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## REAL TIME TEC MONITORING USING TRIPLE FREQUENCY GNSS DATA: A THREE STEP APPROACH

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### Abstract

This paper describes our first attempts to develop and to test an improved real time TEC monitoring technique based on triple frequency GNSS measurements.

First we have developed a software allowing to simulate triple frequency data for Galileo and modernized GPS. This software will enable us to test the technique on realistic GPS and Galileo code and phase measurements. Then we have developed the three steps of the triple frequency technique for absolute TEC monitoring. The goal of the first and second steps is respectively to resolve the extra widelane and widelane ambiguities. In the third step we have to solve the system of two dual frequency Geometric Free phase combinations by using the information we have obtained from the first two steps and by using a rough estimation of the TEC. As a result, we can precisely monitor the TEC.

### Introduction

In the next few years, triple frequency GNSS (Galileo and modernized GPS) will be operational. The availability of a third frequency will not only allow to improve the positioning accuracy but also to exploit his added value in order to develop new techniques.

Our goal here is precisely to develop an improved TEC monitoring technique. In this context, it is important to note that the ionospheric error remains the principal limitation to GNSS precision and reliability. This error depends on the Total Electron Content (TEC), which is the integral of the free electron concentration on the receiver-to-satellite path. In particular, ionospheric variability affects the ambiguity resolution required by real time positioning techniques based on phase measurements. An improved modelling of the ionospheric effects will therefore increase the precision and reliability of these techniques [Lejeune et al., 2006].

## Simulation software

### Principle

As the third frequency is not yet available, the TEC monitoring technique can only be tested on simulated data. For that purpose, we have developed a software allowing to simulate realistic triple frequency code and phase measurements that will be made on the signals which will be emitted by the Galileo system and by the modernized GPS constellation. Table 1 shows the GPS and Galileo civil frequencies used in this work.

| GPS     |        |         | Galileo |         |         |
|---------|--------|---------|---------|---------|---------|
| L1      | L2     | L5      | L1      | E5b     | E5a     |
| 1575.42 | 1227.6 | 1176.45 | 1575.42 | 1207.14 | 1176.45 |

**Table 1 – GPS and Galileo civil frequencies [MHz]**

Some utilities were developed in order to model all the error sources which affect measurements made on these signals. The code measurements  $P_{p,k}^i$  (with  $k$  representing the considered carrier frequency – cf. table 1) equal the geometric distance between the satellite and the receiver plus all those errors. Code measurements can be modelled as follows [in meters]:

$$P_{p,k}^i = D_p^i + T_p^i + I_{p,k}^i - (c\Delta t^i + c\Delta t_{rel}) + c\Delta t_p + \Delta r_p^i + d_k^i + d_{p,k} + M_{p,k}^i + \sigma_{p,k} \quad (1)$$

Let us explain the different terms:

- $D_p^i$  is the geometric distance between the satellite  $i$  and the receiver  $p$  [m];
- $T_p^i$  is the tropospheric delay [m];
- $I_{p,k}^i$  is the ionospheric delay [m] and can be developed as :  $I_{p,k}^i = \frac{40.3}{f_k^2} TEC_p^i$   
with  $f_k$  = the frequency of the signal  $k$  and  $TEC_p^i$  = the Total Electron Content in TECU (1 TECU =  $10^{16}$  electrons/m<sup>2</sup>);
- $c\Delta t^i + c\Delta t_{rel}$  are the satellite clock bias [m] and the associated relativistic effects [m] (with  $c$  = velocity of light = 299 792 458 ms<sup>-1</sup>);
- $c\Delta t_p$  is the receiver clock bias [m];
- $\Delta r_p^i$  is the earth rotation effect [m]. In fact, during the propagation of the signal, the earth is rotating and the satellite position is changing with respect to the terrestrial frame, what induces an effect on the measurements.
- $d_k^i + d_{p,k}$  are respectively the satellite and receiver hardware delays (propagation delays in the electronic hardware) [m];
- $M_{p,k}^i$  is the multipath delay [m] due to signal reflections. In this software, the multipath delay is computed on the basis of a Gaussian time correlated noise multiplied by several characteristics (like amplitude, satellite elevation, etc.);
- $\sigma_{p,k}$  is the measurement noise [m]. It is simulated as a Gaussian noise characterized by its zero-mean and its standard deviation. Table 2 shows the standard deviation values adopted here to simulate GPS and Galileo code measurement noise. We can see that code noise will be lower for Galileo, thanks to a

new modulation scheme and thanks to the fact that the power of the signal will be double. These two features also allow for lower code multipath effects [Simsy, et al. 2005].

