley*, 1995], traffic [*Chowdhury et al.*, 2000], dielectric breakdown [*Vandewalle et al.*, 1999], heart beats [*Ivanov et al.*, 1999], etc... Such observations corroborate the idea that scaling [*Stanley*, 1971] and fractal geometry [*Schuster*, 1984] principles could be useful in such studies on atmospheric turbulence [*Marshak et al.*, 1997]. Thus to derive the FPE [*Risken*, 1984; *Hänggi and Thomas*, 1982; *Gardiner*, 1983] for some meteorological effect is of primary interest.

[5] The aim of this paper is to derive such an FPE, in a first principle statistical approach, from given raw data, i.e. for the Liquid Water Path (LWP) fluctuations in clouds, -in terms of a drift $D^{(1)}$ and a diffusion $D^{(2)}$ coefficient. Much following the lines of thought of [*Friedrich et al.*, 2000] it will be shown that the FPE (also known as the Kolmogorov equation) can be derived up to the first two moments of the (measured) conditional probability distribution functions $p(\Delta x, \Delta t)$, where Δx is a signal increment and Δt a time delay. Thus the method is not anymore based on the conventional phenomenological comparison between models and several stochastic aspects, but is a *model independent* (or first principle, from a statistical point of view) approach. This allows us to examine long and short time scales on the same footing, and suggests new features to be implemented in related weather forecasting. Indeed it is here below verified that the solution of the FPE yields the probability distributions with high accuracy, including the long and high tail Δx and Δt events. In some sense the agreement proves that searching for more improvement with including further event of similar cases is not necessary.

[6] Furthermore the so found analytical form of both drift $D^{(1)}$ and diffusion $D^{(2)}$ coefficients has a simple physical interpretation. It is also shown that a truncation of the FPE expansion in terms of high order joint probabilities to the first two terms is quite sufficient. Moreover the identification of the underlying process leading to the heavy tailed probability density functions (pdf) for the LWP fluctuation correlation with Δt and the volatility clustering (as seen in Figure 3 below) seems to be a new observation leading to an interestingly new set of puzzles for this sort of clouds [*Cahalan et al.*, 1995], and more generally in atmospheric science.

2. Data and Theoretical Analysis

[7] The changes in a time series for a signal $x(t)$ are commonly measured by the quantity $r = x(t + \Delta t)/x(t)$, or increments $\Delta x = x(t + \Delta t) - x(t)$. Results of the analysis of a data set $r(t)$, selecting among others LWP observations, are presented here. The 25 772 data points are taken from microwave radiometer measurements at the Southern Great Plains site of Atmospheric Radiation Measurement (ARM) (http://www.arm.gov) of Department of Energy during the period January 9–14, 1998 as used by [*Ivanova et al.*, 2000]. The microwave radiometer measures the brightness temperature at two frequency channels, one at 23.8 GHz and the other at 31.4 GHz. Then both brightness temperature data series are used to retrieve the vertical columnar amount of water vapor and vertical columnar amount of liquid water in the cloud, i.e. the so-called liquid water path. The raw data r_i (i is equivalent to a time index) of the liquid water path in stratus clouds is shown in Figure 1.

[8] The first thing is to consider whether the distribution function of increments has short or long tails, and whether there is some scaling law involved. Next the central issue is the understanding of the statistics of fluctuations which determine the evolution of the cloud.

