Integrated Two-phase Drift-flux Models for Modeling Sediment Transport

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For the purpose of designing hydraulic structures, engineers perform both impact studies of their effect on the flow and the hydraulic sizing of the structure. In this respect, a key for the sustainable management of natural inland streams and anthropogenic hydraulic structures is the proper identification and assessment of physical processes, which dominate the interaction between the water flow, the sediments, and the heterogeneous environment in which the mixture flows (fractured rocks, macro-roughness, gravel beds, vegetation, porous media,…). Once understood, these processes must be integrated in a unified theoretical framework adapted to multiphase flows and spatial heterogeneities. This is a prerequisite to the development of reliable engineering tools and constitutes a challenge in Environmental Hydraulics.

The research presented in this paper aims at gaining insight into natural sediment-laden flows and their interactions with a heterogeneous environment, as well as to put the new knowledge in a usable form for engineers. For this purpose, we will derive 3D and shallow-water like Navier-Stokes models for two-phase flows and morphodynamics. Consequently, shallow-water like models will be validated and applied in various cases of civil and environmental engineering. Based on the critical discussion of advantages and shortcomings of the previous approach, more advanced multiphase models for sediment transport over rough bed will be derived.

Available model for two-phase flow and morphodynamics.

Although water-sediment mixtures constitute obviously two-phase media, only very few attempts to account explicitly for this multiphase nature have been reported in sediment transport models and hardly none in morphodynamic models. The underlying governing equations usually stem from single-phase flow theory: they involve continuity and momentum equations for clear water, combined with a continuity equation for sediments. They are therefore reasonably valid for low sediment concentrations (<0.1 in volume). Besides, existing morphodynamic models generally rely on simplifications in the water-sediment mixture and global sediment continuity equations, such as ignored time derivative of the bottom elevation. These assumptions become questionable when significantly transient processes take place.

Therefore, an original two-phase flow modeling framework for water-sediment mixtures was developed by the authors. For particular flow configurations, it has been used to derive specific governing equations and appropriate finite volume numerical schemes have been developed accordingly. The resulting models were verified on a number of benchmarks and used for a wide range of applications, including transient hyperconcentrated flows and rapid flowslides. In addition, the models enable to account for bedload transport, as relevant for applications such as prediction of deltaic sedimentation in reservoirs and assessment of flushing operations.

Following an Eulerian description of the flow, the Reynolds-averaged conservation equations were formulated for the *water-sediment mixture*, while conservation of a dispersed phase in the fluid, namely suspended sediments, was expressed by a standard advection-diffusion equation.
Since most flows of interest in civil and environmental engineering are characterized by significantly larger length scales in a reference plane (often almost horizontal) compared to the characteristic depth of the flow, depth-averaged models prove particularly useful. They require far less intricate numerical resolution procedures than needed for general three-dimensional free surface flows and better fit with available data and outputs of interest for most applications in civil and environmental engineering. Therefore, integration of the three-dimensional governing equations over the local flow depth has been conducted, resulting in a set of particularly *general* two-dimensional equations, holding whatever the velocity and concentration distributions across the flow layer, since at this stage neither a particular velocity profile nor a particular concentration distribution has been assumed.
Next, adequate density, velocity and concentration profiles have been introduced. The real concentration profiles are highly case-dependent and, for transient and varied flows, they may considerably deviate from theoretical equilibrium distributions, such as Rouse-type profiles. Therefore, we have simply assumed a uniform concentration profile, which is found close to observations for highly transient flows accompanied by high transport rates. Instead, a piecewise uniform profile may also be assumed, leading to only slight modifications in the governing equations. In applications where the shear layer remains localized near the bed, the velocity profile is also simply assumed uniform, whereas differential advection of momentum may also be considered through the use of Boussinesq coefficients. The pressure is formulated as a function of the sediment relative density and suspended load concentration. Closure of the resulting set of equations requires a resistance formula, a turbulence model, as well as a sediment exchange model. The set of governing equations is solved using a finite volume technique on multiblock Cartesian grids. Fluxes are computed by a stable Flux Vector Splitting developed by the authors.
Since a challenging issue in sediment transport modeling is the need to handle efficiently the wide range of time scales involved in the relevant processes, the resolution procedure of the flow and morphodynamic models may be either synchronous or sequential. Besides, an efficient iterative procedure computing bed equilibrium profile was also developed, as well as a "Lagrangian"-type tracking of sediment particles. As a result of the flexibility offered in the levels of coupling between flow and sediment transport models, stable and accurate numerical solutions are obtained in a realistic CPU time for predictions of erosion and sedimentation patterns in the short, medium or long term, considering both equilibrium and non-equilibrium sediment transport.

The use of enhanced multiphase models remains in its infancy in hydraulic engineering.
If the previous approach succeeds in many situations to assess erosion/deposition rate, it suffers various shortcomings in relation with the simplified description of the sediment motion as well as with crude approximations concerning sediment-water interactions. On the other hand, chemical and mechanical engineering make frequent use of Eulerian two-phase flow models for describing the transport of a dispersed phase. In particular, the drift-flux theory provides a high-fidelity description of phase's interactions without making the mathematical model much more complex than a standard model [1]. This model is indeed fundamentally multiphase and refers to the center of mass of the mixture fluid (water + dispersed phase). Nevertheless, the use of the drift-flux theory remains in its infancy in civil and environmental engineering. Mainly, its use has been circumscribed to the description of air-water pressurized flows [2, 3].

