THE $[\mathbf{56}, 4^+]$ BARYON MULTIPLET IN THE $1/N_C$ EXPANSION OF QCD

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We use the $1/N_c$ expansion of QCD to analyze the spectrum of positive parity resonances with strangeness $S = 0, -1, -2$ and -3 in the 2–3 GeV mass region, supposed to belong to the $[\mathbf{56}, 4^+]$ multiplet. The mass operator is similar to that of $[\mathbf{56}, 2^+]$, previously studied in the literature. The analysis of the latter is revisited. In the $[\mathbf{56}, 4^+]$ multiplet we find that the spin-spin term brings the dominant contribution and that the spin-orbit term is entirely negligible in the hyperfine interaction, in agreement with constituent quark model results. More data are strongly desirable, especially in the strange sector in order to fully exploit the power of this approach.

1. Introduction

Thirty years ago 't Hooft [1] has suggested that the inverse power of the number of colors N_c can be used as an expansion parameter in QCD. Later on, based on general arguments Witten [2] analyzed the properties of mesons and baryon systems in this limit. It turned out that this limit captures the main features of hadron dynamics and allows a unified view of effective theories where the spontaneous breaking of chiral symmetry plays an essential role, such as the Skyrme model, the chiral soliton model, the chiral bag model and the nonrelativistic quark model based on a flavor-spin hyperfine interaction.

Since then, the $1/N_c$ expansion has been proved a useful tool to study baryon spectroscopy. It has been applied to the ground state baryons [3–9] as well as to the negative parity spin-flavor $[\mathbf{70}, 1^-]$ multiplet ($N = 1$ band) [10–16], to the positive parity Roper resonance belonging to the $[\mathbf{56'}, 0^+]$ multiplet ($N = 2$ band) [17] and to the $[\mathbf{56}, 2^+]$ multiplet ($N = 2$ band) [18].

Here we explore the applicability of the $1/N_c$ expansion to the $[\mathbf{56}, 4^+]$ multiplet ($N = 4$ band). The number of experimentally known resonances in the 2–3 GeV region [19], expected to belong to this multiplet is quite restricted. Among the five possible candidates there are two four-star resonances, $N(2220)9/2^+$ and $\Delta(2420)11/2^+$, one three-star resonance $\Lambda(2350)9/2^+$, one two-star resonance $\Delta(2300)9/2^+$ and one one-star resonance $\Delta(2390)7/2^+$. This is an exploratory study which will allow us to make some predictions.

In constituent quark models the $N = 4$ band has been studied so far either in a large harmonic oscillator basis [20] or in a variational basis [21]. We shall show that the present approach reinforces the conclusion that the spin-orbit contribution to the hyperfine interaction can safely be neglected in constituent quark model calculations.

The present paper summarizes the findings of Ref. 22.

2. The Mass Operator

The study of the $[\mathbf{56}, 4^+]$ multiplet is similar to that of $[\mathbf{56}, 2^+]$ as analyzed in Ref. 18, where the mass spectrum is calculated in the $1/N_c$ expansion up to and including $\mathcal{O}(1/N_c)$ effects. The mass operator must be rotationally invariant, parity and time reversal even. The isospin breaking is neglected. The SU(3) symmetry breaking is implemented to $\mathcal{O}(\varepsilon)$, where $\varepsilon \sim 0.3$ gives a measure of this breaking. As the **56** is a symmetric representation of SU(6), it is not necessary to distinguish between excited and core quarks for the construction of a basis of mass operators, as explained in Ref. 18. Then the mass operator of the multiplets has the following structure

$$M = \sum_i c_i O_i + \sum_i b_i \bar{B}_i \tag{1}$$

given in terms of the linearly independent operators O_i and $\bar{B}_i$. Here O_i ($i = 1, 2, 3$) are rotational invariants and SU(3)-flavor singlets [10], $\bar{B}_1$ is the strangeness quark number operator with negative sign, and the operators $\bar{B}_i$ ($i = 2, 3$) are also rotational invariants but contain the SU(6) spin-flavor generators G_{i8} as well. The operators $\bar{B}_i$ ($i = 1, 2, 3$) provide SU(3) breaking and are defined to have vanishing matrix elements for nonstrange baryons. The relation (1) contains the effective coefficients c_i and b_i as parameters. They represent reduced matrix elements that encode the QCD dynamics. The above operators and the values of the corresponding coefficients which we obtained from fitting the experimentally known masses are given in Table 1 both for the $[\mathbf{56}, 4^+]$ and the presently revisited $[\mathbf{56}, 2^+]$.

