Clusters or networks of economies? A macroeconomy study through Gross Domestic Product

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Abstract

We study correlations between web-downloaded gross domestic product (GDP)'s of rich countries. GDP is used as wealth signatures of the country economical state. We calculate the yearly fluctuations of the GDP. We look for forward and backward correlations between such fluctuations. The correlation measure is based on the Theil index. The system is represented by an evolving *weighted* network, nodes being the GDP fluctuations (or countries) at different times.

In order to extract structures from the network, we focus on filtering the time delayed correlations by removing the least correlated links. This percolation idea-based method reveals the emergence of connections, that are visualized by a branching representation. Note that the network is made of weighted *and* directed links when taking into account a delay time. Such a measure of collective habits does not readily fit the usual expectations, except if an economy globalization framework is accepted.

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1. Introduction

Some time ago, the question of economy globalization was studied along the lines of macro-econophysics research [1–3]. Other considerations along related lines can be found in contributions by Miskiewicz [4] and Gligor [5]. The question pertains to well-known observations: there are political, cultural, scientific, economic cooperations all over the world, even though there are even anti-globalization organizations.

In our case, we consider that a *globalization process* in economy should be understood as an increase of similarities within (macroeconomy) development patterns. In so doing we have proposed to study whether the pattern economy of countries can be represented by and discussed through the *distance* between each country normalized gross domestic product (GDP) fluctuations. The notion of distance is somewhat arbitrary, as has been already discussed in Ref. [1], but theoretical considerations are left for further investigation elsewhere. We have chosen to present here a study based on the statistical distance, in Section 2, for the 23 most developed countries for which annual GDP data is available in the interval [1950–2003]. Previous studies using

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Network analysis tools to understand the dynamics of GDP's have been already published, for a few examples see Refs. [6–9].

One might argue [10] that some data detrending is necessary to take into account cycle-like effects [11,12]. In previous studies [1] it has also been observed that there is much noise in the data. We have shown that some average should be taken over optimal time windows to remove such a noise. We will use for averaging a time window of $T = 4$ years which is *a priori* optimizing the statistics but is clearly related to most electoral periods.

One usual *political* question is whether one has to *follow* another country policy in order to improve one's country economy. The question of followers and leaders can be tackled by considering time delayed correlations. In this first attempt we will only consider a one year time lag τ, being aware that other time lags should be involved. Note that positive or negative ones should be considered. Nevertheless it is worthwhile to check whether such an approach is already meaningful for $\tau = 1$ and -1.

The system can be displayed as an evolving network, nodes being the GDP fluctuations or, in a short way, countries at different times. In Section 3, we extract structures from the network through filtering the time delayed correlations (or distance matrix) by removing the least correlated links. This percolation idea-based method reveals the emergence of connections, that are visualized by a branching representation. However, the system is pretty unstable. Yet, expected features (connections) are reproduced, though some others are surprising. This should imply some further consideration in the line of Glansdorff and Prigogine [13] about structure and stability of world economic systems and the intrinsic role of fluctuations. Notice also that the network is much more complex when taking into account delay time. In such a case, the correlation matrix is *NOT* symmetrical, whence the network links are not only weighted but also directed. The number of distances to be considered is also $N(N-1)$, where N is the number of countries, in contrast to $N(N-1)/2$, when $\tau = 0$. In Section 4, we conclude that such a measure of collective habits does not fit the usual expectations, defined by politicians, but that does not imply that they are wrong or right, nor are we.

We stress that the following relevant time instances must be considered, i.e.,

1. Initial observation time t_0,
2. Data acquisition time t,
3. Increment or fluctuation time span Δ,
4. Delay time (between two countries) τ,
5. Window observation (averaging) time T, i.e., $[t_{min}, t_{max}]$,
6. Conclusion time t_N.

Thus the general correlation matrix can be of quite high dimension $(i, j; t_0; t, \tau; ..)$.

2. Methodology

We use data from http://www.ggdc.net/index-series.html for recording the Normalized GDP of the 23 most developed countries. Most of these countries have natural time scales associated to the time lags between elections. Whence we use an average time of $T = 4$ years. For each of the countries, let us define $p_{i;t} = \ln G_{i;t+T}/G_{i;t}$ that is equivalent to the average variation of the GDP over T years.