| GPS           |               |               | Galileo       |               |               |
|---------------|---------------|---------------|---------------|---------------|---------------|
| $\sigma_{L1}$ | $\sigma_{L2}$ | $\sigma_{L5}$ | $\sigma_{L1}$ | $\sigma_{5b}$ | $\sigma_{5a}$ |
| 0.3           | 0.3           | 0.14          | 0.17          | 0.11          | 0.11          |

**Table 2 – Code noise standard deviation for GPS and Galileo [m]**

The phase measurements can be modelled quite similarly in [cycles]:

$$\phi_{p,k}^i = \frac{f_k}{c} (D_p^i + T_p^i - I_{p,k}^i) - f_k (\Delta t^i + \Delta t_{rel}) + f_k \Delta t_p + \frac{f_k}{c} \Delta r_p^i - N_p^i \quad (2)$$

However, equation (2) shows some differences. Firstly, phase measurements are ambiguous. The ambiguity (called integer ambiguity)  $N_p^i$  is the integer constant representing the initial number of cycles between the satellite and the receiver. These ambiguities depend on the frequency and have to be resolved to take advantage of the full phase measurement precision.

Secondly, hardware delays, multipath delays and noise are negligible in phase measurements with respect to one cycle. Therefore, these biases will not be considered in this paper due to the fact that they will not affect our ambiguity resolution method. Nevertheless, phase multipath could have an influence on the accuracy of the reconstructed TEC.

### *Validation*

The next goal of our work is to validate the simulation software with real data. For that purpose, we've first developed the double frequency GPS simulation software. Then we have made the comparison between simulated and real data for different stations and different days. The results show that the differences between simulated and real data can be explained by the uncertainties remaining in the modelling of the perturbations affecting GPS data. Moreover, we can already affirm that these differences will not prevent to correctly develop and evaluate a TEC monitoring technique.

### *Triple frequency software*

This double frequency validation allows to develop the GPS and Galileo triple frequency software. In fact, we have made some adaptations to add the third frequency to the existing software. Then we have developed the Galileo simulation software on the same basis than GPS, except for example that Galileo orbits are circular (GPS's are elliptic). Moreover, the second frequency is different (cf. table 1: E5b  $\neq$  L2) and code measurements are more precise thanks to less multipath and less noise (cf. simulation software). These are two advantages for extra widelane ambiguity resolution (cf. step 1).

### Triple frequency TEC monitoring technique

Afterwards, we have developed a triple frequency technique for absolute TEC monitoring that can be tested on simulated data. When reminding the definition of the TEC, it's obvious that monitoring the TEC requires the use of data coming from one station only, and not double-differenced measurements such as e.g. in Real Time Kinematic positioning techniques.

Our TEC monitoring technique is divided in three steps. These are explained in detail below but it's already important to insist on the fact that code measurements are only used in the first step. As a consequence, the second and third steps are exclusively based on phase measurements, what allows to improve the accuracy of the reconstructed TEC with respect to the usual double frequency reconstruction technique [Warnant, et al. 2000].