3. The $[\mathbf{56}, 2^+]$ multiplet revisited

As mentioned above, the study of the $[\mathbf{56}, 4^+]$ multiplet is similar to that of $[\mathbf{56}, 2^+]$. Here we first revisit the $[\mathbf{56}, 2^+]$ multiplet for two purposes: 1) to get a consistency test of our procedure of calculating matrix elements of the operators in Table 1 and 2) to analyze a new assignement of the $\Delta_{5/2^+}$ resonances.

The matrix elements of O_1, O_2, O_3 and $\bar{B}_1$ are trivial to calculate for both multiplets under study. For the $[\mathbf{56}, 2^+]$ one can find them in Table 2 of Ref. 18 and for the $[\mathbf{56}, 4^+]$ they are given in the next section.

To calculate the diagonal and off-diagonal matrix elements of $\bar{B}_2$ we use the expression

$$G_{i8} = G^{i8} = \frac{1}{2\sqrt{3}} \left(S^i - 3S^i_s \right), \tag{2}$$

where S^i and S^i_s are components of the total spin and of the total strange-quark spin respectively [8]. Using (2) we can rewrite $\bar{B}_2$ from Table 1 as

$$\bar{B}_2 = -\frac{\sqrt{3}}{2N_c} \vec{l} \cdot \vec{S}_s \tag{3}$$

with the decomposition

$$\vec{l} \cdot \vec{S}_s = l_0 S_{s0} + \frac{1}{2} \left(l_+ S_{s-} + l_- S_{s+} \right). \tag{4}$$

The matrix elements were calculated from the wave functions used in constituent quark model studies, where the center of mass coordinate has been removed and only the internal Jacobi coordinates appear (see, for example, Ref. 23). The expressions we found for the matrix elements of $\bar{B}_2$ were identical with those of Ref. 18, based on Hartree wave functions, exact in the $N_c \to \infty$ limit only. This proves that in the Hartree approach no center of mass corrections are necessary for the $[\mathbf{56}, 2^+]$ multiplet. We expect the same conclusion to stand for any $[\mathbf{56}, \ell^+]$. For mixed representations the situation is more intricate [12].

For $\bar{B}_3$, one can use the following relation [14]

$$S_i G_{i8} = \frac{1}{4\sqrt{2}} \left[3I(I+1) - S(S+1) - \frac{3}{4} N_s (N_s + 2) \right] \tag{5}$$

in agreement with [8]. Here I is the isospin, S is the total spin and N_s the number of strange quarks. As for the matrix elements of $\bar{B}_2$, we found identical results to those of Ref. 18. Note that only $\bar{B}_2$ has non-vanishing off-diagonal matrix elements. Their role is very important in the state mixing. We found that the diagonal matrix elements of O_2, O_3, $\bar{B}_2$ and $\bar{B}_3$ of strange baryons satisfy the following relation

$$\frac{\bar{B}_2}{\bar{B}_3} = \frac{O_2}{O_3}, \tag{6}$$

for any state, irrespective of the value of J in both the octet and the decuplet. Such a relation also holds for the multiplet $[\mathbf{56}, 4^+]$ studied in the next section and might possibly be a feature of all $[\mathbf{56}, \ell^+]$ multiplets. It can be used as a check of the analytic expressions in Table 4. In spite of the relation (6) which holds for the diagonal matrix elements, the operators O_i and $\bar{B}_i$ are linearly independent, as it can be easily proved.