Double-averaging methods is a promising integrative up-scaling framework

Most natural open-channels present a rough-bed that originates from the presence of sand, gravels, fractured rocks and concrete, vegetation, bed forms, etc. Although the hydrodynamics of such flows has been studied extensively for many years, there are still unsolved problems awaiting clarification. Notably, most researchers have studied these flows using time-averaged models [4, 5]. The 'double-averaging method' has recently tackled this conceptual shortcoming. DAM relies on the spatial averaging of the time-averaged hydrodynamic variables and equations within a window defined by the longitudinal, transverse, and vertical length scales of roughness elements [5]. These equations explicitly contain important additional terms. This methodology allows scale decomposition and can be viewed as a scaling-up procedure that changes the scale of consideration from one level in time-space-probability domain to another level.

First results: bridging the gap between single- and multiphase models

The 3D drift-flux model is obtained by Eulerian time-averaging the local instant formulation of the conservation laws and by assuming that the multiphase flow may be described as a single-phase flow of mixture variables which refer to the motion of the centre of mass of the system. The motion of the dispersed phase is then treated in terms of diffusion through the mixture. The rigorous demonstration is presented in [1, 6]. Combining simplicity and a rigorous description of interactions between water and dispersed phases, this model seems a good alternative to standard models for describing sediment transport.

Computation of the three-dimensional Drift-flux Model requires a prohibitive computational effort in many cases. In many practical applications in civil engineering, 2D models can be used as the vertical velocities are negligible. In this case, the 3D Drift-flux model is integrated over the flow depth and the vertical momentum equation is cancelled thanks to a dimensional analysis that imposes a hydrostatic pressure. Such a 2D shallow-water-like model offers a valuable alternative to the 3D model since various methods have been developed to accommodate depth-averaged models with vertical distribution of the parameters; namely the addition of Boussinesq/covariance coefficients [1], of moment equations [7] as well as multi-layer integration [8]. The whole integration gives four partial differential equations for each layer. By assuming two layers, the model appears particularly suited for sediment-laden flows, mainly because it describes separately the motion of bed load and suspended load.

In particular applications, we can further assume that the computational domain is essentially one-dimensional. In such cases, equations of momentum along two of the three axes may be simplified and the remaining equations are area-integrated over the flow cross-section. Again, definition of a multi-layer domain of integration and introduction of Boussinesq/covariance coefficients enable to enrich the vertical description of the flow parameters. The integration gives three partial differential equations for each layer. The two layer model describes rigorously the bed load and suspended load motion.

For numerical simulations, we have accommodated the existing modelling system WOLF with the modified mathematical model. WOLF is a computational code developed since the 90s within the University of Liège. It describes flows of pure water and water-sediment mixtures on the basis extended formulations of the Saint-Venant equations (WOLF1D), the shallow-water equations (WOLF2D), or the RANS equations (WOLF3D). WOLF incorporates a series of modules to simulate bed and suspended load as well as morphodynamics. These modules have been designed to handle effectively the wide range of relevant space and time scales involved in

sediment processes. Discretization of the equations is performed by means of the Finite Volume method and an original Flux Vector Splitting [9]. This modeling system constitutes an ideal framework for implementing and testing multiphase models for environmental flows.
The computational codes have been validated by comparison with various analytical, numerical and experimental results. Furthermore, they have been applied for numerous practical applications.

Planned research: Double-averaging of the drift-flux model and experimental investigations
Based on this first success, further researches are scheduled to improve the fidelity of the multiphase model, assess its validity and use it to improve the understanding of sediment laden flows. First, the local instant formulation of the conservation laws holding for a multiphase flow will be averaged over time and space in order to develop a double-averaged drift-flux model. Second, suitable constitutive laws will be derived from experimentations, field observations, and theoretical analyses. The whole set of equations must then be analyzed and assessed by comparison with experimental and field data. Third, experimental investigations are scheduled in various laboratories concerning multiphase constitutive laws and the impact of heterogeneous beds on the multiphase flows. At this occasion, the double-averaged unified mathematical framework will be useful for interpreting the results. Numerical models will be implemented in order to provide for two very different needs, namely scientific research and practical engineering. In this respect, we will rely on a strategy coupling 1D, 2D and 3D models.

Conclusions
In conclusion, this paper proposes original models for describing environmental flows, sediment transport and morphodynamics. These models are proven to enhance the fidelity of the description and constitute a reliable basis to create a computational code. Such code increases the fidelity and decrease the cost in both designing hydraulic structures and performing impact studies. From a wider point of view, this paper presents some result of a larger research project about the use of two-phase flows in civil and environmental engineering. This project relies on the know-how of the HACH (University of Liège) in experimental and numerical hydraulics. It already gave birth to partnerships with other international institutions.

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