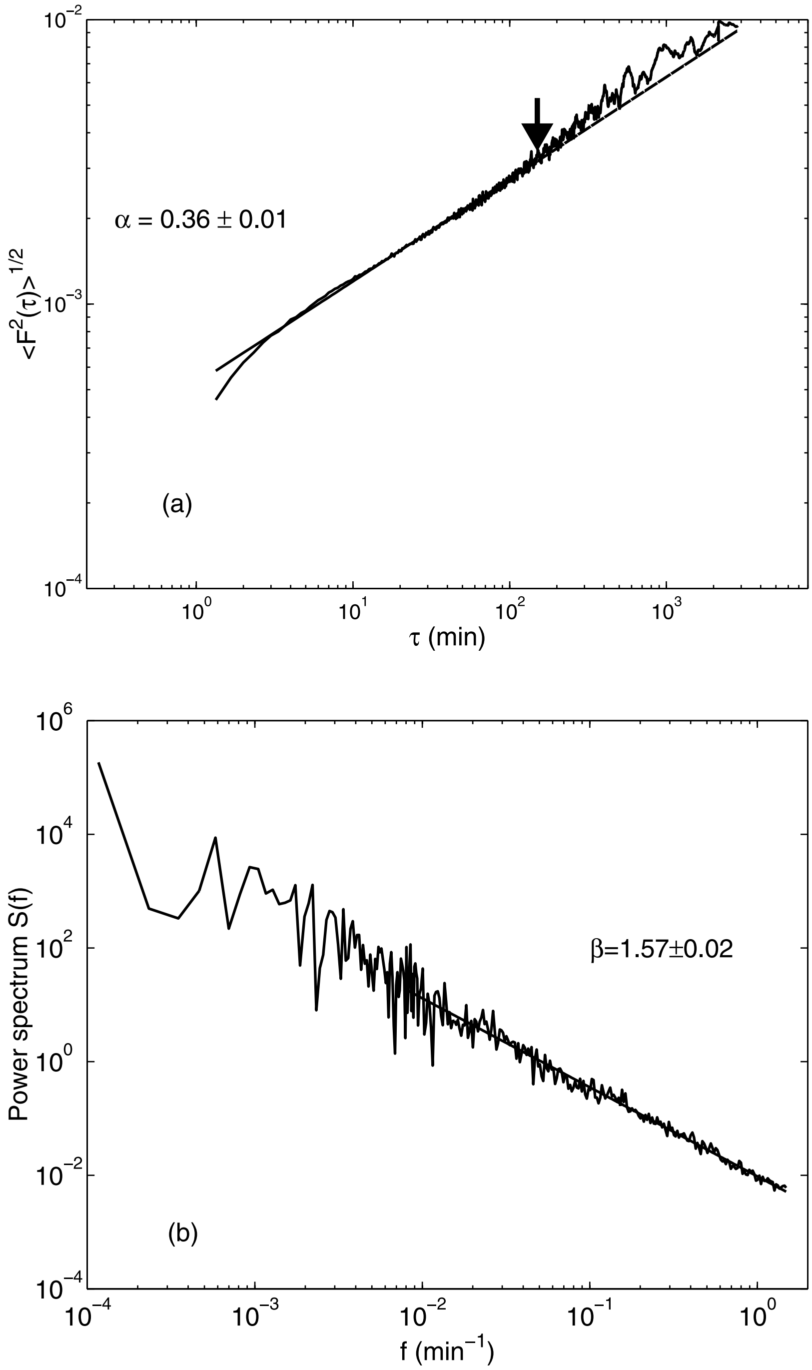

Figure 2. (a) The detrended fluctuation analysis (DFA) function $\langle F^2(\tau)\rangle^{1/2}$ (equation (1)) for the data in Figure 1. Scaling range holds for a time interval between ca. 3 min and ca. 150 min. (b) Power spectrum $S(f)$ for the data in Figure 1.

[9] There are different estimators for the short and/or long range dependence of fluctuations correlations [*Taqqu et al.*, 1995]. Through a detrended fluctuation analysis (DFA) method [*Ausloos and Ivanova*, 1999] we show first that the long range correlations are not brownian. The method has been used previously in the meteorological field [*Ivanova et al.*, 2000; *Ausloos and Ivanova*, 1999; *Koscielny-Bunde et al.*, 1998; *Ivanova and Ausloos*, 1999] and its concepts are not repeated here. We show on Figure 2a the final result, for the

$$\langle F^2(\tau)\rangle^{1/2} = \sqrt{\frac{1}{\tau}\sum_{n=k\tau+1}^{(k+1)\tau}[x(n)-z(n)]^2} \sim \tau^{\alpha} \qquad (1)$$

indicating a scaling law characterized by an exponent $\alpha = 0.36 \pm 0.01$, markedly different from 0.5, thus indicating antipersistence of the signal from about 3 to about 150 minutes. Next the power spectrum $S(f) \sim f^{-\beta}$ with spectral exponent $\beta = 1.57 \pm 0.02$ is also shown (see Figure 2b) [*Davis et al.*, 1996b]. A Kolmogorov-Smirnov test on

