The $[\mathbf{56}, 4^+]$ baryons in the $1/N_c$ expansion

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Abstract

Using the $1/N_c$ expansion of QCD we analyze the spectrum of positive parity resonances with strangeness $S = 0, -1, -2$ and -3 in the 2-3 GeV mass region, supposed to belong to the $[\mathbf{56}, 4^+]$ multiplet. The mass operator is similar to that of $[\mathbf{56}, 2^+]$, previously studied in the literature. The analysis of the latter is revisited. In the $[\mathbf{56}, 4^+]$ multiplet we find that the spin-spin term brings the dominant contribution and that the spin-orbit term is entirely negligible in the hyperfine interaction, in agreement with constituent quark model results. More data are strongly desirable, especially in the strange sector in order to fully exploit the power of this approach.

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I. INTRODUCTION

The $1/N_c$ expansion of QCD [1, 2, 3, 4] has been proved a useful approach to study baryon spectroscopy. It has been applied to the ground state baryons [5, 6, 7, 8, 9, 10, 11] as well as to excited states, in particular to the negative parity spin-flavor $[\mathbf{70}, 1^-]$ multiplet ($N = 1$ band) [12, 13, 14, 15, 16, 17], to the positive parity Roper resonance belonging to $[\mathbf{56'}, 0^+]$ ($N = 2$ band) [18] and to the $[\mathbf{56}, 2^+]$ multiplet ($N = 2$ band) [19]. In this approach the main features of the constituent quark model emerge naturally and in addition, new information is provided, as for example, on the spin-orbit problem.

In this study we explore its applicability to the $[\mathbf{56}, 4^+]$ multiplet ($N = 4$ band) for the first time. The number of experimentally known resonances in the 2-3 GeV region [20], expected to belong to this multiplet is quite restricted. Among the five possible candidates there are two four-star resonances, $N(2220)9/2^+$ and $\Delta(2420)11/2^+$, one three-star resonance $\Lambda(2350)9/2^+$, one two-star resonance $\Delta(2300)9/2^+$ and one one-star resonance $\Delta(2390)7/2^+$. This is an exploratory study which will allow us to make some predictions.

In constituent quark models the $N = 4$ band has been studied so far either in a large harmonic oscillator basis [21] or in a variational basis [22]. We shall show that the present approach reinforces the conclusion that the spin-orbit contribution to the hyperfine interaction can safely be neglected in constituent quark model calculations.

The properties of low energy hadrons are interpreted to be a consequence of the spontaneous breaking of chiral symmetry [23]. For highly excited hadrons, as the ones considered here, there are phenomenological arguments to believe that the chiral symmetry is restored. This would imply a weakening (up to a cancellation) of the spin-orbit and tensor interactions [24]. Then the main contribution to the hyperfine interaction remains the spin-spin term.

II. THE WAVE FUNCTIONS

The N = 4 band contains 17 multiples having symmetries (56), (70) or (20) and angular momenta ranging from 0 to 4 [22]. Among them, the $[\mathbf{56}, 4^+]$ multiplet has a rather simple structure. It is symmetric both in SU(6) and O(3), where O(3) is the group of spatial rotation. Together with the color part which is always antisymmetric, it gives a totally antisymmetric wave function. In our study of the $[\mathbf{56}, 4^+]$ multiplet, we have to couple the

symmetric orbital part ψ_{4m}^{Sym} with $\ell = 4$ (see Table 2 of Ref. [22]) to a spin-flavor symmetric wave function. This gives

$$|4, S; d, Y, II_z; JJ_z\rangle = \sum_{m, S_z} \left(\begin{array}{cc|c} 4 & S & J \\ m & S_z & J_z \end{array} \right) \psi_{4m}^{\mathrm{Sym}} \times |SS_z; d, Y, II_z\rangle_{\mathrm{Sym}}, \qquad (1)$$

where S, S_z are the spin and its projection, d labels an SU(3) representation (here 8 and 10), Y, I, I_z stand for the hypercharge, isospin and its projection and J, J_z for the total angular momentum and its projection. Expressing the states (1) in the obvious notation $^{2S+1}d_J$, they are as follows: two SU(3) octets $^2 8_{\frac{7}{2}}$, $^2 8_{\frac{9}{2}}$ and four decuplets $^4 10_{\frac{5}{2}}$, $^4 10_{\frac{7}{2}}$, $^4 10_{\frac{9}{2}}$, $^4 10_{\frac{11}{2}}$.

