

# Long-range potential and the fine structure of the diffraction peak

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The possibility of oscillations in the differential elastic cross section of hadron scattering at small momentum transfer is studied. It is shown that string-like quark potentials at large distances can lead to such small oscillations, and an analysis of the experimental data at small  $|t|$  allows the determination of the parameters of the potential.

The AKM theorem [1] predicts that the differential elastic cross section will contain a structure periodic in the scale  $q = \sqrt{|t|}$  for  $t \rightarrow 0$  [2]. We study this effect through models for the hadron-hadron interaction at large distances. This will lead to a new parametrisation of experimental data with a high confidence level in a wide energy region. We consider many experimental data on nucleon-nucleon elastic scattering at small momentum transfer in a very large energy interval from  $\sqrt{s} = 2$  GeV to 1800 GeV. We find that the oscillations are present in most of the datasets.

The differential cross sections are given by the formula

$$\begin{aligned} d\sigma/dt = \pi \Big\{ & F_C^2(t) + \left[1 + \rho^2(s, t)\right] \operatorname{Im} F_N^2(s, t) \\ & \mp 2[\rho(s, t) + \alpha\varphi(s, t)] F_C(t) \operatorname{Im} F_N(s, t) \Big\}, \end{aligned} \quad (1)$$

where  $F_C$  is the Coulomb scattering amplitude,  $F_N$  the hadronic amplitude,  $\rho(s, t)$  the cotangent of its phase, and  $\varphi(s, t)$  the Coulomb-hadron interference phase [3]. Here we neglect the hadron spin-flip amplitudes and take

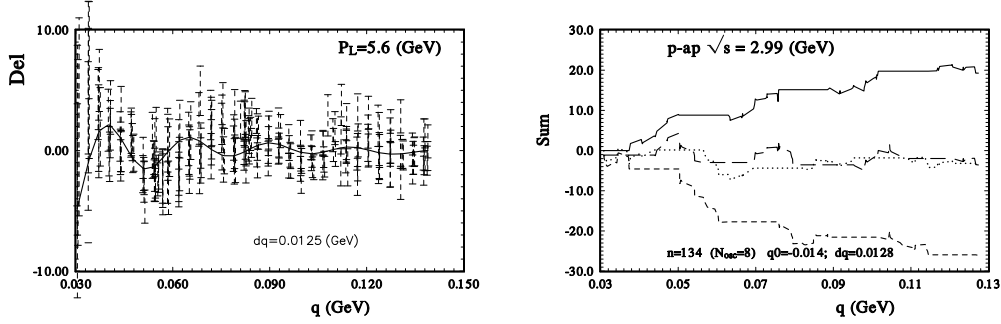


Figure 1: a) [left] Proton-antiproton scattering at  $P_L = 5.6$  GeV/c: points - the difference between  $d\sigma/dt$  and an exponential description; line - the fitted additional function (3) with  $dq = 0.0125$  GeV/c. b) [right] Proton-antiproton scattering at  $P_L = 3.7$  GeV/c: the sum of the deviations of the experimental points from an exponential curve: the solid and the short dashed lines are the sums on the even and odd intervals of width  $dq = 0.0128$  GeV; the middle curves are the same but the beginning of the intervals is moved by  $dq/2$ .

into account all parts of the electromagnetic amplitudes. The spin-non-flip amplitudes are written in the standard form for small momentum transfer:  $F_C = \mp 2\alpha G^2(t)/|t|$ ;  $F^N(s, t) = h(s) [1 + \rho(s, t)] \exp(B(s)t/2)$ , where  $\alpha$  is the fine-structure constant and  $G(t)$  the proton electromagnetic form factor squared. This formula is used by experimentalists to extract  $\rho$  from their data to obtain the value of  $\rho(s, t)$ . If an additional periodic amplitude has a sizeable real part  $ReF_{osc}(s, t)$ , the oscillation in the differential cross sections will be proportional to

$$\Delta[d\sigma/dt]_{osc} \sim 2 ReF_{osc}(s, t) [ReF^C(t) + \rho(s, t)ImF^h(s, t)]. \quad (2)$$

The determination of  $\rho(s, t)$  and  $ReF_{osc}(s, t)$  then clearly depend on each other.

We considered several forms for  $F_{osc}$  (Bessel functions  $J_0$  or  $J_1$ , sines, functions of  $q$  or  $q^2$ ...), and the best results are obtained for

$$F_{osc} = h_{osc} \sin[\pi(\phi(s) + q/dq)]. \quad (3)$$

We considered experimental data for  $\bar{p}p$  scattering at low energies ( $\sqrt{s} = 3.1 - 6.2$  GeV) and high energies ( $\sqrt{s} = 52.6, 541, 546, 1800$  GeV) [4]. The

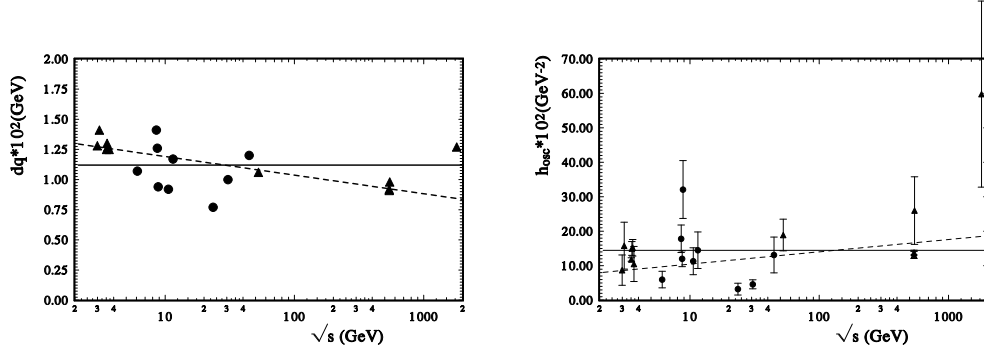


Figure 2: a) [left] Half-period of the oscillation; b) [right] Amplitude of the oscillation.

