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Measuring educational efficiency at student level with parametric stochastic distance functions: an application to Spanish PISA results

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The aim of the present paper is to examine the observed differences in Students' test performance across public and private-voucher schools in Spain. For this purpose, we explicitly consider that education is a multi-input multi-output production process subject to inefficient behaviors, which can be identified at student level using a parametric stochastic distance function approach. The empirical application of this model, based on Spanish data from the Programme for International Student Assessment implemented by the Organization for Economic Co-operation and Development in 2003, allows us to identify different aspects of the underlying educational technology. Among other things, the results provide insights into how student background, peer group, school characteristics and personal circumstances interact with educational outputs. Moreover, our findings suggest that, once educational inputs and potential bias due to school choice endogeneity are taken into account, no further unexplained difference remains between students' efficiency levels across public and private-voucher schools.

Keywords: public schools; educational efficiency; stochastic frontier; distance function

1. Introduction

Since the early 1960s (Carroll 1963), a wide range of studies (for example, Coleman 1966; Jenks 1972; Summers and Wolfe 1977; Hanushek 1986, 1996, 1997, 2003; Pritchett and Filmer 1999) have sought to define the relationship between school inputs, student background and achievement at school. Despite all the research devoted to this issue, the well-known debate 'Does school matter? – Does money matter?' remains open. A great deal of evidence has established that a student's education takes place both at home and at school. However, the way that a student's own characteristics, home, peer group and school interact with educational outputs continues to be largely unknown, and this is a serious drawback for policy-makers taking decisions about the allocation of the scarce public resources devoted to education.

We can roughly summarize as follows the main reasons put forward in the literature as to why empirical research does not find systematic relationships between school inputs and outputs. First, education is a highly complex process with variables, such as organization or non-monetary inputs, implied in production (for a review, see

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Vandenberghe 1999). Second, there is the inconsistency of the use of Cobb–Douglas specifications for the estimation of educational production functions (see, for example, Eide and Showalter 1998; Figlio 1999; or Baker 2001). Third, most production function studies into the economics of education do not consider the theoretically potential role of the efficiency component (Farrell 1957; Leibenstein 1966). And, last but not least, in empirical research, student results are typically aggregated at school or district level, imposing a considerable limitation on disentangling the effect of a student's own background from peer group and school inputs on student achievement. For example, Hanushek, Rivkin and Taylor (1996) showed how aggregation can dramatically influence upwards the statistical significance of inputs in the educational process.

In order to tackle the inefficiency issue in education, many studies use deterministic non-parametric data envelopment analysis in empirical evaluations. Pioneer studies applying data envelopment analysis in education originate with Bessent and Bessent (1980), Charnes, Cooper, and Rhodes (1981) and Bessent et al. (1982) (for an empirical survey of frontier efficiency techniques in education, see Worthington 2001). Other studies have considered parametric methodologies, mainly using the Cobb–Douglas specifications, but also the translog functional form proposed by Christensen, Jorgenson, and Lau (1971). These studies have included Jiménez (1986), Callan and Santerre (1990), Gyimah-Brempong and Gyapong (1992), Deller and Rudnicki (1993) and Grosskopf et al. (1997). The main advantage of the translog function is its highly flexible nature, which allows the study of interactions in the production process. Summers and Wolfe (1977) and Figlio (1999) used student-level data in their econometric studies; both concluded that the student level is more appropriate than higher levels of aggregation. Their findings show that school inputs matter but that their impact on different types of student varies considerably.

In order to overcome simultaneously the difficulties underlined here above, in the present paper we propose the use of frontier analysis techniques; more precisely, a parametric stochastic distance function. Under this specification, we explicitly consider that education is a process in which students use their own and school inputs in order to transform these inputs into academic results, subject to inefficient behaviors that can be identified at both student and school levels. Moreover, parametric stochastic distance functions, flexible by definition, allow us to deal simultaneously with multiple outputs (e.g. mathematics and reading test scores) and multiple inputs (including school inputs, student background and peer-group characteristics) within a stochastic framework. We adopt here a translog specification to estimate the parametric stochastic distance function at the student level. This allows us to calculate several aspects of educational technology, mainly output elasticities with respect to inputs and outputs.

Another general issue widely debated in the economics of education is whether or not private-voucher schools are more efficient than their public counterparts once student and school characteristics are taken into account (for example, Witte 1992; Neal 1997; Grimes 1994; Krueger and Zhu 2004; Vandenberghe and Robin 2004; Duncan and Sandy 2007). In most situations, student attendance is not randomly distributed between private-voucher and public schools. This is the case in Spain, where the public educational system finances both private-voucher schools (government-financed private school) and public schools. However, students opting for the public-financed educational system are distributed between schools (private-voucher or public schools) through a competitive process of point gains regarding geographical location of family home or parents' workplace (the most important factor) and the number of siblings at school. Nevertheless, we do believe that other non-explicit factors could be affecting school choices, even though the Educational Spanish Law establishes that education is free in public and private-voucher schools for students ranging from three to 16 years old.

In practice, we detect two main driving factors in favor of a selection process against low-income and large-size families. On the one hand, most private-voucher schools also supply childcare and education from zero to three years old, but on a privately paid basis. Children starting their education at this level in a private-voucher school automatically receive an extra point,[1] which is needed to continue their education in the same institution. On the other hand, practically all private-voucher schools ask families for a *voluntary monthly fee* in order to offer extra-curricular and sport activities, foreign-language reinforcement and other complementary services. As a consequence, families not able to afford these monthly fees (they can vary from 50 to 300 per month and per child) self-select themselves, sending their children to public schools.

Summing up, as shown in most of the studies mentioned previously dealing with the public versus private issues in education, comparative analyses of student performance face a potential endogeneity bias. In other words, school choices are probably affected by the same variables that explain students' results. In order to overcome this potential bias, we follow a two-stage instrumental variable strategy. In the first stage, we estimate the expected individual probability of attending a private-voucher school, which, in the second stage, is included as an explanatory variable in the parametric distance function.