The other issue for the $[\mathbf{56}, 2^+]$ multiplet is that the analysis performed in Ref. 18 is based on the standard identification of resonances due to the pioneering work of Isgur and Karl [24]. In that work the spectrum of positive parity resonances was calculated from a Hamiltonian containing a harmonic oscillator confinement and a hyperfine interaction of one-gluon exchange type. The mixing angles in the $\Delta_{5/2^+}$ sector turned out to be

State	Mass (MeV)	Mixing angles
$^4\Delta[\mathbf{56}, 2^+]\frac{5}{2}^+$	1940	$\begin{bmatrix} 0.94 & 0.38 \\ -0.38 & 0.94 \end{bmatrix}$
$^4\Delta[\mathbf{70}, 2^+]\frac{5}{2}^+$	1975	

which shows that the lowest resonance at 1940 MeV is dominantly a $[\mathbf{56}, 2^+]$ state. As a consequence, the lowest observed F_{35} $\Delta(1905)$ resonance was interpreted as a member of the $[\mathbf{56}, 2^+]$ multiplet.

In a more realistic description, based on a linear confinement [25], the structure of the $\Delta_{5/2^+}$ sector appeared to be different. The result was

State	Mass (MeV)	Mixing angles
$^4\Delta[\mathbf{56}, 2^+]\frac{5}{2}^+$	1962	$\begin{bmatrix} 0.408 & 0.913 \\ 0.913 & -0.408 \end{bmatrix}$
$^4\Delta[\mathbf{70}, 2^+]\frac{5}{2}^+$	1985	

which means that in this case the higher resonance, of mass 1985 MeV, is dominantly $[\mathbf{56}, 2^+]$. Accordingly, here we interpret the higher experimentally observed resonance F_{35} $\Delta(2000)$ as belonging to the $[\mathbf{56}, 2^+]$ multiplet instead of the lower one. Thus we take as experimental input the mass 1976 $\pm$ 237 MeV, determined from the full listings of the PDG [19] in the same manner as for the one- and two-star resonances of the $[\mathbf{56}, 4^+]$ multiplet (see below). The results for $[\mathbf{56}, 2^+]$ multiplet based on this assignement are shown in Table 1. The $\chi^2_{\rm dof}$ obtained is 0.58, as compared to $\chi^2_{\rm dof} = 0.7$ of Ref. 18. The contribution of the spin-orbit operator O_2 is slightly smaller here than in Ref. 18. Although $\Delta(2000)$ is a two-star resonance only, the incentive of making the above choice was that the calculated pion decay widths of the $\Delta_{5/2^+}$ sector were better reproduced [26] with the mixing angles of the model [25] than with those of the standard model of Ref. 24. It is well known that decay widths are useful to test mixing angles. Moreover, it would be more natural that the resonances $\Delta_{1/2}$ and $\Delta_{5/2}$ would have different masses, contrary to the assumption of Ref. 18 where these masses were identical.

Table 1. Operators of Eq. (1) and coefficients resulting from the fit with $\chi^2_{\rm dof} \simeq 0.58$ for $[\mathbf{56}, 2^+]$ and $\chi^2_{\rm dof} \simeq 0.26$ for $[\mathbf{56}, 4^+]$.

Operator	Fitted coef. (MeV)	
	$[\mathbf{56}, 2^+]$	$[\mathbf{56}, 4^+]$
$O_1 = N_c \, \mathbb{1}$	$c_1 = 540 \pm 3$	$c_1 = 736 \pm 30$
$O_2 = \frac{1}{N_c} l_i S_i$	$c_2 = 14 \pm 9$	$c_2 = 4 \pm 40$
$O_3 = \frac{1}{N_c} S_i S_i$	$c_3 = 247 \pm 10$	$c_3 = 135 \pm 90$
$\bar{B}_1 = -\mathcal{S}$	$b_1 = 213 \pm 15$	$b_1 = 110 \pm 67$
$\bar{B}_2 = \frac{1}{N_c} l_i G_{i8} - \frac{1}{2\sqrt{3}} O_2$	$b_2 = 83 \pm 40$	
$\bar{B}_3 = \frac{1}{N_c} S_i G_{i8} - \frac{1}{2\sqrt{3}} O_3$	$b_3 = 266 \pm 65$	