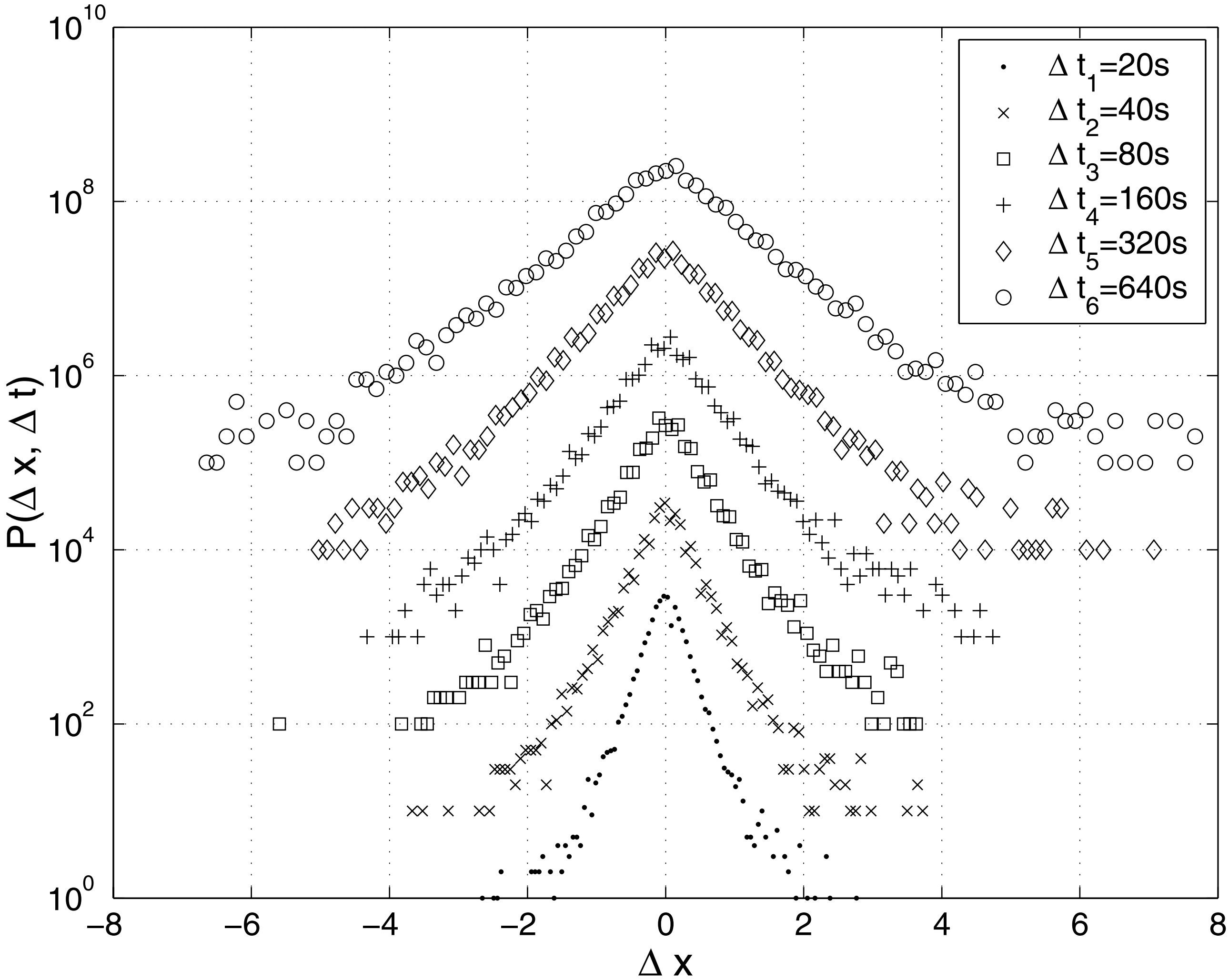

Figure 3. (a) Frequency $p(\Delta x, \Delta t)$ of the LWP increments Δx for different time delays Δt; the units of Δx are taken as multiples of the standard deviation $\sigma = 0.0036$ at $\Delta t = 640$ s.

surrogate data has indicated the statistical validity of the value and error bars.

[10] Thereafter we focus on the LWP changes measured as increments Δx which are given in Figure 3 in units of the standard deviation σ of Δx at $\Delta t = 640$ s. In order to characterize the statistics of these LWP changes, LWP increments Δx_1, Δx_2 for delay times Δt_1, Δt_2 at the same time t are considered. The corresponding joint probability density functions are evaluated for various time delays $\Delta t_1 < \Delta t_2 < \Delta t_3 < \ldots$ directly from the given data set. One example of a contour plot of these functions is exhibited in Figure 4. If two LWP changes, i.e. Δx_1 and Δx_2 are statistically independent, the joint pdf should factorize into a product of two probability density functions:

$$p(\Delta x_1, \Delta t_1; \Delta x_2, \Delta t_2) = p(\Delta x_1, \Delta t_1) p(\Delta x_2, \Delta t_2). \quad (2)$$

leading to a camelback or an isotropic single hill landscape. The tilted anisotropic form of the joint probability density (Figure 4) clearly shows that such a factorization does not hold for small values of $|\log(\Delta t_1/\Delta t_2)|$, whence both LWP changes are statistically dependent. The same is found to be true for other $(\Delta t_i/\Delta t_j)$ ratios. This is in agreement with the hint taken from the above observations for the long range cross-correlation functions with the DFA.