Let us now introduce a correlation measure and associated distance measure for the countries. The correlation measure is based on the Theil index and is defined as follows:

$$C_{i,j;t,\tau} = \frac{\langle p_{i;t'} p_{j;t'+\tau} \rangle_t - \langle p_{i;t'} \rangle_t \langle p_{j;t'+\tau} \rangle_t}{\sqrt{\langle p_{i;t'}^2 - \langle p_{i;t'} \rangle^2 \rangle_t \langle p_{j;t'+\tau}^2 - \langle p_{j;t'+\tau} \rangle^2 \rangle_t}}, \tag{1}$$

where the averages are defined as follows:

$$\langle F(t') \rangle_t = \frac{1}{\Delta} \sum_{t'=t}^{t+\Delta} F(t'). \tag{2}$$

The "averaging time" Δ has been hereby chosen to be $\Delta = 4$ in the following. The distance is obtained from the correlation from the definition (dropping indices here):

$$d = \sqrt{\tfrac{1}{2}(1 - C)}. \tag{3}$$

Let us stress that this short hand writing accounts for time delays between the signals. Let us also note that other measures of distance between different countries can be defined, e.g.

$$D_{i,j;t,\tau} = |p_{i;t} - p_{j;t+\tau}|. \tag{4}$$

The comparison of results obtained for different measures will be considered elsewhere.

When $\tau = 0$, the above quantities are at fixed time. When $\tau \neq 0$, these quantities measure the correlations with a delay time between the economies of countries. If the evolutions were completely identical, one would find $D = 0$. Let us stress again that $D_{i,j;t,\tau}$ is symmetric in i, j, when $\tau = 0$. For each pair of countries, this quantity defines therefore a surface whose statistical properties can be studied.

The first relevant quantity to report is the rank analysis of $D_{i,j;t,\tau}$, at different values of t (1974, 1984, 1994) and $\tau = 0, 1, -1$ (Figs. 1–3), like on a Zipf plot. Notice the non-trivial behavior, with various crossing of the curves, on one hand; moreover, the data does not seem to lead to a fine straight line fit. Because the GDP increment distance between countries is rather uniform, the results suggest non-trivial correlations in evolving
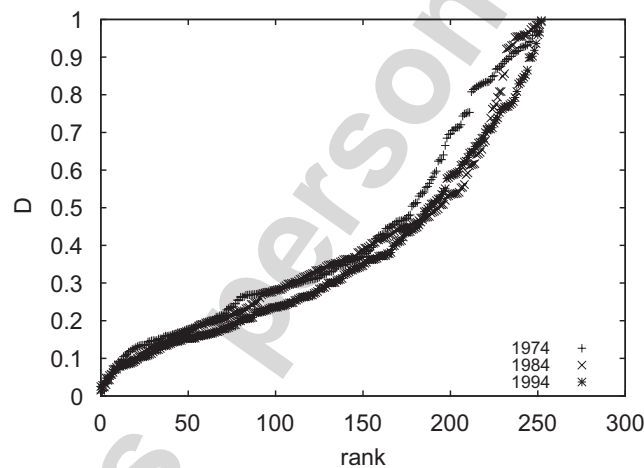

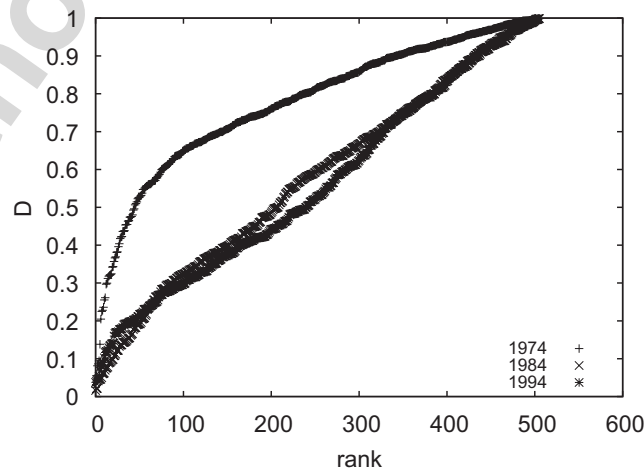Fig. 1. Rank analysis of $D_{i,j;t,\tau}$ for 1974, 1984, 1994 years when $\tau = 0$.