In order to illustrate the potentialities of the approach proposed here, we provide an application to Spanish educational data from the Programme for International Student Assessment (PISA), implemented in 2003 by the Organization for Economic Co-operation and Development (OECD). Spanish student performance has been shown to be poor, both in PISA 2000 and PISA 2003 and in other international studies. For example, in the Third International Mathematics and Science Study (TIMSS) carried out in 1995, Spain's performance in science and mathematics ranked as 27 and 31, respectively, in a survey of 41 countries. For an extensive review of all results of the TIMSS, see Gonzalez and Smith (1997). In PISA 2003, Spain reached 26th position out of 40 OECD evaluated countries in mathematics, reading and science. Furthermore, Spain is one of the European Countries (EU-27) with the highest percentage (29.0%) among the 18-year-old to 25-year-old population either without a high school diploma or who did not follow any other educational training (MEC 2004). These are puzzling results if we consider that 16 years old corresponds to the end of compulsory-age schooling in Spain. For this reason, the PISA evaluations offer an exciting framework in which to analyze and to identify, as much as possible, the factors at work. We investigate differences in student performance across Spanish public and private-voucher schools. We conclude that, once educational inputs and potential bias due to school choice endogeneity are taken into account, no further unexplained, or statistically significant, difference exists in students' efficiency levels across public and private-voucher schools.

The paper is organized as follows. Section 2 provides an overview of educational production functions and presents the parametric stochastic distance function and the estimation strategy. Section 3 is devoted to describing data, and provides results and a discussion of our empirical analysis. Section 4 presents the calculation of the empirical elasticities of educational inputs. Finally, the paper ends with a summary and explores directions for further research.

2. Education and inefficiency

2.1. Estimating an educational production function through distance functions

In most studies, a common conceptual framework for estimating the educational production function might take the following form (Levin 1974; Hanushek 1979):

$$A_{is} = f(B_{is}, S_s, I_{is}) \tag{1}$$

where A_{is} equals the achievement of student i at school s, B_{is} is the student's background, S_s are school inputs, G_{is} denotes the peer-group effect, and I_{is} are student innate abilities. Most often, Equation (1) is estimated at school level. This analysis typically aggregates student inputs and achievements belonging to each school as an average by school, or even by school district when some non-controllable inputs are not observable at school level.

In this paper, we propose to use parametric stochastic distance functions at student level in order to go further in the analysis of production functions in education. For this purpose, Equation (1) becomes:

$$D_{is} = g(A_{is}, B_{is}, S_s, G_{is}).I_i \tag{2}$$

where g represents the best practice technology used in the transformation of educational inputs to outputs, and D_{is} is the distance that separates each student from the technological boundary. Unobservable student innate abilities, I_i, are assumed to be randomly distributed in the population and to influence individual performance in a multiplicative way. This simple transformation places the empirical estimation of Equation (2) naturally within the framework of parametric stochastic frontier analysis, which, under specific distributional assumptions, allows the disentangling of random effects from efficiency (distance to the frontier).

In the particular case of educational production analyzed here, distance functions D_{is} are expected to capture individual student performance measured with respect to the estimated frontier benchmark. However, disentangling student and school sources of poor performance is an identification issue. Several factors could be responsible for observed differences in performance – among them the effort and motivation put into education by teachers on the one hand, and by parents and students on the other. Other factors relate to the overall role of institutions, including main pedagogical choices, organizational structure and incentive schemes, among others. Within the context of this study, we are particularly interested in the comparison between public and private-voucher school performance. The empirical question is whether being educated in a private school causally yields significantly better children's test scores. This hypothesis will be tested including an institutional variable V_{is}, indicating that the student attends a private-voucher school, alongside the input variables in Equation (2), as follows:

$$D_{is} = g(A_{is}, B_{is}, S_s, G_{is}, V_{is}).I_i. \tag{3}$$

Note that, given that school choices may be driven by the same factors explaining student performance, for estimation purposes, V_{is} will be replaced in Equation (3) by $\hat{V}_s$, an instrumented variable estimated in a previous stage, in order to avoid a potential endogeneity bias.

2.2. The parametric stochastic distance function approach

Defining a vector of inputs $x = (x_1, \ldots, x_K) \in \Re^{K+}$ and a vector of outputs $y = (y_1, \ldots, y_M) \in \Re^{M+}$, a feasible multi-input multi-output production technology can be defined using the output possibility set $P(x)$, which can be produced using the input vector x:

$$P(x) = \{y\!: x \text{ can produce } y\}$$

which is assumed to satisfy the set of axioms described in Färe and Primont (1995). This technology can also be defined as the output distance function proposed by Shephard (1970):

$$D_o(x, y) = \mathit{inf}\{\theta : \theta > 0, (x, y / \theta) \in P(x)\}$$

If $D_o\,(x, y) \leq 1$, then (x, y) belongs to the production set $P(x)$. In addition, $D_o\,(x, y) = 1$ if y is located on the outer boundary of the output possibility set.

Figure 1 illustrates these concepts in a simple two-output setting. Let us assume that two decision-making units in frontier analysis terminology, A and C, dispose of equal input endowments to perform outputs y_1 (mathematics) and y_2 (reading). Then C is efficient, $D_o\,(x, y_A) \equiv, \theta_c = 1$, because it lies on the boundary of the output possibility set, whereas A, an interior point, is inefficient at a rate given by the radial distance function $D_o\,(x, y_A) \equiv \theta_A = OA/OB$ where $D_o\,(x, y) \equiv \theta \in [0;1]$.

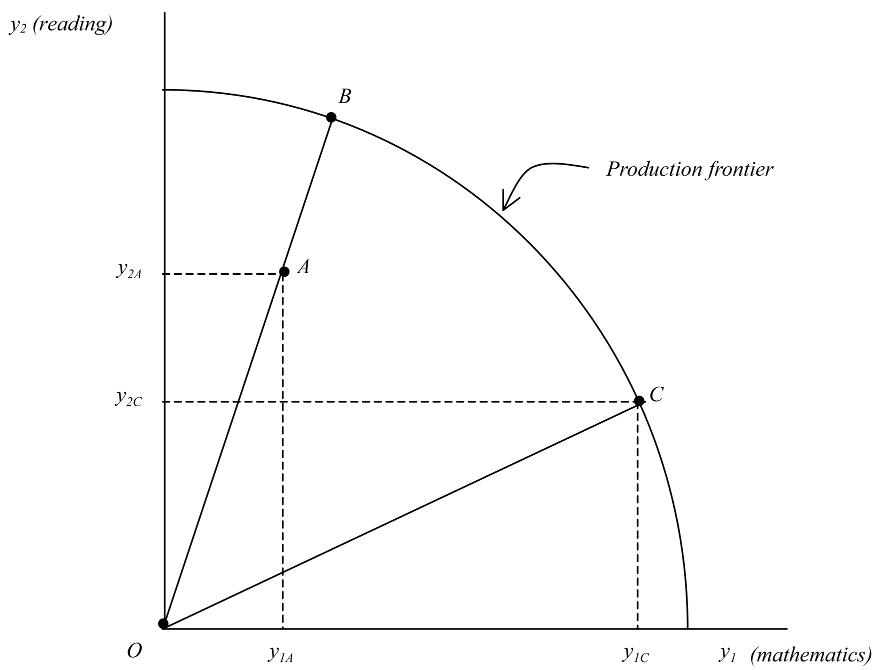

Figure 1. Output possibility set $P(x)$.