inclusion of the term (3) in the fits leads to a decrease of the  $\chi^2$  of the order of 20%, as can clearly be seen in Fig. 1a, where we show the deviation of the data from an exponential. For  $pp$  elastic scattering we examined low energies ( $\sqrt{s} = 8.55, 8.7, 8.83, 10.6$  GeV) and high energies ( $\sqrt{s} = 30.6, 44.7$  GeV). We again obtain an improvement in the  $\chi^2$  between 15% and 25%. More remarkably, the half period of the oscillation for all these experiments lies near 12 MeV (see Fig. 2a). The normalization constant of the additional term fluctuates around  $0.15 \text{ GeV}^{-2}$ , and it may grow with energy as  $\ln s$ , as shown in Fig. 2b.

In all cases we obtain a substantial decrease in the value of the  $\chi^2$ . It is clear that for such a high frequency, the improvement is unlikely to be accidental, or to correspond to fluctuations of the data. We believe that it is evidence for the existence of such oscillations in the real part of the scattering amplitude.

Note that to obtain the above fits, we also resorted to an unusual method. Indeed, the direct minimization of the  $\chi^2$  works poorly, as one should first fix the model producing oscillations for the small- $|t|$  part of the scattering amplitude, and as the effect is small, so that outside of the exact fit, it will give an insignificant change to the  $\chi^2$ .

Therefore, we also used a method comparing two statistically independent sets of data, for example [5]. The whole interval of  $q$  is divided into small intervals  $\Delta q_i$ ,  $i = 1, \dots, n$ , equal to  $dq$ . The deviations of the experimental data from an exponential, weighted by the inverse the experimental errors,

are added in two separate sums for  $i$  even or odd. If the intervals correspond to the period of the cross section, then these two sums will be significantly different in sign. If the starting point of the first interval is moved by one half period, the two sums will become identical, and close to zero (see the middle lines of Fig. 1b and [6]). We can calculate the significance level by comparing the two fits, with the interval moved or not. Most remarkably this method leads to a similar value for the half period,  $dq \simeq 12$  MeV. Note that the effect comes mostly from the lowest  $q$ , where it gets enhanced by the Coulomb-hadron interference term.

The convergence of the two methods to equivalent fits, and the significant improvement in the quality of the fit for many independent experiments, suggest that the oscillations are not due to statistical fluctuations, but rather reflect a physical phenomenon.

## 1 The hadron potential at large distance

Such a small period of oscillation may be related with the properties of the hadron interaction at large distances.

In some calculations of the scattering amplitude, determined by the gravitational potential in an  $n + 4$  dimensional world in the framework of the ADD-scenario [7], one obtains a periodical structure (our calculations and [8]).

In general, we can assume that an additional potential has a small constant value at large distances and is sharply screened at a given distance, which provides a cut-off for the integrals. In the  $q$ -representation, the corresponding amplitude will oscillate with a period that depends on the distance at which the potential is cut, as shown in Fig. 3a.

Such a scenario could also be realized during fireball processes via the screening of the electromagnetic interactions at large distances.

Indeed, let us take the additional potential in our case in  $b$ -space in the following form:

$$F_{ad}(s, t) \sim \int_0^\infty b db J_0(bq) h_{ad}(s) [b_{scr}^2 - b^2]^{-2} = iqb_{scr} K_1(iqb_{scr}), \quad (4)$$

where  $b_{scr}$  is the distance at which the additional potential has a screening effect, and  $K_1(ix)$  is the MacDonald function of imaginary arguments. If  $b_{scr}$  is sufficiently large we obtain an oscillating amplitude with a small period in  $q$  (see Fig. 3b). Note that in this case the integration has no specific cut.

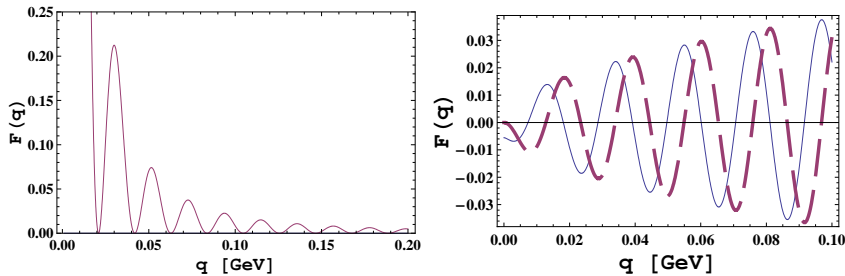


Figure 3: a) [left] The amplitude corresponds to a small constant potential cut at large distance; b) [right] The additional amplitude obtained in the model of the rigid string (solid and dashed lines - the real and imaginary parts).

However, the oscillations can be characterized by an amplitude and a period that do not depend on energy, or depend on it very weakly. This, together with the small size of the coupling and the long range of the interaction, may point to an electromagnetic origin of this effect.

## 2 Conclusion

We have shown that oscillations, periodic in  $\sqrt{-t}$ , exist in many experimental data sets at a significant level. The confirmation of the existence of such a periodic structure in the elastic-scattering amplitude at the LHC would give us important information about the behavior of the hadron interaction potential at large distances which may be connected with the problem of confinement.

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