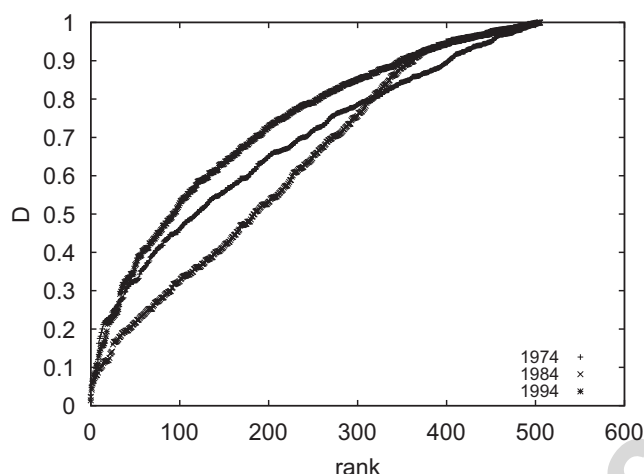Fig. 2. Rank analysis of $D_{i,j;t,\tau}$ for 1974, 1984, 1994 years, when $\tau = 1$.

Fig. 3. Rank analysis of $D_{i,j;t,\tau}$ for 1974, 1984, 1994 years, when $\tau = -1$.

economy schemes, which likely result from complex interactions of country economies. A globalization framework is hardly proved from the above. However, it is clear that the ranking pattern does not change during the last part of the century, when $\tau = 0$, indicating a rather common economy philosophy and similar results for all countries. The drastic change between the 1974 epoch and the later ones (1984,1994) if a small time delay is introduced in order to study the economy patterns seems also to indicate a tendency toward globalization and a similar economic policy for all.

3. Clustering analysis

In order to extract structures from the distance matrix, we can build country networks through filtering the time delayed correlation matrix by removing the least correlated links, e.g. moving a threshold down. To do so, we define the filtered matrix D^F, where

$$
\begin{aligned}
D_{ij}^F &= 0 \quad \text{if } D_{ij} > f, \\
D_{ij}^F &= 1 \quad \text{if } D_{ij} < f.
\end{aligned}
\tag{5}
$$

The resulting matrix therefore connects nodes that are very close to each other, and removes the weak links. By decreasing the values of f, one observes therefore the breaking of the *continent* into several islands that correspond to very connected "communities of countries". Let us stress that this method has already been successfully applied in order to reveal structures in online communities [14]. Contrary to this previous study, the distance matrix may be asymmetric, i.e., when $\tau \neq 0$. Consequently, the network representation will be made of directed links from i to j, if the matrix element D_{ij} verifies $D_{ij} < f$. This percolation idea-based method reveals the emergence of connections, that are interestingly visualized by a branching representation. It seems best due to lack of space in these proceedings (other displays are available form the authors) to illustrate the study through the case $t = 1984$, $\tau = 1$ (Fig. 4); the four levels allow clusters to appear, but the figure obviously indicates correlations at odd with geography or trivial political economy minded expectations.