#### First step

The objective of the first step is to resolve the extra widelane (EWL) ambiguities  $N_{25}$  :

$$N_{25} = N_5 - N_2 \quad (3)$$

For the GPS case, these ambiguities are estimated by subtracting the L2/L5 code combination (resp. E5b/E5a for Galileo) from the L2/L5 phase combination. The corresponding equation is:

$$\Phi_2 - \Phi_5 - \frac{f_2 - f_5}{f_2 + f_5} \left( \frac{f_2}{c} P_2 + \frac{f_5}{c} P_5 \right) = N_{25} - \frac{f_2 - f_5}{f_2 + f_5} (d_{25} + M_{25} + \sigma_{25}) \quad (4)$$

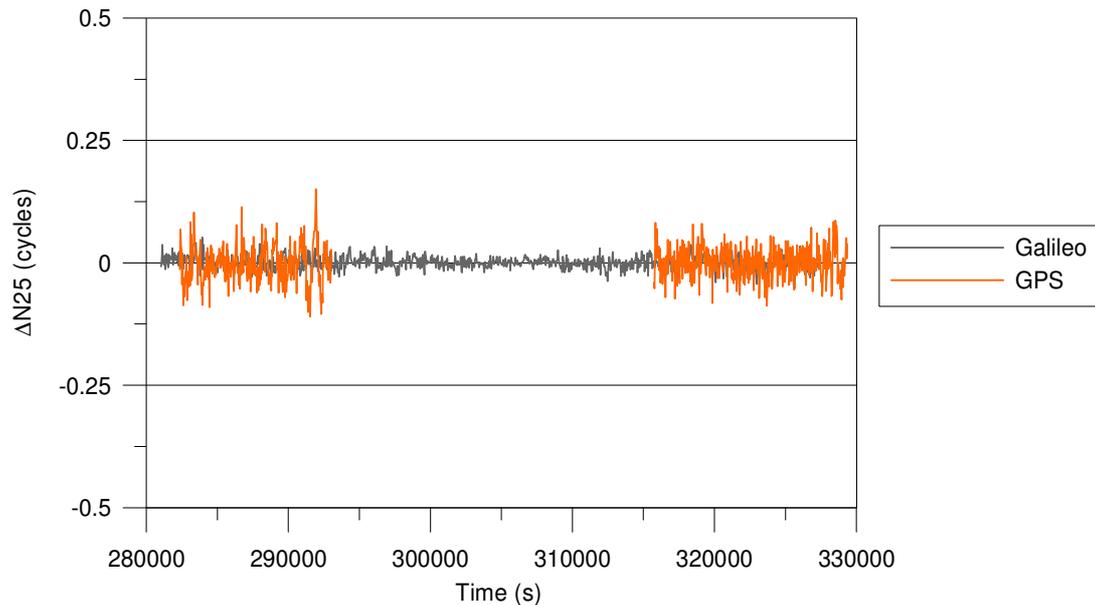
The wavelength of this EWL combination is given by

$$\lambda_{25} = \frac{c}{f_2 - f_5} \quad (5)$$

This equals 5.861 m for GPS and 9.768 m for Galileo. It's important to mention here that how large is the wavelength how easier it is to fix the ambiguities at their correct integer values. It can be explained as follows: the same budget of errors (or residuals effects – see below) will have a lower impact (with respect to one cycle) on a combination with a larger wavelength, and thus the resolution of the ambiguities is easier. In this particular case, the EWL combination wavelengths are relatively large, although Galileo's is about twice GPS's.

In this step, the problem is that computing the left hand side of equation (4) do not exactly provide the values of the EWL ambiguities  $N_{25}$  because several code effects do not disappear when subtracting the code combination from the phase combination. These residual effects are a combination on both frequencies of code hardware delays (satellite and receiver –  $d_{25}$ ), code multipath delays ( $M_{25}$ ) and code noise ( $\sigma_{25}$ ).