In order to estimate the distance function in a parametric setting, a translog functional form is assumed. According to Coelli and Perelman (1999, 2000), this specification fulfills a set of desirable characteristics: flexible, easy to derive and allowing the imposition of homogeneity. The translog distance function specification herein adopted for the case of K inputs and M outputs is:

$$ln\,D_{oi}(x,y)=\alpha_0+\sum_{m=1}^{M}\alpha_m ln\,y_{mi}+\frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M}\alpha_{mn} ln\,y_{mi} ln\,y_{ni}+\sum_{k=1}^{K}\beta_k ln\,x_{ki}$$
$$+\frac{1}{2}\sum_{k=1}^{K}\sum_{t=1}^{K}\beta_{kl} ln\,x_{ki} ln\,x_{li}+\sum_{k=1}^{K}\sum_{m=1}^{M}\delta_{km} ln\,x_{ki} ln\,y_{mi},\qquad i=1,2,...,N, \tag{4}$$

where i denotes the ith unit in the sample. In order to obtain the production frontier surface, we set $D_o\,(x, y) = 1$, which implies $ln\ D_o\,(x, y) = 0$. The parameters of the above distance function must satisfy a number of restrictions, among them symmetry and homogeneity of degree + 1 in outputs. This latter restriction indicates that distances with respect to the boundary of the production set are measured by radial expansions, as illustrated in Figure 1.

According to Lovell et al. (1994), normalizing the output distance function by one of the outputs is equivalent to imposing homogeneity of a degree +1. Therefore, Equation (4) can be represented as:

$$ln(D_{oi}(x,y)/y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta),\qquad i=1,2,\ ...,N, \tag{5}$$

where

$$TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta)=\alpha_0+\sum_{m=1}^{M-1}\alpha_m ln(y_{mi}/y_{Mi})+\frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1}\alpha_{mn} ln(y_{mi}/y_{Mi})$$
$$ln(y_{mi}/y_{Mi})+\sum_{k=1}^{K}\beta_k ln\,x_{ki}+\frac{1}{2}\sum_{k=1}^{K}\sum_{t=1}^{K}\beta_{kl} ln\,x_{ki} ln\,x_{li}+\sum_{k=1}^{K}\sum_{m=1}^{M-1}\delta_{km} ln\,x_{ki} ln(y_{mi}/y_{Mi})$$

Rearranging terms, the function above can be rewritten as follows:

$$-\ln(y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta)-\ln D_{oi}(x,y),\qquad i=1,2,\ ...,N, \tag{6}$$

where $-ln\ D_{oi}\ (x, y)$ corresponds to the radial distance function from the boundary. Hence we can set $u = -ln\ D_{oi}\ (x, y)$ and add a term v_i capturing for noise to obtain the Battese and Coelli (1988) version of the traditional stochastic frontier model proposed by Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977):

$$-\ln(y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta,\delta)+\varepsilon_i\qquad \varepsilon_i=v_i+u_i \tag{7}$$

where $u = -ln\ D_{oi}\ (x, y)$, the distance to the boundary set, is a negative random term assumed to be independently distributed as truncations at zero of the $N\left(0,\sigma_u^2\right)$ distribution, and the v_i term is assumed to be a two-sided random (stochastic) disturbance designated to account for statistical noise and is distributed as *iid* $N\left(0,\sigma_v^2\right)$. Both terms are independently distributed, $\sigma_{uv} = 0$. In order to estimate this model, we used the computer program Frontier V4.1 developed by Coelli (1994), assuming that u has a half-normal distribution. That is, it has a distribution equal to the upper half of the $N\left(0,\sigma_u^2\right)$ distribution.

But these are the normal assumptions given to u_i and v_i error terms in the frontier analysis literature dealing with the technical efficiency of firms in production. What is the interpretation we can give to these error terms in the particular case of student performance as discussed here? As indicated in Section 1, we think that they allow for a straightforward interpretation.

On the one hand, the stochastic term v_i is expected to capture unobserved student characteristics, mainly innate abilities, but also aptitude regarding the performance of tests and luck, as well as family-specific circumstances (e.g. parents' workplace status or family problems at home, potentially affecting a student's results, but not captured by the model). All of these characteristics are assumed to be distributed normally at random in the population.

On the other hand, the distance function term u_i is expected to capture students' and teachers' efforts and motivation as well as school performance and organization, not explained by input endowments, to be included in the distance function.

The emergence of inefficiencies in education can be outlined in the following way. Firstly, different methodologies exist to teach different subjects. However, not all pedagogical tools are equally productive for all students or all circumstances. Secondly, teachers are not homogeneous or perfectly interchangeable for transforming educational inputs into academic results. Moreover, their efforts and motivation are likely to depend on financial incentives, as is the case in other production and service activities. Thirdly, factors such as effort, preferences, motivation or interest in learning and further education are not the same for all students, nor is the level of parental surveillance evenly distributed (controlling homework, skipping classes, assessment of education, conflicts and activities outside school). Last but not least, there is the role of educational institutions, which are nowadays considered the main explanatory factor for observed international differences in student achievement. This is particularly the case for institutions with a system that favors homogeneity in classroom composition, by means such as tracking students at an early age (Hanushek and Wossmann 2005) or allowing private–public school competition (Nechyba 2000).

Coming back to Equation (5), note that, in practice, the parameters of the model are estimated together with two other parameters, σ^2 and γ, using a maximum likelihood analysis where, according to Battese and Corra (1977), $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \sigma_u^2 / \left(\sigma_v^2 + \sigma_u^2\right)$.

Using this information, it is worthwhile analyzing the variance decomposition of the estimated endogenous variable $-\ln\left(\hat{y}_{Mi}\right)$ in Equation (7) denoted by $\hat{S}_{y_M}^2$ as follows:

$$S_{y_M}^2 = \hat{S}_{TL}^2 + \hat{S}_v^2 + \hat{S}_u^2, \tag{8}$$

where $\hat{S}^2_{TL}$ denotes the percentage of variance of the left-hand term in Equation (8) explained by the estimated translog educational production model $\left[TL(x_i, y_i / y_{Mi}, \hat{\alpha}, \hat{\beta}, \hat{\delta})\right]$ in the mean inefficiency value, $\hat{S}^2_v$ denotes the percentage of variance explained by the random error term and, finally, $\hat{S}^2_u$ captures the percentage of variance in the response corresponding to the inefficiency term [$-\ ln\hat{D}$]. The latter term is computed as $\hat{S}^2_u = S^2_{y_M} - (\hat{S}^2_{TL} + \hat{S}^2_v)$.