[11] This implies a (fractal-like) hierarchy of *time* scales. If fluctuations in LWP went up over a certain Δt_2, then it is more likely that, on a shorter Δt_1 within the larger one, the LWP went down instead of up. To analyze these correlations in more detail, the question on what kind of statistical process underlies the LWP changes over a *series* of nested time delays Δt_i of decreasing duration should be raised. A complete characterization of the statistical properties of the data set in general requires the evaluation of joint pdf's $p^N(\Delta x_1, \Delta t_1; \ldots; \Delta x_N, \Delta t_N)$ depending on N variables (for arbitrarily large N). In the case of a Markov process (a process without memory) [*Schuster*, 1984], an important simplification arises: The N-point pdf p^N is generated by a product of the conditional probabilities $p(\Delta x_{i+l}, \Delta t_{i+l}|\Delta x_i, \Delta t_i) = p(\Delta x_{i+l}, \Delta t_{i+l}; \Delta x_i, \Delta t_i)/p(\Delta x_i, \Delta t_i)$ for i = 1, ..., N-1. The conditional probability is given by the probability of finding Δx_{i+1} values for fixed

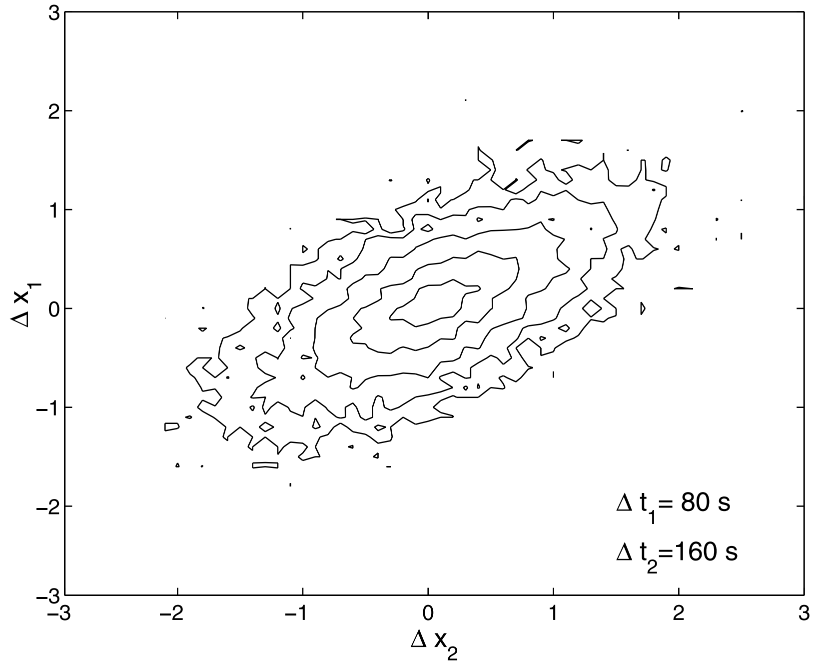

Figure 4. Contour plot of the joint LWP increment pdf $p(\Delta x_1, \Delta t_1; \Delta x_2, \Delta t_2)$ for the simultaneous occurrence of LWP fluctuations $\Delta x_1(\Delta t_1)$ and $\Delta x_2(\Delta t_2)$; $\Delta t_1 = 80$ s and $\Delta t_2 = 160$ s. The contour lines correspond to $log_{10}p = -2, -2.5, -3, -3.5, -4$.