4. The $[\mathbf{56}, 4^+]$ multiplet

Tables 3 and 4 give all matrix elements needed for the octets and decuplets belonging to the $[\mathbf{56}, 4^+]$ multiplet. They are calculated following the prescription of the previous section. This means that the matrix elements of O_1, O_2, O_3 and $\bar{B}_1$ are straightforward and for $\bar{B}_3$ we use the formula (5). The matrix elements of $\bar{B}_2$ are calculated from the wave functions given explicitly in Ref. 22, firstly derived and employed in constituent quark model calculations [26]. One can see that the relation (6) holds for this multiplet as well.

As mentioned above, only the operator $\bar{B}_2$ has non-vanishing off-diagonal matrix elements, so $\bar{B}_2$ is the only one which induces mixing be-

Table 2. Masses (in MeV) of the $[\mathbf{56}, \mathbf{2}^+]$ multiplet predicted by the $1/N_c$ expansion as compared with the empirically known masses. The partial contribution of each operator is indicated for all masses. Those partial contributions in blank are equal to the one above in the same column.

	Partial results						Total	Empirical
	O_1	O_2	O_3	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$		
$N_{3/2}$	1621	-7	62	0	0	0	1675 ± 10	1700 ± 50
$\Lambda_{3/2}$				213	36	-58	1867 ± 29	1880 ± 30
$\Sigma_{3/2}$				213	-12	19	1895 ± 20	(1840)
$\Xi_{3/2}$				425	48	-77	2072 ± 44	
$N_{5/2}$	1621	5	62	0	0	0	1687 ± 10	1683 ± 8
$\Lambda_{5/2}$				213	-24	-58	1818 ± 26	1820 ± 5
$\Sigma_{5/2}$				213	8	19	1927 ± 19	1918 ± 18
$\Xi_{5/2}$				425	-32	-77	2004 ± 40	
$\Delta_{1/2}$	1621	-21	309	0	0	0	1908 ± 21	1895 ± 25
$\Sigma_{1/2}$				213	36	-96	2061 ± 39	
$\Xi_{1/2}$				425	72	-192	2214 ± 69	
$\Omega_{1/2}$				638	108	-288	2367 ± 101	
$\Delta_{3/2}$	1621	-14	309	0	0	0	1915 ± 18	1935 ± 35
$\Sigma'_{3/2}$				213	24	-96	2056 ± 35	(2080)
$\Xi'_{3/2}$				425	48	-192	2197 ± 63	
$\Omega_{3/2}$				638	72	-288	2338 ± 93	
$\Delta_{5/2}$	1621	-2	309	0	0	0	1927 ± 16	1976 ± 237
$\Sigma'_{5/2}$				213	4	-96	2048 ± 32	(2070)
$\Xi'_{5/2}$				425	8	-192	2169 ± 58	
$\Omega_{5/2}$				638	12	-288	2289 ± 86	
$\Delta_{7/2}$	1621	14	309	0	0	0	1944 ± 18	1950 ± 10
$\Sigma_{7/2}$				213	-24	-96	2037 ± 35	2033 ± 8
$\Xi_{7/2}$				425	-48	-192	2129 ± 63	
$\Omega_{7/2}$				638	-72	-288	2222 ± 93	

tween the octet and decuplet states of $[\mathbf{56}, 4^+]$ with the same quantum numbers, as a consequence of the SU(3)-flavor breaking. Thus this mixing affects the octet and the decuplet Σ and Ξ states. As there are four off-diagonal matrix elements (Table 4), there are also four mixing angles, namely, θ_J^{Σ} and θ_J^{Ξ}, each with $J = 7/2$ and $9/2$. In terms of these mixing angles, the physical Σ_J and Σ'_J states are defined by the following basis states

$$|\Sigma_J\rangle = |\Sigma_J^{(8)}\rangle \cos\theta_J^{\Sigma} + |\Sigma_J^{(10)}\rangle \sin\theta_J^{\Sigma}, \tag{7}$$