Furthermore, assuming that differences across school performance are independent of differences among student performance only detected at the intra-school level, $\hat{S}^2_u$ can be decomposed through an analysis of variance as follows:

$$\hat{S}^2_u = \hat{S}^2_{uB} + \hat{S}^2_{uW} \tag{9}$$

where $\hat{S}^2_{uB}$ and $\hat{S}^2_{uW}$ indicate the between-schools and within-schools inefficiency variance, respectively. Through this explanation of the variance of the dependent variable, we mainly seek to estimate the role of inefficiency in educational production.

3. Application to Spanish secondary schools

The quality of public schools has recently been under scrutiny in Spain, with voices calling for the reform of the Spanish Law for Education. As in most other countries, there are two possible ways of publicly financing a school in Spain: private schools financed by a public voucher system (the so-called *escuela concertada*), and public schools. The argument calling for more private schools and for a greater percentage of public expenditure in education to be monitored by private hands is usually based on the simple observation of aggregate results, like those we obtained from PISA 2003, as presented in Table 1.

A *naïve* interpretation of the descriptive results presented in Table 1 would bring us to conclude that there is a higher level of performance by private schools in terms of average scores. The study presented here, based on the estimation of a parametric

Table 1. Mathematics and reading scores by school type in Spain in PISA 2003.

School type	*n*	Mean	Standard deviation	Minimum	Maximum
Mathematics scores					
Private, government dependent	145	524.5	34.4	393.8	588.3
Government	198	506.8	37.8	365.3	607.5
All	343	514.3	37.4	365.3	607.5
Reading scores					
Private, government dependent	145	524.6	38.6	382.2	589.8
Government	198	501.4	37.3	350.8	574.4
All	343	511.2	39.5	350.8	589.8

Note: Mean differences are statistically significant, at the 95% level, with F-test = 19.72 and 31.47 for mathematics and reading, respectively. We cannot reject the hypothesis that variances are distributed homogenously, at the 95% level, with Levene's test = 0.453 and 0.526 for mathematics and reading, respectively.

stochastic distance function, can provide additional information to enable the confirmation or rejection of this assumption.

3.1. Data

In our empirical analysis, we used data from PISA, implemented in 2003 by the OECD. The aim of PISA is to measure how well 15-year-old students are prepared to face up to the challenges of modern society. PISA tests students in the subjects of reading, mathematics and science. Because the home, school, and national contexts can play an important role in how students learn, PISA also collects extensive information about such background factors. The entire database comprises 40 countries, but the present study is limited to the Spanish case. Given that the target 15-year-old population tends to be enrolled in different grades, we selected for this study upper 10th-grade students. To sum up, the analysis is based on a homogenous population composed of 6997 students (3663 in public schools and 3334 in private-voucher schools) attending 10th-grade (4° ESO in the Spanish educational system) at 343 different schools (145 private-voucher schools and 198 public schools), who, in the year 2003, completed the mathematics and reading PISA tests. It is worth noting that PISA 2003 is methodologically highly complex and it would exceed the aims of this paper to present a complete explanation of the procedures followed in both sampling design and index construction. Nevertheless, for a complete review, OECD (2004, 2005a, 2005b) may be consulted. We classify inputs in three categories – student background, school variables and peer-group effects – together with a set of control variables. Table 2 presents a rough description of each variable.

3.1.1. Background

The index of economic, social and cultural status (ESCS) was derived from three variables: the highest, father or mother, index of occupational status; the highest, father or mother, level of education (OECD 1999); and an index of educational resources at home. The variable LATE is the student's direct answer to the following question: 'In the last two full weeks you were in school, how many times did you arrive late for school?' We think that arriving late could be a sign of lack of communication between the student's teachers and parents.

3.1.2. School

SCMATBUI is the PISA index of the quality of the school physical infrastructure, and was derived from three items answered by the principals' perceptions of potential factors hindering instruction at school: school building and grounds; heating/cooling and lighting systems; and instructional space. More positive values indicate a more positive evaluation of the building. The index of quality of school educational resources (SCMATEDU) was derived from seven items measuring principals' perceptions of potential factors hindering instruction at school: instructional materials, computers, computer software, calculators, library materials, audio-visual resources, and science laboratory equipment and materials. Higher values point to positive evaluations of this aspect. Another school variable is the index of autonomy of the school (SCHAUTON). School principals were asked to report who had the main responsibility for: selecting teachers for hire; dismissing teachers; establishing

Table 2. Variable definitions.

Variable	Description
Background	
ESCS	Index of economic, social and cultural status
LATE	Number of times student has arrived late for school in the last two weeks (1 = none, 2 = one or two times, 3 = three or four times, 4 = five times or more)
School	
SCMATBUI	Index of the quality of the school's physical infrastructure
SCMATEDU	Index of the quality of the school's educational resources
SCHAUTON	Index of school autonomy
CLASS-SIZE	Average number of students in class
Peer group	
DISCLIM	Index of disciplinary climate
ESCS_MEAN	Average ESCS index of the student's peer group
Control variables	
GENDER	Student's gender (1 = male; 0 = female)
CONSOLE	Possession of video-console (Play-Station, X-Box or similar) (1 = yes; 0 = no)
NATIVE	The student and at least one of the parents was born in Spain (1 = native; 0 = non-native)
PRESCHOOL	The student attended non-compulsory preschool education for at least one year (1 = yes; 0 = no)
NUCLEAR	Family structure (1 = nuclear (the student lives with both biological parents); 0 = other families)
PRIVATE	School ownership (1 = student attends a private-voucher school; 0 = student attends a public school)
Instrumental variable	
CITY	School location (1 = student attending a school located in a city of more than 100,000 inhabitants; 0 = other locations)

teachers' starting salaries; determining teachers' salary increases; formulating school budgets; deciding on budget allocation within the school; establishing student disciplinary policies; establishing student assessment policies; approving students for admission to school; choosing what textbooks to use; determining course content; and deciding which courses to offer. The index is constructed in such a way that more positive values correspond to more direct school responsibility. The last variable directly related with the school is the students' answer to the following question: 'On average, about how many students attend your mathematics class?' Because the answer to this question is an individual student's perception, we average all the answers given to this question by all of the students belonging to the same classroom.

