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An Iterative Finite Element Perturbation Method for Computing Electrostatic Field Distortions

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Abstract

A finite element (FE) perturbation method for computing electrostatic field distortions due to moving conductors is presented. First, an unperturbed problem (in the absence of conductors) is solved with the conventional FE method in the complete domain. Then, a perturbed problem is computed in a reduced region using the solution of the unperturbed problem as a source field. When the source is close to the perturbing regions, an iterative computation may be required. The developed procedure allows for solving sub-problems in reduced domains with independent problem-adapted meshes.

I. Introduction

The finite element (FE) modeling of an electrostatically driven moving conductor needs successive computations for each new position. This may be computationally expensive specially when dealing with 3D models of complex geometries. It is worth then benefiting from previous computations instead of starting a new complete FE solution for every new position.

A perturbation method is developed for solving the electrostatic formulation with an electric scalar potential. An unperturbed problem is first solved in a large mesh taking advantage of any symmetry and excluding additional regions and thus avoiding their mesh [1][2]. Its solution is applied as a source to the further computations of the perturbed problems when conductive regions are added [1]. For some positions where the coupling between regions is significant, an iterative procedure is used to obtain an accurate solution. Successive perturbations in each region are thus calculated not only from the initial source region to the added conductor but also from the latter to the former.

As test cases, both a 1D analytical solution (perturbing region with no fringing effects) and a 2D FE computation (perturbing region with significant fringing effects) of electrostatic field distortions between the plates of a capacitor are carried out.

II. Unperturbed and Perturbed Electrostatic Problems

We consider an electrostatic problem in a domain Ω with boundary $\partial\Omega$ (possibly at infinity), of 2D or 3D Euclidean space. The conducting parts of Ω are denoted Ω_c and dielectric ones Ω_d, with $\Omega=\Omega_c \cup \Omega_d$. The governing differential equations and constitutive law of the electrostatic problem in Ω are

$$\mathbf{curl}\, \boldsymbol{e} = 0\,,\ \ \mathrm{div}\, \boldsymbol{d} = q\,,\ \ \boldsymbol{d} = \varepsilon\, \boldsymbol{e}\,, \qquad \text{(1a-b-c)}$$

where $\boldsymbol{e}$ is the electrostatic field, $\boldsymbol{d}$ is the electric flux density, q is the electric charge density and ε is the electric permittivity. In charge free regions, we obtain from (1a-b-c) the following equation in terms of the electric scalar potential v

$$\mathrm{div}(-\varepsilon\, \mathbf{grad}\, v) = 0. \qquad (2)$$

An unperturbed problem is first solved in the complete domain Ω without considering the perturbing domain $\Omega_{c,p}\subset\Omega_p$. At the discrete level, the mesh of Ω_p, the union of $\Omega_{c,p}$ and its neighborhood, is not described in the mesh of Ω. Hereafter, the subscripts u and p will refer to the unperturbed and perturbed quantities, respectively. The process of the resolution and the projection of fields from one mesh to another are represented in Fig. 1.

Field distortions appear when a perturbing conducting region $\Omega_{c,p}$ is added to the initial configuration. The perturbed problem is defined as an electrostatic problem in Ω_p. Particularising (1a-b-c) for both the unperturbed and perturbed problems, and subtracting the unperturbed equations from the perturbed ones, one gets

$\mathbf{curl}\,\boldsymbol{e} = 0$, $\operatorname{div}\boldsymbol{d} = 0$, $\boldsymbol{d} = \varepsilon_p\,\boldsymbol{e} + (\varepsilon_p - \varepsilon_u)\,\boldsymbol{e}_u$,(3a-b-c)
with the field distortions: $\boldsymbol{e} = \boldsymbol{e}_p - \boldsymbol{e}_u$ and $\boldsymbol{d} = \boldsymbol{d}_p - \boldsymbol{d}_u$.

Note that if $\varepsilon_p \neq \varepsilon_u$, an additional source term given by the unperturbed solution $(\varepsilon_p - \varepsilon_u)\,\boldsymbol{e}_u$ is considered in (3c). The zero tangential electric field on the boundary $\partial\Omega_{c,p}$ of the so-considered perfect conductors leads to a condition on the electric scalar potential $v = -v_u$ in $\partial\Omega_{c,p}$

III. APPLICATION

In order to illustrate and validate the iterative perturbation method for electrostatic field distortions, we consider a parallel-plate capacitor (length of plates = 200 µm, distance between plates: d = 200µm). The difference of electric potential between electrodes is ΔV = 1V (one electrode fixed to V_0 = 1V, the other to 0V). Two perturbing regions are considered.

A. Conductor without fringing fields
First, we consider a conductor with the same length as the plates of the capacitor and thickness of 1 µm. This perturbing region is placed at a distance d_1 of the electrode at V_0. An iterative procedure is required for getting an accurate solution.