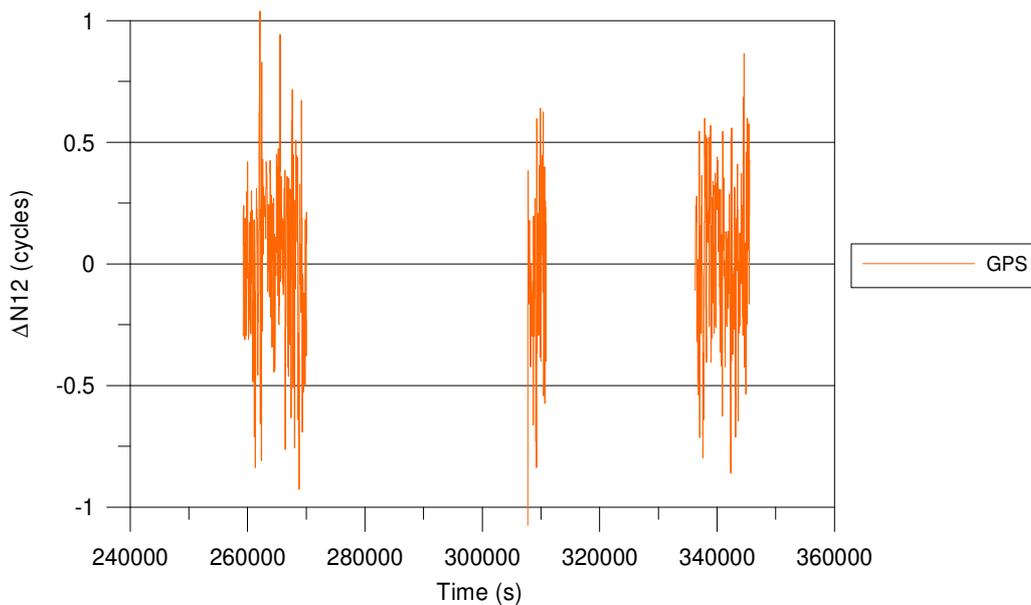
Then the condition to fix the EWL ambiguities at their correct integer values is that the influence of these residuals terms should not exceed half a cycle. We have tested this condition on simulated data. In particular, we have estimated the influence of code multipath and code noise on the EWL ambiguities. Figure 1 shows that this influence is negligible with respect to half a cycle, particularly for Galileo, thanks to the fact that the EWL combination wavelength is larger and that there's less multipath effects and noise on code measurements.



**Figure 1 – Influence of multipath and noise on the EWL ambiguities [cycles]**

Moreover, we suppose that the influence of code hardware delays on the EWL ambiguities should be negligible within the remaining margin. Finally, the sum of the three influences should not exceed half a cycle and it will be possible to resolve the EWL ambiguities.

Before moving to the second step, it can be useful to notice that it's not possible to resolve the widelane ambiguities with the same method, because the residuals effects are too important with respect to the widelane combination wavelength, which is shorter than 1 meter (cf. step 2). For example, figure 2 shows that the only influence of GPS code noise is already larger than half a cycle. That's why we have to use a method exclusively based on phase measurements which is described below.



**Figure 2 – Influence of noise on the WL ambiguities [cycles]**

*Second step*

The objective of the second step is to resolve the widelane (WL) ambiguities  $N_{12}$  :

$$N_{12} = N_2 - N_1 \quad (6)$$

These ambiguities are estimated by using the EWL fixed ambiguities and the phase measurements to form the following combination:

$$(\Phi_1 - \Phi_2) - (\Phi_2 - \Phi_5 + N_{25}) \frac{\lambda_{25}}{\lambda_{12}} = N_{12} - bI_1 \quad (7)$$

The wavelength of this combination is given by

$$\lambda_{12} = \frac{c}{f_1 - f_2} \quad (8)$$

This equals 0.862 m for GPS and 0.814 m for Galileo. This is much shorter than in the EWL case, and here the values are approximately similar for GPS and Galileo.

As in the EWL case, there are residual effects. The left hand side of equation (7) gives the WL ambiguities plus a term  $bI_1$  depending on the ionospheric effects. Namely,  $b \sim 0.5$  depends on the three frequencies and  $I_1$  is the ionospheric delay on L1. It can be shown that these ionospheric effects cause an error of several cycles on the WL ambiguities. As a consequence, the resolution of the WL ambiguities is not possible. However, we obtain a first estimation of those ambiguities which will be used in the third step.