Δx_i. As a necessary condition, the Chapman-Kolmogorov equation

$$p(\Delta x_1, \Delta t_1 \mid \Delta x_2, \Delta t_2) = \int d(\Delta x_i) p(\Delta x_1, \Delta t_1 \mid \Delta x_i, \Delta t_i) p(\Delta x_i, \Delta t_i \mid \Delta x_2, \Delta t_2) \quad (3)$$

should hold for any value of Δt_i, with $\Delta t_1 < \Delta t_i < \Delta t_2$. We checked the validity of the Chapman-Kolmogorov equation for different Δt_i triplets by comparing the directly evaluated conditional probability distributions $p(\Delta x_1, \Delta t_1 | \Delta x_2, \Delta t_2)$ with the ones calculated (P_{cal}) according to (3), in Figure 5. The solutions of (4) usually give rise to exponential laws, but initial conditions must be taken into account for adjusting the constants of the integration. Starting with a stretched exponential, the latter is conserved after successive integrations, with a width varying with Δt. The power law probability density function which is observed is thus a signature of fractal properties of the signal increments because the tails of the pdf are not truly exponential ones. In Figure 5a, the contour lines of the two corresponding pdf's for all values of Δx_1 are shown. There are mild deviations, probably resulting from a finite resolution of the statistics. Cuts for some exemplarily chosen values of Δx_1 are shown in addition in Figures 5b–5d).

[12] As is well known, the Chapman-Kolmogorov equation yields an evolution equation for the change of the distribution functions $p(\Delta x, \Delta t | \Delta x_1, \Delta t_1)$ and $p(\Delta x, \Delta t)$ across the scales Δt [*Reichl*, 1980; *Ernst et al.*, 1969; *Risken*, 1984; *Hänggi and Thomas*, 1982; *Gardiner*, 1983]. For the following it is convenient (and without loss of generality) to consider a normalized logarithmic time scale $\tau = ln\,(640/\Delta t)$. The limiting case $\Delta t_i \to 0$ corresponds to $\tau \to \infty$.

[13] The Chapman-Kolmogorov equation formulated in differential form yields a master equation, which can take the form of a Fokker-Planck equation (for a detailed discussion, we refer the reader to [*Reichl*, 1980; *Ernst et al.*, 1969; *Risken*, 1984; *Hänggi and Thomas*, 1982; *Gardiner*, 1983]) (The change in variable from Δt to τ is non linear and might be debatable if the following equations (4) and (9) are integrated with respect to time. This is not done here. The integrations discussed in the text are made at fixed Δt or τ.):

$$\frac{d}{d\tau} p(\Delta x, \tau) = \left[-\frac{\partial}{\partial \Delta x} D^{(1)}(\Delta x, \tau) + \frac{\partial}{\partial^2 \Delta x^2} D^{(2)}(\Delta x, \tau) \right] p(\Delta x, \tau) \quad (4)$$

in terms of a drift $D^{(1)}(\Delta x, \tau)$ and a diffusion coefficient $D^{(2)}(\Delta x, \tau)$. Note that the Fokker-P1anck equation results from a truncation of the master equation expanded in terms of high order joint probabilities. According to Pawula's theorem [*Risken*, 1984], such a truncation is valid provided that the fourth order coefficient $D^{(4)}$ vanishes. We did check the $D^{(4)}$ coefficient for $\Delta x \in [-0.3, 0.3]$ and obtained values that are two orders of magnitude smaller than $D^{(2)}$ and three decades smaller than $D^{(1)}$, which we consider justifies the truncation. The functional dependence of the drift and diffusion coefficients can be estimated directly from the moments $M^{(k)}$ of the conditional probability distributions (cf. Figure 5):

$$M^{(k)} = \frac{1}{\Delta \tau} \int d\Delta x' (\Delta x' - \Delta x)^k p(\Delta x', \tau + \Delta \tau \mid \Delta x, \tau) \quad (5)$$

for different small $\Delta\tau$'s (Figure 6), such that

$$D^{(k)}(\Delta x, \tau) = \frac{1}{k!} \lim M^{(k)} \quad (6)$$

for $\Delta \tau \to 0$.