The starting point of the iterative process is to determine the electrostatic field between the plates of the capacitor in the absence of the conductor. The solution of this problem is then evaluated on the added conductor and used as a source for the so-called perturbation problem. This way, we obtain a potential in the conductor V_c that counterbalances the potential on the electrode and must be applied as a boundary condition in our initial configuration, i.e. the new potential on the electrode becomes V_0–V_c. This iterative process is repeated until convergence.

Potentials in the conductor and the considered electrode are computed for each iteration and shown in Fig. 2 (top). The relative error on the electric field with respect to the analytical solution is depicted in Fig. 2 (bottom). When the conductor is too close to the electrode (d_1 = 20 µm), more iterations are needed for a given accuracy.

B. Conductor with significant fringing fields
As a 2D FE test case, a square conductor (side = 20 µm) is considered as a perturbing region inside the previous capacitor. Its distance from the electrode at V_0 = 1V is d_1 = 20 µm. For this position, iterative perturbation sequences have been carried out. An Adapted mesh, specially fine in the vicinity of the corners of the conductor and near the electrode at V_0 = 1V is used. The y-component of the electric field, near the conductor and electrode at V_0 = 1V, obtained with both the conventional FE model and the perturbation technique for a different number of iterations is depicted in Fig. 3.

At iteration 0, the unperturbed electric field is computed in the whole domain Ω and projected in the domain Ω_p where field perturbation $\boldsymbol{e}_p$ is then calculated. The difference between the y-components of $\boldsymbol{e}_p$ and $\boldsymbol{e}$ (reference solution) is considerable at this iteration (relative error up to 14%) which is due to a significant coupling between these regions. At iteration 1, $\boldsymbol{e}_p$ is projected in Ω where a new perturbation problem is solved and its solution is projected again in Ω_p (at iteration 2). Note that the relative error at iteration 4 is already reduced to 2%.

IV. CONCLUSION

In this paper, an iterative finite element perturbation method for computing electrostatic field distortions due to the presence of moving conductors is presented. In order to illustrate and validate this method, we consider both a 1D analytical solution and a 2D FE method with and without fringing field effects, respectively. When conductors are close to the electrostatic field source, several iterations are required. Details on the iterative perturbation method, more results and discussion on the acceleration of the convergence will be included in the extended paper.

V. REFERENCES

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Fig. 1. *Unperturbed (left: mesh of Ω and distribution of the unperturbed electric field* $\boldsymbol{e}_u$ *to be projected on* $\partial\Omega_{c,p}$*, i.e. the boundary of* $\Omega_{c,p}$*) and perturbed problems (right: distribution of e and adapted mesh of* Ω_p*).*

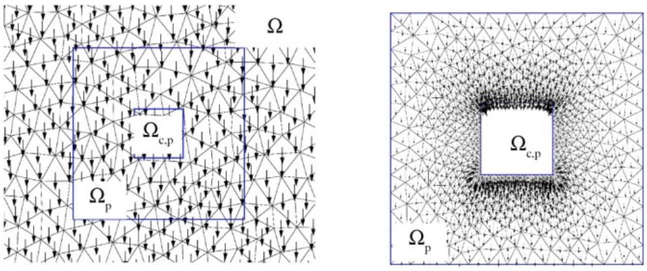

Fig. 2. *Iterations between unperturbed and perturbed problems (di= 100 µm) (top). Relative error with respect to the reference solution of the electric field for each iteration (bottom).*

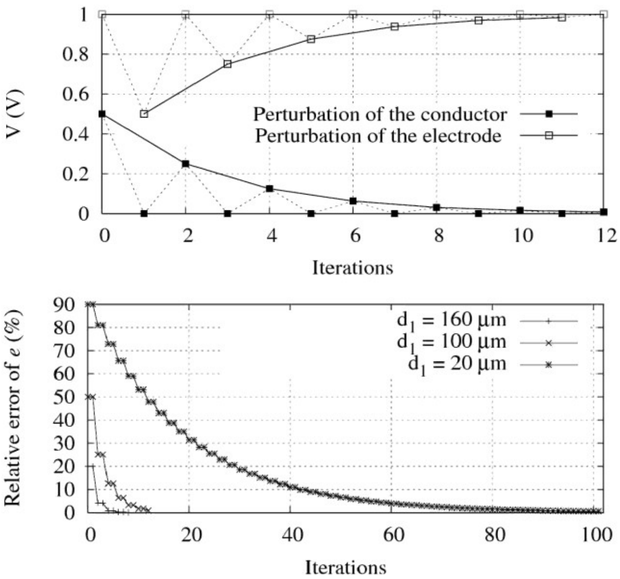

Fig. 3. *Electric field (y-component) computed near the conductor (top) and electrode at* V_0 *= 1V (bottom) in each iteration.*