*Third step*

The objective of the third step is to resolve the system of two dual frequency Geometric Free (GF) phase combination in order to monitor the TEC. In fact, the availability of three frequencies allows to form two independent double frequency phase combinations. Let us recall that in the geometric free combination all the terms relative to the geometry and clocks disappear, so that the TEC and the ambiguities are the only unknowns remaining. The system can be written as follows:

$$\begin{cases} \Phi_2 - c_{25}\Phi_5 = a_{25}TEC - N_2 + c_{25}N_5 \\ \Phi_1 - c_{12}\Phi_2 = a_{12}TEC - N_1 + c_{12}N_2 \end{cases} \quad (9)$$

In a first time, the resolution of this system is impossible because there are four unknowns: the TEC and the ambiguities on the three frequencies ( $N_1, N_2$  and  $N_5$ ). In a second time however, the values of the WL and EWL ambiguities can be introduced in equation (9) by replacing  $N_1$  by  $N_2 - N_{12}$  (equation (6)) and  $N_5$  by  $N_2 + N_{25}$  (equation (3)), so that it remains only two unknowns: the TEC and  $N_2$  :

$$\begin{cases} \Phi_2 - c_{25}\Phi_5 - c_{25}N_{25} = a_{25}TEC + N_2(c_{25} - 1) \\ \Phi_1 - c_{12}\Phi_2 - N_{12} = a_{12}TEC + N_2(c_{12} - 1) \end{cases} \quad (10)$$

The system can be written in a simple matrix form:

$$\begin{pmatrix} \Phi_2 - c_{25}\Phi_5 - c_{25}N_{25} \\ \Phi_1 - c_{12}\Phi_2 - N_{12} \end{pmatrix} = \begin{pmatrix} a_{25} & c_{25} - 1 \\ a_{12} & c_{12} - 1 \end{pmatrix} \begin{pmatrix} TEC \\ N_2 \end{pmatrix} \quad (11)$$

Then the unknowns are computed as follows:

$$\begin{pmatrix} TEC \\ N_2 \end{pmatrix} = \begin{pmatrix} a_{25} & c_{25} - I \\ a_{12} & c_{12} - I \end{pmatrix}^{-1} \begin{pmatrix} \Phi_2 - c_{25}\Phi_5 - c_{25}N_{25} \\ \Phi_1 - c_{12}\Phi_2 - N_{12} \end{pmatrix} \quad (12)$$

But here the problem is that the unfixed WL ambiguities  $N_{12}$  prevent from resolving the system. If we introduce the unfixed values of the WL ambiguities in equation (12), the values of TEC and ambiguities that we obtain are biased. To avoid this bias, we have to fix the WL ambiguities at their correct integer values, which cannot directly made. Let's explain the method used in our work.

Let us first recall that WL ambiguities are integer numbers, which will make their resolution easier.

Let's then compute the change in the TEC values caused by a change of 1 cycle in the WL ambiguities  $N_{12}$  (equation (12)). We have computed it equals  $\pm 12.2$  TECU for GPS and  $\pm 11.3$  TECU for GPS, what's relatively large.

In the next step, we can use the first estimation of the WL ambiguities (cf. step 2) to make a search on the possible WL integer ambiguities ( $N_{12} - 1$ ,  $N_{12} - 2$ , etc.). The “-” sign can be justified by the sign of the residual terms in equation (7). So, at each possible value of the WL ambiguities corresponds a value of the TEC that is approximately 12 TECU different from the previous value.

As the usual dual frequency TEC monitoring technique [Warnant, et al. 2000] can provide us a rough estimation of the TEC (better than 6 TECU), we can compare it with the successive values of the computed TEC. The value that is more or less equivalent to the estimated value is the correct one. At this moment, the corresponding WL ambiguity is the correct integer value. Finally, the system is solved and the TEC is precisely monitored. Moreover, the system provides the values of  $N_2$ , from which the values of  $N_1$  and  $N_5$  can be extracted.

## Conclusions

The availability of a third frequency will allow to solve phase ambiguities in real time using one station data only. As a consequence, it will be possible to reconstruct absolute TEC with the accuracy of phase measurements. In fact, contrary to the dual frequency method, codes hardware delays do not affect the accuracy of the reconstructed TEC. Another advantage of the method is that we have to solve integer ambiguities (WL and EWL) instead of non-integer ambiguities coming from the GF combination, what's easier and more efficient.

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