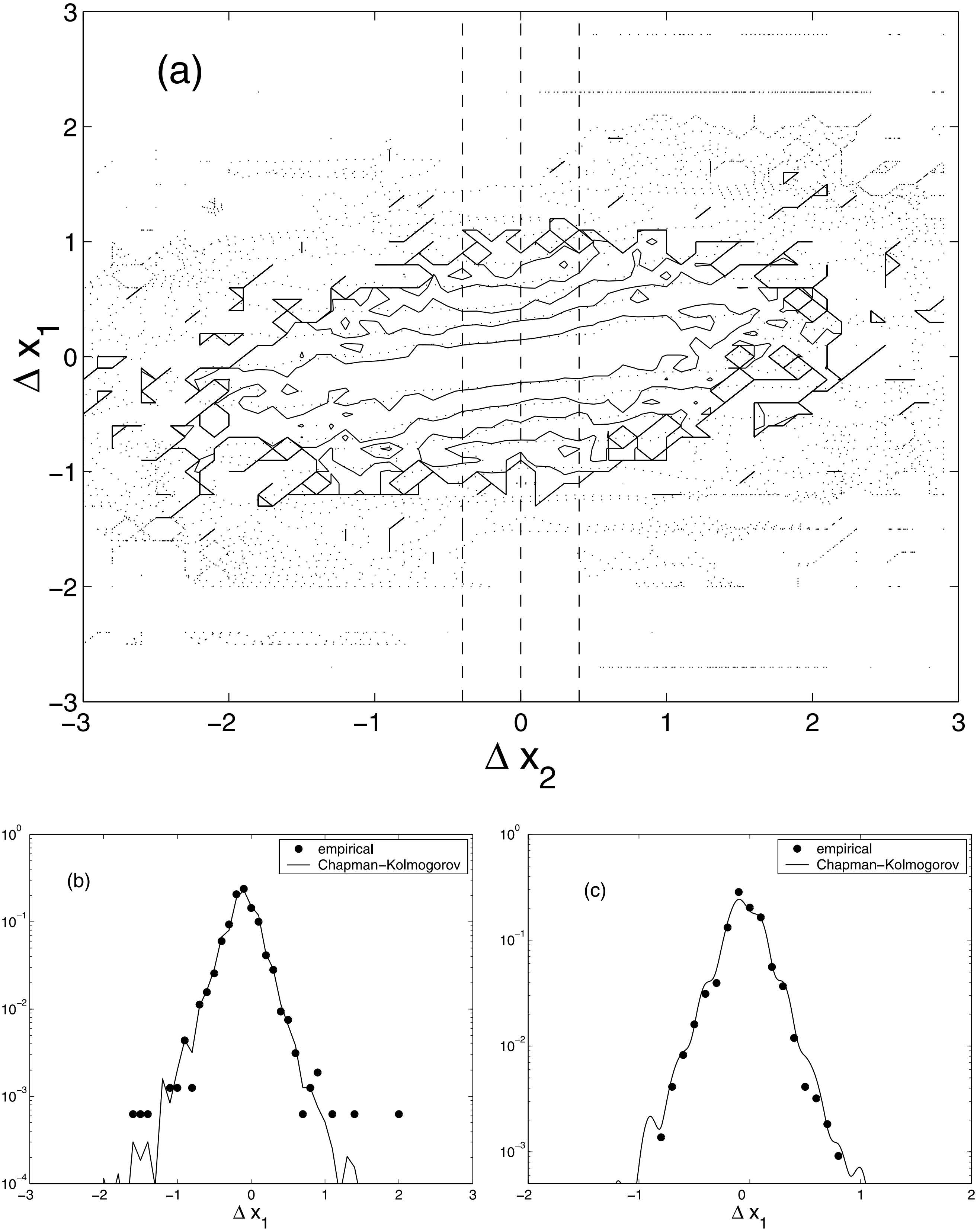

Figure 5. (a) Empirical, and theoretical contour plots of the conditional LWP increment pdf $p(\Delta x_1, \Delta t_1 - \Delta x_2, \Delta t_2)$ for Δt_2 = 180 s and Δt_1 = 20 s. In order to verify the Chapman-Kolmogorov equation, the directly evaluated pdf (solid) is compared with the integrated pdf (dotted). Assuming a statistical error of the square root of the number of events of each bin, both pdfs are found to be statistically identical; (b), (c), and (d) directly evaluated pdf's (dots) and results of the numerical integration of the Chapman-Kolmogorov equation (line) for cuts at $\Delta x_1 = -0.4$, 0.0, 0.4.