4. Conclusions

In this short paper, we have introduced techniques in order to study the development of structures in macro-economical systems. To do so, we have considered the GDP's of the 23 most developed countries, and defined statistical distances between their time evolution. A possible delay between economies has been accounted for through the parameter τ, thereby allowing for some possible distinction between *leading* and *following* countries.
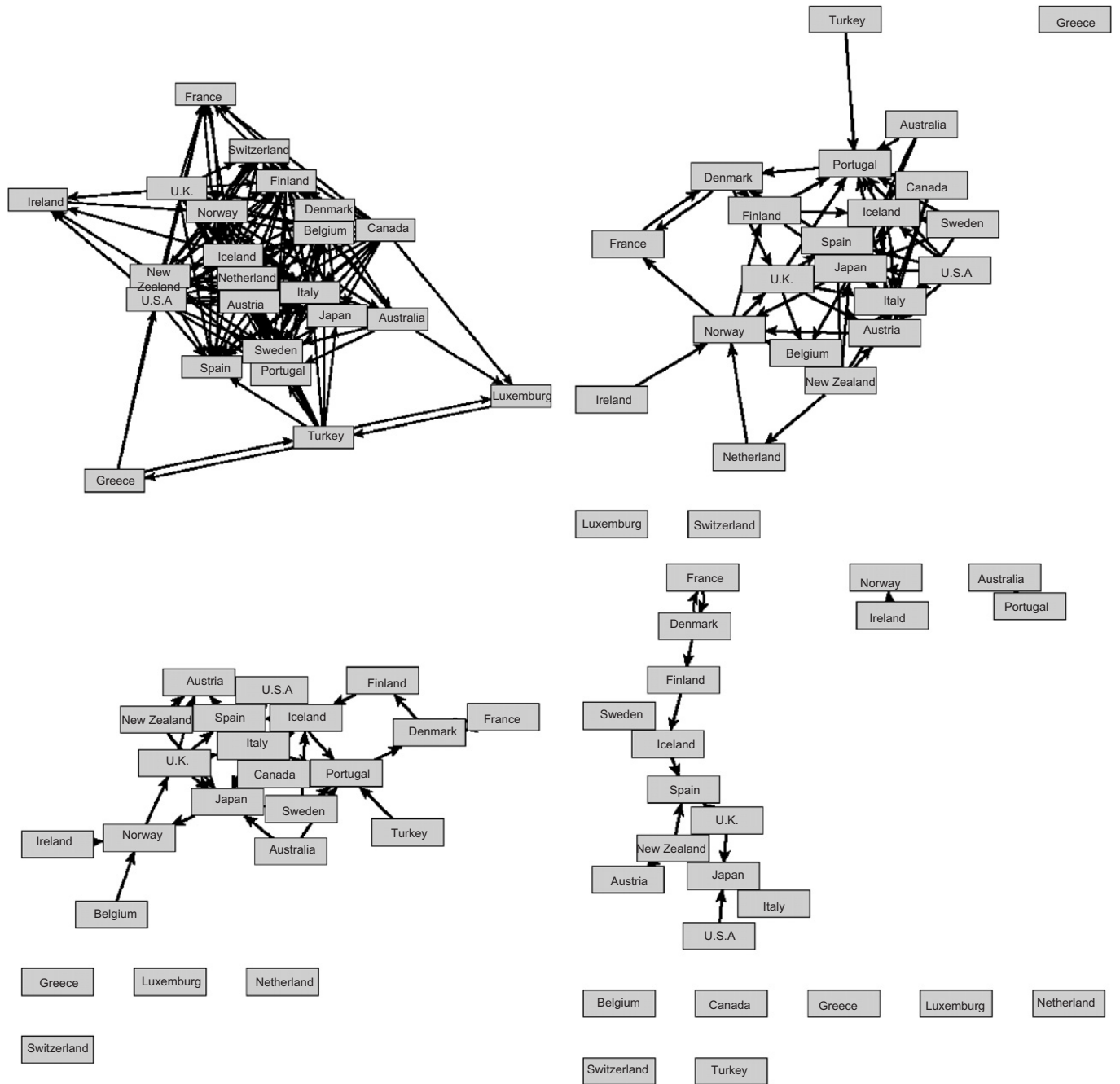
Fig. 4. Clusters resulting from filtering the distance matrix for $t = 1984$ and $\tau = 1$. The values of the filtering f are respectively 0.45, 0.20, 0.15 and 0.1.

In the above analysis, we have observed that the rank distribution of GDP *fluctuation* correlations is unexpectedly giving a law different from the classical Zipf power law (with an exponent$= -1$); even after time averaging the data contains much noise [15]. Nevertheless, patterns are found, under the form of clusters in the countries' network [15–17]. These are somewhat unexpected from geography. Therefore, this structuring seems to be in line with economy globalization, that tends to homogenize the economic development of countries; it may be related to the influence of political considerations.

Clearly, other values of the time lag should be examined, and not so rich countries as well. There is no reason that the same time lag should be considered for different countries, in order to find some strong influence of one on another, if any. In fact one could look for higher order time delayed correlation functions as well.

Finally, let us stress that there are many possible approaches, that could help the understanding of globalization processes in economy. A classical way in searching for clusters is to find subgraphs with a (high) clustering coefficient or to investigate the graph topology.

Acknowledgments

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[15] An anonymous referee commented that the noise inherent to such small sample invalidates any possible outcome of the analysis. Moreover temporal series of fluctuations in economy are usually autocorrelated, like when two such series are correlated with a common process, e.g. traders in the stock market all influenced by the same inputs, such as news or index markets, or countries within the EU who follow common european policies. Under these conditions, it is easy to see that both series are cross correlated at different time lags; this fact does not imply that one series is driving the other one. We quote *In fact, one finds cross correlation for positive and negative lags and therefore it is not possible to say who is influencing who*. We do not disagree; in fact we claim that the networks generated in this paper might not reflect the real (if any) pattern of influences among countries. However the clustering can be accepted if one accepts economy (empirical) theoreticians that globalization *exists*. In summary, we believe that our possible interpretation of the results is meaningful.

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