3.1.3. Peer group

In order to control for potential *peer-group* effects, we employed two variables. The ESCS_MEAN input is the average ESCS of the schoolmates. Although this variable measures the potential of the group, we also consider the real control that the teacher has over his/her students. For this purpose we used the PISA index of disciplinary climate (DISCLIM), which was derived from students' reports on the frequency with which the following items occur in their mathematics lessons: students do not listen to what the teacher says; there is noise and disorder; the teacher has to wait a long time for students to quieten down; students cannot work well; and students do not start working for a long time after the lesson begins. A four-point scale with the responses 'every 'lesson', 'most lessons', 'some lessons' and 'never or hardly ever' was used. Higher values of this index indicate perceptions of a more positive disciplinary climate.

3.1.4. Controls

Other variables cannot be automatically classified as inputs but they could have a significant influence over some students. As control variables, we chose students' gender (1 = male; 0 = female); possession of a video-console (1 = yes; 0 = no); attendance at preschool for more than one year (1 = Yes; 0 = no); and family structure (1 = nuclear family; 0 = other families including single-parent, mixed and others).

3.1.5. Other variables

Together with these variables, we defined the variable PRIVATE (1 = student attending a private-voucher school; 0 = student attending a public school). As mentioned before, an instrumented variable corresponding to the expected probability of attending a private-voucher school would be estimated in a first stage using a Probit model with PRIVATE as the dependent variable. As an explanatory variable in this model, other than all those present in the distance function, we included as the instrument a binary variable CITY – equal to one if a student lives in a city of more than 100,000 inhabitants, and zero otherwise. We follow here Vandenberghe and Robin (2004), who showed that, in the case of Spain, and using PISA 2000 data, this variable appears to be positively correlated with school choices, private-voucher versus public, but not with student performance. In other words, CITY appears to be the best candidate variable to correct for endogeneity bias in educational efficiency models.

Several systematic differences between the public and private-voucher schools are presented in Table 3.

All mean values across the three groups of input variables – background, school and peer group – are more favorable for private-voucher schools. Moreover, standard deviations are lower in private schools than in their public counterparts, pointing to fewer differences between students in private schools than between students in public schools. One exception is the average lower class-size in public schools. On the one hand, in small villages, there are not enough children to merit the building of private-voucher schools. On the other hand, in the past decade, Spanish people have tended to increase their demand for private education, understanding that public education is significantly worse. Regarding the control variables, we can emphasize that private-voucher schools are mostly located in big cities.

Table 3. Descriptive statistics of variables at student level in Spain by school ownership.

Input	Private-voucher (n = 3334)				Public (n = 3663)			
	Mean	Standard deviation	Minimum	Maximum	Mean	Standard deviation	Minimum	Maximum
Scores								
MATHEMATICS	527.8	71.1	249.8	731.6	509.1	71.8	241.6	723.8
READING	527.9	71.7	235.7	700.9	503.2	73.1	140.1	706.8
Background								
ESCS	4.88	0.80	1.72	6.83	4.47	0.97	1.51	6.83
LATE	1.47	0.79	1.00	4.00	1.56	0.85	1.00	4.00
School								
SCMATBUI	3.94	0.83	1.00	4.80	3.15	1.01	1.00	4.80
SCMATEDU	4.54	0.94	1.73	6.43	3.97	1.00	1.00	6.43
SCHAUTON	3.34	0.65	2.02	4.47	2.10	0.33	1.00	3.20
CLASS-SIZE	23.60	4.57	10.78	32.11	18.84	4.71	7.97	29.18
Peer group								
DISCLIM	3.83	0.99	1.00	6.09	3.80	0.98	1.00	6.09
ESCS_MEAN	4.80	0.45	3.76	5.76	4.34	0.46	2.90	5.62
Control								
GENDER	0.44	0.50	0.00	1.00	0.44	0.50	0.00	1.00
CONSOLE	0.73	0.45	0.00	1.00	0.69	0.46	0.00	1.00
NATIVE	0.98	0.15	0.00	1.00	0.97	0.17	0.00	1.00
PRESCHOOL	0.88	0.32	0.00	1.00	0.84	0.37	0.00	1.00
NUCLEAR	0.85	0.36	0.00	1.00	0.85	0.36	0.00	1.00
Instrumental variable								
CITY	0.50	0.50	0.00	1.00	0.30	0.46	0.00	1.00

3.2. Model estimation

A parametric output distance function controlling for potential endogeneity of school choice was estimated assuming a stochastic translog technology, as indicated in Section 2. Homogeneity of degree +1 was imposed by selecting one of the outputs, the students' scores in mathematics y_1 as the dependent variable, and the ratio y_2/y_1 as the explanatory variable, instead of y_2, as described by Equations (5)–(7). However, for presentation purposes, Table 4 reports the parameters corresponding to y_1, as calculated by the application of the homogeneity condition as in Equation (4).

Previously, three different specifications were estimated – translog assuming non-separability between inputs and outputs, quadratic form with non-separability, and quadratic form with separability – in order to test the most accurate specification for the output distance function in the educational production process. For this purpose, following Coelli et al. (2005, 223–224), we conducted two generalized likelihood ratio tests (*LR*), which enable the contrasting of whether or not input and output cross-effect parameters are statistically significant. Firstly, for the quadratic functional form with non-separability, the null hypothesis is rejected if the *LR* test exceeds $\chi^2_8(\alpha)$. For

Table 4. Parametric output distance function estimation correcting for endogeneity bias.