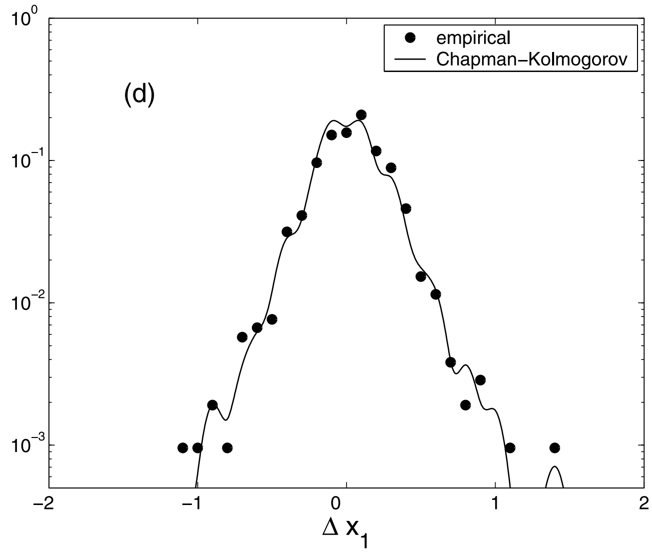

Figure 5. (continued)

[14] The coefficient $D^{(1)}$ shows a linear dependence on Δx, while $D^{(2)}$ can be approximated by a polynomial of degree two in Δx. This behavior was found for all scales τ and $\Delta\tau$. Therefore the drift term $D^{(1)}$ is well approximated by a linear function of Δx, whereas the diffusion term $D^{(2)}$ follows a function quadratic in Δx. For large values of Δx the statistics becomes poorer and the uncertainty increases. From a careful analysis of the data based on the functional dependences of $D^{(1)}$ and $D^{(2)}$ (Figures 6a–6b), the following approximations hold true:

$$D^{(1)} = -0.0078\Delta x, \tag{7}$$

$$D^{(2)} = 0.00259(\Delta x - 0.0305)^2 + 0.00009. \tag{8}$$

[15] It may be worthwhile to remark that the observed quadratic dependence of the diffusion term $D^{(2)}$ is essential for the logarithmic scaling of the intermittency parameter in previous studies on turbulence. Finally, the FPE for the distribution function is known to be equivalent to a Langevin equation for the variable, i.e. Δx here, (within the Ito interpretation [*Reichl*, 1980; *Ernst et al.*, 1969; *Risken*, 1984; *Hänggi and Thomas*, 1982; *Gardiner*, 1983])

$$\frac{d}{d\tau}\Delta x(\tau) = D^{(1)}(\Delta x(\tau),\tau) + \eta(\tau)\sqrt{D^{(2)}(\Delta x(\tau),\tau)}, \tag{9}$$

where $\eta(\tau)$ is a fluctuating δ-correlated force with Gaussian statistics, i.e. $\langle\eta(\tau)\,\eta(\tau')\rangle = 2\delta(\tau-\tau')$.

3. Conclusions

[16] At least since the pioneering work of Lorenz [*Lorenz*, 1963] stochastic problems in turbulence are commonly treated as processes running in time t with long time correlations. Inspired by the idea of an existing energy cascade process [*Cahalan*, 1994] we present here a new approach, namely, we investigate among other related phenomena, how LWP changes are correlated on different time steps Δt. The pdf shape expresses an unexpected high probability (compared to a Gaussian pdf) of large LWP changes which is of utmost importance for forecasting. In recent works [*Friedrich et al.*, 2000; *Friedrich and Peinke*, 1997] this finding leads to postulating the existence of hierarchical features, i.e. an energy cascade process from large to small time scales. The existence of finite non Gausssian tails for large events is thought to be due or to imply drastic evolutions, as for earthquakes, financial crashes or heart attacks.

[17] One unexpected result is the time dependence of the tails (Figure 3) which seems quite smooth for the LWP case. This puzzle should initiate new research with a goal toward forecasting, in particular to find cases in which such a smoothness does not hold. This evolution shows how the pdf's deviate more and more from a Gaussian shape as Δt increases or τ decreases. This definitely is a new quality in describing the hierarchical structure of such data sets, - not seen in the DFA nor spectral result. Now it becomes clear that one must not require stationary probability distributions