$$|\Sigma'_J\rangle = -|\Sigma_J^{(8)}\rangle \sin\theta_J^{\Sigma} + |\Sigma_J^{(10)}\rangle \cos\theta_J^{\Sigma}, \tag{8}$$

and similar relations hold for Ξ. The masses of the physical states become

$$M(\Sigma_J) = M(\Sigma_J^{(8)}) + b_2\langle\Sigma_J^{(8)}|\bar{B}_2|\Sigma_J^{(10)}\rangle \tan\theta_J^{\Sigma}, \tag{9}$$

$$M(\Sigma'_J) = M(\Sigma_J^{(10)}) - b_2\langle\Sigma_J^{(8)}|\bar{B}_2|\Sigma_J^{(10)}\rangle \tan\theta_J^{\Sigma}, \tag{10}$$

where $M(\Sigma_J^{(8)})$ and $M(\Sigma_J^{(10)})$ are the diagonal matrix of the mass operator (1), here equal to $c_1O_1 + c_2O_2 + c_3O_3 + b_1\bar{B}_1 + b_2\bar{B}_2 + b_3\bar{B}_3$, for Σ states and similarly for Ξ states (see Table 5). If replaced in the mass operator (1), the relations (9) and (10) and their counterparts for Ξ, introduce four new parameters which should be included in the fit. Actually the procedure of Ref. 18 was simplified to fit the coefficients c_i and b_i directly to the physical masses and then to calculate the mixing angle from

$$\theta_J = \frac{1}{2}\arcsin\left(2\ \frac{b_2\langle\Sigma_J^{(8)}|\bar{B}_2|\Sigma_J^{(10)}\rangle}{M(\Sigma_J) - M(\Sigma'_J)}\right). \tag{11}$$

for Σ_J states and analogously for Ξ states.

Due to the scarcity of data in the 2–3 GeV mass region, even such a simplified procedure is not possible at present in the $[\mathbf{56}, 4^+]$ multiplet.

The fit of the masses derived from Eq. (1) and the available empirical values used in the fit, together with the corresponding resonance status in the Particle Data Group [19] are listed in Table 5. The values of the coefficients c_i and b_1 obtained from the fit are presented in Table 1, as already mentioned. For the four and three-star resonances we used the empirical masses given in the summary table. For the others, namely the one-star resonance $\Delta(2390)$ and the two-star resonance $\Delta(2300)$ we adopted the following procedure. We considered as "experimental" mass the average of all masses quoted in the full listings. The experimental error to the mass was defined as the quadrature of two uncorrelated errors, one being the average error obtained from the same references in the full listings and the other was the difference between the average mass relative to the farthest off observed mass. The masses and errors thus obtained are indicated in the before last column of Table 5.

Table 3. Matrix elements of $SU(3)$ singlet operators.

	O_1	O_2	O_3
$^28_{7/2}$	N_c	$-\frac{5}{2N_c}$	$\frac{3}{4N_c}$
$^28_{9/2}$	N_c	$\frac{2}{N_c}$	$\frac{3}{4N_c}$
$^410_{5/2}$	N_c	$-\frac{15}{2N_c}$	$\frac{15}{4N_c}$
$^410_{7/2}$	N_c	$-\frac{4}{N_c}$	$\frac{15}{4N_c}$
$^410_{9/2}$	N_c	$\frac{1}{2N_c}$	$\frac{15}{4N_c}$
$^410_{11/2}$	N_c	$\frac{6}{N_c}$	$\frac{15}{4N_c}$

Table 4. Matrix elements of $SU(3)$ breaking operators. Here, $a_J = 5/2, -2$ for $J = 7/2, 9/2$, respectively and $b_J = 5/2, 4/3, -1/6, -2$ for $J = 5/2, 7/2, 9/2, 11/2$, respectively.