Variable and coefficient			*t*-ratio	Variable and coefficient			*t*-ratio
Intercept	α_0	0.071	−6.81	$(\ln x_3)(\ln x_5)$	β_{35}	−0.016	0.67
Outputs				$(\ln x_3)(\ln x_6)$	β_{36}	−0.014	0.81
$\ln y_1$ (mathematics)	α_1	$\underline{-0.490}$		$(\ln x_3)(\ln x_7)$	β_{37}	−0.014	1.33
$\ln y_2$ (reading)	α_2	−0.510	37.70	$(\ln x_3)(\ln x_8)$	β_{38}	0.022	−1.15
$(\ln y_1)^2$	α_{11}	$\underline{-3.102}$		$(\ln x_4)(\ln x_5)$	β_{45}	0.103	−3.26
$(\ln y_2)^2$	α_{22}	−3.102	25.82	$(\ln x_4)(\ln x_6)$	β_{46}	0.008	−0.34
$(\ln \mathrm{y}_1)(\ln \mathrm{y}_2)$	α_{12}	$\underline{3.102}$		$(\ln x_4)(\ln x_7)$	β_{47}	0.017	−1.07
Inputs				$(\ln x_4)(\ln x_8)$	β_{48}	−0.025	0.85
$\ln x_1$ (ESCS)	β_1	0.100	12.75	$(\ln x_5)(\ln x_6)$	β_{56}	−0.018	1.02
$\ln x_2$ (MEAN_ESCS)	β_2	0.203	12.39	$(\ln x_5)(\ln x_7)$	β_{57}	−0.029	2.46
$\ln x_3$ (SCMATBUI)	β_3	0.003	0.43	$(\ln x_5)(\ln x_8)$	β_{58}	0.022	−0.96
$\ln x_4$ (SCMATEDU)	β_4	−0.003	0.44	$(\ln x_6)(\ln x_7)$	β_{67}	0.015	−1.60
$\ln x_5$ (SCHAUTON)	β_5	−0.005	0.58	$(\ln x_6)(\ln x_8)$	β_{68}	−0.036	1.89
$\ln x_6$ (DISCLIM)	β_6	0.060	10.77	$(\ln x_7)(\ln x_8)$	β_{78}	0.011	−0.87
$\ln x_7$ (LATE)	β_7	−0.036	7.30	*Inputs–outputs*			
$\ln x_8$ (CLASS_SIZE)	β_8	0.019	2.73	$(\ln x_1)(\ln y_1)$	δ_{11}	$\underline{0.145}$	
$(\ln x_1)^2$	β_{11}	−0.014	0.30	$(\ln x_1)(\ln y_2)$	δ_{12}	−0.145	2.50
$(\ln x_2)^2$	β_{22}	−0.206	0.78	$(\ln x_2)(\ln y_1)$	δ_{21}	$\underline{0.252}$	
$(\ln x_3)^2$	β_{33}	−0.060	2.55	$(\ln x_2)(\ln y_2)$	δ_{22}	−0.252	1.95
$(\ln x_4)^2$	β_{44}	−0.044	1.04	$(\ln x_3)(\ln y_1)$	δ_{31}	$\underline{-0.065}$	
$(\ln x_5)^2$	β_{55}	0.020	0.69	$(\ln x_3)(\ln y_2)$	δ_{32}	0.065	−1.52
$(\ln x_6)^2$	β_{66}	0.066	4.32	$(\ln x_4)(\ln y_1)$	δ_{41}	$\underline{0.149}$	
$(\ln x_7)^2$	β_{77}	0.052	3.11	$(\ln x_4)(\ln y_2)$	δ_{42}	−0.149	2.41
$(\ln x_8)^2$	β_{88}	−0.032	1.07	$(\ln x_5)(\ln y_1)$	δ_{51}	$\underline{-0.063}$	
$(\ln x_1)(\ln x_2)$	β_{12}	0.090	1.12	$(\ln x_5)(\ln y_2)$	δ_{52}	0.063	−1.35
$(\ln x_1)(\ln x_3)$	β_{13}	−0.005	0.20	$(\ln x_6)(\ln y_1)$	δ_{61}	$\underline{0.021}$	
$(\ln x_1)(\ln x_4)$	β_{14}	0.049	1.45	$(\ln x_6)(\ln y_2)$	δ_{62}	−0.021	0.52
$(\ln x_1)(\ln x_5)$	β_{15}	−0.023	0.83	$(\ln x_7)(\ln y_1)$	δ_{71}	$\underline{0.016}$	
$(\ln x_1)(\ln x_6)$	β_{16}	0.016	0.63	$(\ln x_7)(\ln y_2)$	δ_{72}	−0.016	0.61
$(\ln x_1)(\ln x_7)$	β_{17}	0.007	0.43	$(\ln x_8)(\ln y_1)$	δ_{81}	$\underline{0.073}$	
$(\ln x_1)(\ln x_8)$	β_{18}	0.016	0.55	$(\ln x_8)(\ln y_2)$	δ_{82}	−0.073	1.50
$(\ln x_2)(\ln x_3)$	β_{23}	0.038	0.71	*Control*			
$(\ln x_2)(\ln x_4)$	β_{24}	−0.151	1.90	GENDER	λ_1	0.005	1.84
$(\ln x_2)(\ln x_5)$	β_{25}	−0.150	2.22	NATIVE	λ_2	0.047	5.84
$(\ln x_2)(\ln x_6)$	β_{26}	−0.052	1.02	CONSOLE	λ_3	−0.011	3.81
$(\ln x_2)(\ln x_7)$	β_{27}	0.010	0.29	PRESCHOOL	λ_4	0.026	6.70
$(\ln x_2)(\ln x_8)$	β_{28}	0.115	1.63	NUCLEAR	λ_5	0.011	2.84
$(\ln x_3)(\ln x_4)$	β_{34}	0.034	1.39	PRIVATE_HAT	λ_6	−0.003	0.42
Other ML parameters	σ^2	0.030	34.86				
	γ	0.857	74.38	Mean inefficiency		0.875	

Notes: Underlined parameters are calculated by applying homogeneity conditions; all the parameters are multiplied by −1. Maximum likelihood function = 5253.2; $LR\chi^2$ on one-sided error = 399.4.

α = 0.01 the critical value is 20.09, and we obtained LR = 39.71 for the quadratic with non-separability. We therefore rejected the quadratic functional form with separability. Secondly, we decided to contrast the quadratic with non-separability functional form (the null hypothesis) with a translog functional form also with non-separability. In this case, for $\chi^2_{28}(\alpha)$ with α =0.01 the critical value is 48.28, and we obtained LR = 67.08. We therefore also rejected the quadratic functional form with non-separability. For these reasons, the results presented in Table 4 are those corresponding to the translog output distance function with non-separability between inputs and outputs.

Once we had determined the educational functional form, the next stage was to estimate the school choice model, private-voucher versus public, using the variable CITY as the instrument. As expected, the effect of living in a city of more than 100,000 inhabitants on the probability of attending a private-voucher school is highly significant and indicates a marginal effect of 0.318. That is an effect higher than those estimated by Vandenberghe and Robin (2004), who reported values ranging from 0.18 to 0.25 using PISA 2000 data for Spain. The estimated probability, the PRIVATE_HAT variable, replaces the observed PRIVATE variable in the estimation of the distance function (Equation (3)).

As is usual for the estimation of translog functions, the original variables, y_m (m = 1,2) and x_k (k = 1,…,8), were transformed in deviations to mean values. Therefore, the parameters in Table 4 must be interpreted as distance function partial elasticities at mean values. For instance, those corresponding to the reading and mathematics scores are negative and indicate that student performance or efficiency increase (distance functions increase) when, *ceteris paribus*, their reading and mathematics scores increase. The opposite effect is observed for coefficients on inputs that are positive. This indicates that student performance decreases (distance functions decreases) when inputs increase.