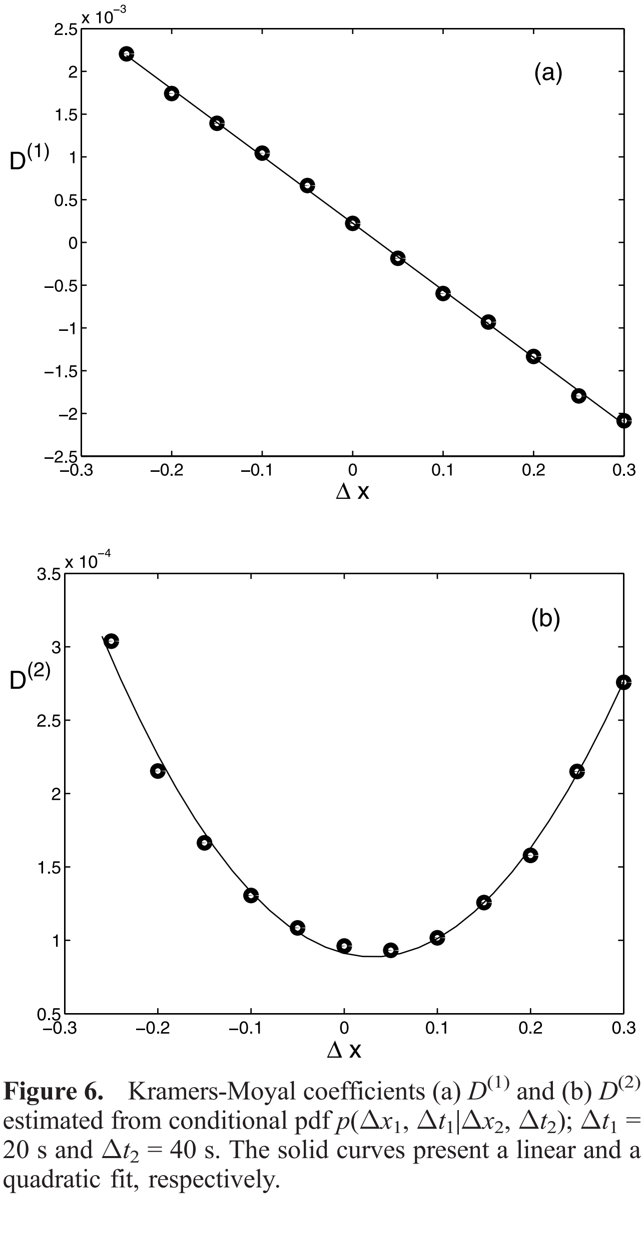

Figure 6. Kramers-Moyal coefficients (a) $D^{(1)}$ and (b) $D^{(2)}$ estimated from conditional pdf $p(\Delta x_1, \Delta t_1|\Delta x_2, \Delta t_2)$; Δt_1 = 20 s and Δt_2 = 40 s. The solid curves present a linear and a quadratic fit, respectively.

for LWP differences for different τ; on the contrary the coupling between different scales τ via a Markov process is essential. Thus also a proper modeling of the time evolution of LWP differences for a fixed time delay must take into account the coupling of these quantities to LWP differences on different time delays.

[18] Furthermore, it is stressed that in contrast to the use of phenomenological fitting functions, the above method provides the evolution process of pdf's from small time delays to larger ones. Interestingly this is through an analogy with two physically meaningful coefficients, a drift term $D^{(1)}$ and a diffusion term $D^{(2)}$. The first one linearly behaves, thus looks like a "restoring force", the second behaving quadratically in Δx, is obviously like an autocorrelation function as for bona fide (chemical) diffusion. Of course further theoretical work is needed before understanding the numerical values in (7)–(8).

[19] Finally, the present report presents a method on how to derive an underlying mathematical (statistical or model free) equation for a LWP cascade directly. The method yields an effective stochastic equation in the form of a FPE in the variable Δt. The excellent agreement between the experimental and the statistical approach removes the need for much further proof based on examining many other cases for the same type of clouds. The FPE provides the complete knowledge as to how the statistics of LWP distribution change correlations on different delay times. Since this includes an autocorrelation analysis in time t for a scalar Δx, it is suggesting that the findings could be implemented in atmospheric weather low dimensional *vector - models* [*Grabowski and Smolarkiewicz*, 1999; *Ragwitz and Kantz*, 2000]. Consequently, even though for the first time in atmospheric science, a stochastic equation (of LWP evolution) is here by introduced from first principles, several interesting statements can be presented and new questions opened.

[20] **Acknowledgments.** We thank C. Nicolis, T.P. Ackerman, H.N. Shirer and E.E. Clothiaux for stimulating discussions and comments. Email correspondence with J. Peinke, Ch. Renner and R. Friedrich is specifically appreciated. This investigation was partially supported by Battelle grant number 327421-A-N4.

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K. Ivanova, Department of Meteorology, Pennsylvania State University, University Park, PA 16802, USA. (kristy@essc.psu.edu)
M. Ausloos, SUPRAS and GRASP, B5, Sart Tilman Campus, B-4000 Liège, Belgium.