	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$
N_J	0	0	0
Λ_J	1	$\frac{\sqrt{3}\ a_J}{2N_c}$	$-\frac{3\sqrt{3}}{8N_c}$
Σ_J	1	$-\frac{\sqrt{3}\ a_J}{6N_c}$	$\frac{\sqrt{3}}{8N_c}$
Ξ_J	2	$\frac{2\sqrt{3}\ a_J}{3N_c}$	$-\frac{\sqrt{3}}{2N_c}$
Δ_J	0	0	0
Σ_J	1	$\frac{\sqrt{3}\ b_J}{2N_c}$	$-\frac{5\sqrt{3}}{8N_c}$
Ξ_J	2	$\frac{\sqrt{3}\ b_J}{N_c}$	$-\frac{5\sqrt{3}}{4N_c}$
Ω_J	3	$\frac{3\sqrt{3}\ b_J}{2N_c}$	$-\frac{15\sqrt{3}}{8N_c}$
$\Sigma^8_{7/2} - \Sigma^{10}_{7/2}$	0	$-\frac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Sigma^8_{9/2} - \Sigma^{10}_{9/2}$	0	$-\frac{\sqrt{11}}{\sqrt{3}N_c}$	0
$\Xi^8_{7/2} - \Xi^{10}_{7/2}$	0	$-\frac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Xi^8_{9/2} - \Xi^{10}_{9/2}$	0	$-\frac{\sqrt{11}}{\sqrt{3}N_c}$	0

Due to the lack of experimental data in the strange sector it was not possible to include all the operators $\bar{B}_i$ in the fit in order to obtain some reliable predictions. As the breaking of SU(3) is dominated by $\bar{B}_1$ we included only this operator in Eq. (1) and neglected the contribution of the operators $\bar{B}_2$ and $\bar{B}_3$. At a later stage, when more data will hopefully be available, all analytical work performed here could be used to improve the fit. That is why Table 1 contains results for c_i ($i = 1$, 2 and 3) and b_1 only. The $\chi^2_{\rm dof}$ of the fit is 0.26, where the number of degrees of freedom (dof) is equal to one (five data and four coefficients).

The first column of Table 5 contains the 56 states (each state having a $2I + 1$ multiplicity from assuming an exact SU(2)-isospin symmetry [a]). The columns two to five show the partial contribution of each operator included in the fit, multiplied by the corresponding coefficient c_i or b_1. The column six gives the total mass according to Eq. (1). The errors shown in the predictions result from the errors on the coefficients c_i and b_1 given in Table 1. As there are only five experimental data available, nineteen of these masses are predictions. The breaking of SU(3)-flavor due to the operator $\bar{B}_1$ is 110 MeV as compared to 200 MeV produced in the $[\mathbf{56}, 2^+]$ multiplet.

The main question is, of course, how reliable is this fit. The answer can be summarized as follows:

- The main part of the mass is provided by the spin-flavor singlet operator O_1, which is $\mathcal{O}(N_c)$.
- The spin-orbit contribution given by c_2O_2 is small. This fact reinforces the practice used in constituent quark models where the spin-orbit contribution is usually neglected.
- The breaking of the SU(6) symmetry keeping the flavor symmetry exact is mainly due to the spin-spin operator O_3. This hyperfine interaction produces a splitting between octet and decuplet states of approximately 130 MeV which is smaller than that obtained in the $[\mathbf{56}, 2^+]$ case [18], which gives 240 MeV.
- The contribution of $\bar{B}_1$ per unit of strangeness, 110 MeV, is also smaller here than in the $[\mathbf{56}, 2^+]$ multiplet [18], where it takes a value of about 200 MeV. That may be quite natural, as one expects a shrinking of the spectrum with the excitation energy.
- As it was not possible to include the contribution of $\bar{B}_2$ and $\bar{B}_3$ in our fit, a degeneracy appears between Λ and Σ.

For completeness in Table 6 we give the eighteen mass relations which

[a] Note that the notation Σ_J, Σ'_J is consistent with the relations (9), (10) inasmuch as the contribution of $\bar{B}_2$ is neglected (same remark for Ξ_J, Ξ'_J and corresponding relations).