Some general conclusions can be drawn from Table 4. The expected mean efficiency, computed as $E[\exp(-u_i|\varepsilon)]$, is equal to 0.8752 (bottom of Table 4), which indicates average student efficiency measured with respect to the stochastic frontier model. Furthermore, we can directly examine the coefficients of the control variables. The variables NATIVE, CONSOLE, PRESCHOOL and NUCLEAR are all significant regard to the expected coefficient. Therefore, if one student simultaneously fulfills the variables of being a Spanish native, of not having a console, of attending preschool and of belonging to a nuclear family, he will improve his efficiency by 0.095 points over a non-native, console-possessing, non-preschool and non-nuclear friend. This represents an improvement in both scores of a 9.5%. Moreover, the variable GENDER obtains a positive but only significant coefficient at 90% level, indicating that males are slightly more efficient than females. Finally, the instrumental variable PRIVATE_HAT is not significant after controlling for selection bias. This means that we can identify more and less efficient students and schools across the whole population. However, being a private-voucher school does not guarantee better employment of educational resources once we control for school choice endogeneity and educational inputs.

Using the estimated parameters, the notation presented in Equations (8) and (9) and after proceeding to some further calculations, we obtained the variance decomposition of the estimated response presented in Table 5.

This decomposition is also illustrative of the model potentialities to be used in the policy orientation debate. It shows us that student achievements can mainly be

Table 5. Estimated variance decomposition.

Variance component		Share (%)	
Model	$\hat{S}^2_{TL}$		44.0
Random term	$\hat{S}^2_{v}$		7.6
School inefficiency	$\hat{S}^2_{uB}$	8.8	
Student inefficiency	$\hat{S}^2_{uW}$	39.6	
Total inefficiency	$\hat{S}^2_{u}$		48.4
Total variance	$\hat{S}^2_{y_M}$		100.0

explained by the model (44.03%); that is, by school, by student endowments introduced as input factors, by peer-group effects and control variables and, finally, by the ratio of actual outputs. The random term that we attributed to non-observable factors, such as individuals' innate abilities, other family circumstances or simply luck, account for only 7.58% of the total variance. Finally, estimated inefficiencies play an important role (48.39%), as expected, but these are mostly attributed to students (39.58%) rather than to schools (8.81%). Remember, however, this last decomposition is obtained under the strong assumption that schools are only responsible for differences in mean efficiency across the institutions, and not at all for inefficiencies within them at student level.

4. Computation of elasticities

One of the major advantages of flexible parametric output distance function analysis at student level is that it can provide additional insights into the educational production process, overcoming at the same time model misspecification problems and the estimations of non-parametric techniques.

Once the parameters of Equation (4) are estimated, it is interesting to calculate meaningful elasticities. In education, we are concerned with exploring three results: the relative facility in substitution between outputs; the elasticity of each output with respect to each input; and the elasticity between the outputs themselves. Note that, in the general case under study here, the units of observation i are the students, the outputs y_{mi} are the students' performance in M subjects and the x_{ki} are individual input variables corresponding to family background, peer-group characteristics and school factors (note that the i subscripts are suppressed in this section in order to simplify the presentation).

First, the distance function elasticity with respect to each input (and each output) provides information about how increases in one input (output) translate into more (less) inefficiency for each student. These values can be obtained using the following expressions:

$$r_{D,x_k} = \frac{\partial D}{\partial x_k} = \frac{\partial \ln D(x,y)}{\partial \ln x_k}\frac{D(x,y)}{x_k};\quad r_{D,y_m} = \frac{\partial D}{\partial y_m} = \frac{\partial \ln D(x,y)}{\partial \ln y_m}\frac{D(x,y)}{y_m} \tag{10}$$

Positive values of r_{D,x_k} (r_{D,y_m}) indicate that greater input (output) implies higher (lower) distance values or, in other words, more inefficiency (efficiency). According to Grosskopf et al. (1996), it is also meaningful to measure the Allen elasticity of output substitution, which can be defined in terms of distance functions as:

$$A_{y_n y_m} == \left[D_o(x,y) \cdot \frac{\partial^2 D_o(x,y)}{\partial y_n \partial y_m} \right] \Big/ \left[\frac{\partial D_o(x,y)}{\partial y_n} \cdot \frac{\partial D_o(x,y)}{\partial y_m} \right] \qquad (11)$$

Negative values of the Allen elasticity reflect output substitutability and positive value output complementarity between y_n and y_m. Finally, it is interesting to measure how one output is marginally influenced by changes in inputs. Partial elasticities between output m and input k are obtained in the following way:

$$s_{y_m, x_k} \equiv \frac{dy_m / y_m}{dx_k / x_k} = \frac{r_{D, x_k}}{r_{D, y_m}} = \frac{\beta_k + \sum_{k=1}^{K} \beta_{kl} \ln x_k + \sum_{m=1}^{M} \delta_{km} \ln y_m}{\alpha_m + \sum_{m=1}^{M} \alpha_{mn} \ln y_m + \sum_{k=1}^{K} \delta_{km} \ln x_l} \qquad (12)$$

In addition we can compute how one particular output varies with another output.

$$s_{y_m, y_n} \equiv \frac{dy_n / y_n}{dy_m / y_m} = -\frac{r_{D, y_m}}{r_{D, y_n}} - \frac{\alpha_m + \sum_{m=1}^{M} \alpha_{mn} \ln y_m + \sum_{k=1}^{K} \delta_{km} \ln x_k}{\alpha_n + \sum_{n=1}^{M} \alpha_{mn} \ln y_n + \sum_{k=1}^{K} \delta_{kn} \ln x_k} \qquad (13)$$

4.1. Empirical elasticity estimations

In this section we present the results obtained by applying the elasticity Equations (11)–(13) shown above. Given the flexible nature of the translog distance function, elasticities vary at each point and must be calculated for all observations in order to obtain a more convenient appraisal of the way they vary across the sample population. For presentation purposes only, inter-quartile elasticity values are reported here.

The output Allen elasticity of substitution $A_{y_n y_m}$ was estimated as shown in Equation (11). In most cases, the computed values were negative, with a median value of –5.10, and must be interpreted as meaning that outputs (reading and mathematics scores) are substitutes. This result suggests that the choice of how much effort students devote to each subject could damage results in the study of the other output. This occurs, for instance, if the student dedicates too much time to reading at the expense of time spent on mathematics. Changes in individual motivations and preferences or in teachers' and parents' requirements can make the proportion of time devoted to educational instruction become more balanced. Output–output and output–input elasticities are presented in Table 6.

With respect to output–output elasticities, we can confirm the result of substitutability obtained by the Allen elasticity between the outputs. Moreover, the results indicate that a 1% gain in the mathematics score can be made with a median loss of 3.121% in reading performance. On the other hand, an augmentation of a 1% reading result can be made with a median cost of a 0.320% score in mathematics. As a

Table 6. Output/output and output/input elasticities.