Table 5. Masses (in MeV) of the $[\mathbf{56}, 4^+]$ multiplet predicted by the $1/N_c$ expansion as compared with the empirically known masses. The partial contribution of each operator is indicated for all masses. Those partial contributions in blank are equal to the one above in the same column.

	$1/N_c$ expansion results						
	Partial contribution (MeV)				Total (MeV)	Empirical	Name, status
	c_1O_1	c_2O_2	c_3O_3	$b_1\bar{B}_1$		(MeV)	
$N_{7/2}$	2209	-3	34	0	2240 ± 97		
$\Lambda_{7/2}$				110	2350 ± 118		
$\Sigma_{7/2}$				110	2350 ± 118		
$\Xi_{7/2}$				220	2460 ± 166		
$N_{9/2}$	2209	2	34	0	2245 ± 95	2245 ± 65	N(2220)****
$\Lambda_{9/2}$				110	2355 ± 116	2355 ± 15	Λ(2350)***
$\Sigma_{9/2}$				110	2355 ± 116		
$\Xi_{9/2}$				220	2465 ± 164		
$\Delta_{5/2}$	2209	-9	168	0	2368 ± 175		
$\Sigma_{5/2}$				110	2478 ± 187		
$\Xi_{5/2}$				220	2588 ± 220		
$\Omega_{5/2}$				330	2698 ± 266		
$\Delta_{7/2}$	2209	-5	168	0	2372 ± 153	2387 ± 88	Δ(2390)*
$\Sigma'_{7/2}$				110	2482 ± 167		
$\Xi'_{7/2}$				220	2592 ± 203		
$\Omega_{7/2}$				330	2702 ± 252		
$\Delta_{9/2}$	2209	1	168	0	2378 ± 144	2318 ± 132	Δ(2300)**
$\Sigma'_{9/2}$				110	2488 ± 159		
$\Xi'_{9/2}$				220	2598 ± 197		
$\Omega_{9/2}$				330	2708 ± 247		
$\Delta_{11/2}$	2209	7	168	0	2385 ± 164	2400 ± 100	Δ(2420)****
$\Sigma_{11/2}$				110	2495 ± 177		
$\Xi_{11/2}$				220	2605 ± 212		
$\Omega_{11/2}$				330	2715 ± 260		

hold for this multiplet. They can be easily derived from Eq. (1). Presently one cannot test the accuracy of these relations due to lack of data.

Table 6. The 18 independent mass relations include the GMO relations for the two octets and the two EQS for each of the four decuplets.

$9(\Delta_{7/2} - \Delta_{5/2})$	$=$	$7(N_{9/2} - N_{7/2})$
$9(\Delta_{9/2} - \Delta_{5/2})$	$=$	$16(N_{9/2} - N_{7/2})$
$9(\Delta_{11/2} - \Delta_{9/2})$	$=$	$11(N_{9/2} - N_{7/2})$
$8(\Lambda_{7/2} - N_{7/2}) + 14(N_{9/2} - \Lambda_{9/2})$	$=$	$3(\Lambda_{9/2} - \Sigma_{9/2}) + 6(\Delta_{11/2} - \Sigma_{11/2})$
$\Lambda_{9/2} - \Lambda_{7/2} + 3(\Sigma_{9/2} - \Sigma_{7/2})$	$=$	$4(N_{9/2} - N_{7/2})$
$\Lambda_{9/2} - \Lambda_{7/2} + \Sigma_{9/2} - \Sigma_{7/2}$	$=$	$2(\Sigma'_{9/2} - \Sigma'_{7/2})$
$11\ \Sigma'_{7/2} + 9\ \Sigma_{11/2}$	$=$	$20\ \Sigma'_{9/2}$
$20\ \Sigma_{5/2} + 7\ \Sigma_{11/2}$	$=$	$27\ \Sigma'_{7/2}$
$2(N + \Xi)$	$=$	$3\ \Lambda + \Sigma$
$\Sigma - \Delta$	$=$	$\Xi - \Sigma = \Omega - \Xi$

In conclusion we have studied the spectrum of highly excited resonances in the 2–3 GeV mass region by describing them as belonging to the $[\mathbf{56}, 4^+]$ multiplet. This is the first study of such excited states based on the $1/N_c$ expansion of QCD. A better description should include multiplet mixing, following the lines developed, for example, in Ref. 27.

We support previous assertions that better experimental values for highly excited non-strange baryons as well as more data for the Σ^* and Ξ^* baryons are needed in order to understand the role of the operator $\bar{B}_2$ within a multiplet and for the octet-decuplet mixing. With better data the analytic work performed here will help to make reliable predictions in the large N_c limit formalism.

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References

1. G. 't Hooft, *Nucl. Phys.* **B72**, 461 (1974).
2. E. Witten, *Nucl. Phys.* **B160** 57 (1979).
3. R. Dashen and A. V. Manohar, *Phys. Lett.* **B315** (1993) 425; *Phys. Lett.* **B315** (1993) 438.

4. R. Dashen, E. Jenkins, and A. V. Manohar, *Phys. Rev.* **D49** (1994) 4713.
5. R. Dashen, E. Jenkins, and A. V. Manohar, *Phys. Rev.* **D51** (1995) 3697.
6. C. D. Carone, H. Georgi and S. Osofsky, *Phys. Lett.* **B322** (1994) 227.
M. A. Luty and J. March-Russell, *Nucl. Phys.* **B426** (1994) 71.
M. A. Luty, J. March-Russell and M. White, *Phys. Rev.* **D51** (1995) 2332.
7. E. Jenkins, *Phys. Lett.* **B315** (1993) 441.
8. E. Jenkins and R. F. Lebed, *Phys. Rev.* **D52** (1995) 282.
9. J. Dai, R. Dashen, E. Jenkins, and A. V. Manohar, *Phys. Rev.* **D53** (1996) 273.
10. J. L. Goity, *Phys. Lett.* **B414**, (1997) 140.
11. D. Pirjol and T. M. Yan, *Phys. Rev.* **D57** (1998) 1449.
D. Pirjol and T. M. Yan, *Phys. Rev.* **D57** (1998) 5434.
12. C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, *Phys. Lett.* **B438** (1998) 327; *Phys. Rev.* **D59** (1999) 114008.
13. C. D. Carone, H. Georgi, L. Kaplan and D. Morin, *Phys. Rev.* **D50** (1994) 5793.
14. C. E. Carlson and C. D. Carone, *Phys. Lett.* **B441** (1998) 363; *Phys. Rev.* **D58** (1998) 053005.
15. C. L. Schat, J. L. Goity and N. N. Scoccola, *Phys. Rev. Lett.* **88** (2002) 102002; J. L. Goity, C. L. Schat and N. N. Scoccola, *Phys. Rev.* **D66** (2002) 114014.
16. D. Pirjol and C. Schat, *Phys. Rev.* **D67** (2003) 096009.
17. C. E. Carlson and C. D. Carone, *Phys. Lett.* **B484** (2000) 260.
18. J. L. Goity, C. L. Schat and N. N. Scoccola, *Phys. Lett.* **B564** (2003) 83.
19. S. Eidelman et al., *Phys. Lett.* **B592** (2004) 1.
20. S. Capstick and N. Isgur, *Phys. Rev.* **D34** (1986) 2809.
21. P. Stassart and Fl. Stancu, *Z. Phys.* **A359** (1997) 321.
22. N. Matagne and Fl. Stancu, *Phys. Rev.* **D**, to appear [arXiv:hep-ph/0409261]
23. Fl. Stancu, *Group Theory in Subnuclear Physics*, Clarendon Press, Oxford, 1996, Ch. 10.
24. N. Isgur and G. Karl, *Phys. Rev.* **D19** (1979) 2653.
25. R. Sartor and Fl. Stancu, *Phys. Rev.* **D34** (1986) 3405.
26. P. Stassart and Fl. Stancu, *Z. Phys.* **A351** (1995) 77.
27. J. L. Goity, these proceedings.