Variable	Mathematics inter-quartile values 25%	50%	75%	Reading inter-quartile values 25%	50%	75%
Output with respect to output ($S_{y_m,y_{mk}}$)						
Mathematics score (y_1)	–	–	–	−4.855	−3.121	−2.244
Reading score (y_2)	−0.446	−0.320	−0.201	–	–	–
Output with respect to input (S_{y_m,x_k})						
Background						
ESCS	0.096	0.198	0.398	0.059	0.076	0.102
LATE	−0.197	−0.100	−0.024	−0.053	−0.041	−0.021
School						
SCMATBUI	−0.281	−0.154	−0.076	−0.071	−0.057	−0.045
SCMATEDU	0.076	0.193	0.398	0.053	0.075	0.104
SCMATAUTON	−0.353	−0.183	−0.087	−0.091	−0.067	−0.050
CLASS-SIZE	0.023	0.040	0.073	0.012	0.014	0.017
Peer group						
MEAN ESCS	−0.046	0.083	0.263	0.017	0.047	0.086
DIS	0.175	0.312	0.565	0.094	0.112	0.137

consequence of this asymmetric relationship, output/input elasticities reported in Table 6 vary on behalf of the output considered. The size of variations in outputs over inputs has a different and greater effect on mathematics than on reading at student level. Firstly, the median elasticity of the individual ESCS on mathematics is around 0.2. So a 10% increase in ESCS home background returns a 2% better result in mathematics and a 0.7% better result in reading. We consider this a considerable effect, remembering that these are non-repeating 15-year-old students. Therefore, we conclude that, at 15 years old, this remains a significant family background effect. Regarding the LATE variable, each change from value one to value two and so on yields a median loss of 1% of the result in mathematics and 0.5% in reading. Regarding the peer-group effect, we observe few gains due to the average ESCS_MEAN in both subjects. Nevertheless, the variable with the biggest elasticity is the behaviors of classmates. At the age of 15 years, it is crucial, above all other inputs, to maintain a disciplinary climate in class. The school variable elasticities do not point to a straight direction. On the one hand, as expected, the index of the quality of the school educational resources SCMATEDU is positive, but on the other, unexpectedly, SCMATBUI and SCHAUTON are negative and CLASS_SIZE is positive. This is a puzzle whose explanation exceeds the purpose of the paper; however, we provide two possible interpretations to this result. The first reason we can provide to explain such a result is that a high index in SCMATBUI and SCHAUTON is related to the new private-voucher schools built in the past few years, revealing management problems related to the beginnings of any enterprise (staff selection, children and teacher movement, lack of experience, and so on). A second reason can be argued if we add up the coefficients of the four school variables. The results show a range from slightly negative to slightly positive values in both scores. This fact could indicate that, once family and peer group are taken into account, school variables have only a little or even an insignificant effect over mathematics and reading scores (Hanushek 1986, 2003).

5. Concluding remarks

A comprehensive review of the literature of educational economics shows that the process of transforming educational inputs into test results is highly complex and little understood. Despite this generalized result, most studies continue to apply the traditional Cobb–Douglas analysis at school level. On the other hand, whether or not public or private-voucher schools are equally efficient constitutes a current debate in the Spanish educational system.

In order to address the problem of the estimation of an educational production function in this paper, we have proposed the use of frontier analysis techniques – more precisely, a flexible parametric stochastic distance function – in order to overcome the main criticisms directed at these studies. Moreover, we explicitly consider that education is a process in which students use their own and school inputs in order to transform these inputs into academic results, subject to inefficient behaviors that can be identified at student level. With respect to the case of the private–public debate, we suspected possible school choice selection bias. In order to control for potential endogeneity bias in this model, we used a two-stage instrumental variable regression approach. We applied this methodology to the Spanish case using the test scores of 15-year-old students and background data available from the PISA Project, implemented by the OECD in 2003. The main results of this study can be summarized as follows:

(1) Individual student achievements are explained, at a rate of 44.03%, by the education production model itself. The random term that we attributed to non-observable factors, such as an individual's innate abilities or family circumstances, accounts for only 7.58% of the total variance. Finally, 48.39% was the share attributed to the inefficiency component. According to this result, we think the potential role of inefficiency in education should not be omitted from educational production models, nor from educational policies. Direct campaigns of information to the students and their families regarding how the length and the quality of education lead to higher life-cycle earnings could yield better results than public policies based solely on a growth in educational expenditure.
(2) Several factors could be responsible for the observed differences in performance, among them the effort and motivation put into education by both teachers and students. Within the context of this study, we were particularly interested in a comparison between public and private school scores. The results showed that, in the case of Spain, the observed differences in favor of private-voucher schools scores were mainly accounted for by differences in school inputs, student background and peer-group characteristics, considered production factors in the education process. All in all, once school inputs, student background and peer group, and the school choice are taken into account, the observed differences across schools, as distinguished by ownership, vanish. This does not mean that the information about performance was wrong, but that

if students attending private-voucher schools obtained better results, this was as a direct consequence of more favorable conditions: better family background, peer groups and school inputs. Furthermore, a well-known selection process is at work in Spain, as well as in other countries, which offers the choice between public and private-voucher schools. As a consequence, public schools accept a higher percentage of students with less favorable backgrounds (e.g. foreigner populations with language difficulties and special needs). On the other hand, the 'voluntary' fee demanded for most private-voucher schools in Spain could be an entry barrier, with the poorest families being self-selected into the public schools. As the estimated two-stage parametric stochastic distance function model takes into account the choice of school and other student characteristics, public schools take as a benchmark this less favorable context and, as expected, their efficiency scores are better than when directly compared with private school scores in Table 1.

(3) The analysis conducted here reveals that input effects are better captured with a translog specification, a non-linear second-order approximation, which takes into account the multi-output multi-input nature of education production. A noteworthy finding is that the debate about which is the best way to spend public resources in school needs to take into account these interactions. Our estimations indicate that the climate of the school classroom is the variable with the most influence over test scores. However, in general we cannot conclude that more money devoted to school resources is always effective whatever its allocation.

To sum up, we think that the conceptual framework presented in this paper, based on the estimation of parametric stochastic output distance function at student level, provides an appealing methodology for enhancing our understanding of the education process, as it is subject to inefficiency.

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Note

1. A private-voucher school can freely assign one point to those children who have been attending the kindergarten attached to that school. Since all preschool education is run on a fully private basis, parents paying (by necessity) for these years of education are, in effect, 'buying' this extra point, thereby giving their child a headstart in their school career.

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