In the following, we need the explicit form of the wave functions. They depend on J_z but the matrix elements of the operators that we shall calculate in the next sections do not depend on J_z due to the Wigner-Eckart theorem. So, choosing $J_z = \dfrac{1}{2}$, we have for the octet states

$$|^2 8[\mathbf{56}, 4^+] \frac{7}{2}^+ \frac{1}{2}\rangle = \sqrt{\frac{5}{18}} \psi_{41}^S \left(\chi_-^\rho \phi^\rho + \chi_-^\lambda \phi^\lambda \right) - \sqrt{\frac{2}{9}} \psi_{40}^S \left(\chi_+^\rho \phi^\rho + \chi_+^\lambda \phi^\lambda \right), \qquad (2)$$

$$|^2 8[\mathbf{56}, 4^+] \frac{9}{2}^+ \frac{1}{2}\rangle = \sqrt{\frac{2}{9}} \psi_{41}^S \left(\chi_-^\rho \phi^\rho + \chi_-^\lambda \phi^\lambda \right) + \sqrt{\frac{5}{18}} \psi_{40}^S \left(\chi_+^\rho \phi^\rho + \chi_+^\lambda \phi^\lambda \right), \qquad (3)$$

and for the decuplet states

$$|^4 10[\mathbf{56}, 4^+] \frac{5}{2}^+ \frac{1}{2}\rangle = \left(\sqrt{\frac{5}{21}} \psi_{42}^S \chi_{\frac{3}{2} - \frac{3}{2}} - \sqrt{\frac{5}{14}} \psi_{41}^S \chi_{\frac{3}{2} - \frac{1}{2}} + \sqrt{\frac{2}{7}} \psi_{40}^S \chi_{\frac{3}{2} \frac{1}{2}} - \sqrt{\frac{5}{42}} \psi_{4-1}^S \chi_{\frac{3}{2} \frac{3}{2}} \right) \phi^S, \quad (4)$$

$$|^4 10[\mathbf{56}, 4^+] \frac{7}{2}^+ \frac{1}{2}\rangle = \left(\sqrt{\frac{3}{7}} \psi_{42}^S \chi_{\frac{3}{2} - \frac{3}{2}} - \sqrt{\frac{2}{63}} \psi_{41}^S \chi_{\frac{3}{2} - \frac{1}{2}} - \sqrt{\frac{10}{63}} \psi_{40}^S \chi_{\frac{3}{2} \frac{1}{2}} + \sqrt{\frac{8}{21}} \psi_{4-1}^S \chi_{\frac{3}{2} \frac{3}{2}} \right) \phi^S, \quad (5)$$

$$|^4 10[\mathbf{56}, 4^+] \frac{9}{2}^+ \frac{1}{2}\rangle = \left(\sqrt{\frac{3}{11}} \psi_{42}^S \chi_{\frac{3}{2} - \frac{3}{2}} + \sqrt{\frac{49}{198}} \psi_{41}^S \chi_{\frac{3}{2} - \frac{1}{2}} - \sqrt{\frac{10}{99}} \psi_{40}^S \chi_{\frac{3}{2} \frac{1}{2}} - \sqrt{\frac{25}{66}} \psi_{4-1}^S \chi_{\frac{3}{2} \frac{3}{2}} \right) \phi^S, (6)$$

$$|^4 10[\mathbf{56}, 4^+] \frac{11}{2}^+ \frac{1}{2}\rangle = \left(\sqrt{\frac{2}{33}} \psi_{42}^S \chi_{\frac{3}{2} - \frac{3}{2}} + \sqrt{\frac{4}{11}} \psi_{41}^S \chi_{\frac{3}{2} - \frac{1}{2}} + \sqrt{\frac{5}{11}} \psi_{40}^S \chi_{\frac{3}{2} \frac{1}{2}} + \sqrt{\frac{4}{33}} \psi_{4-1}^S \chi_{\frac{3}{2} \frac{3}{2}} \right) \phi^S. \quad (7)$$

with ϕ^λ, ϕ^ρ, ϕ^S and χ given in Appendix A.

III. THE MASS OPERATOR

The study of the $[\mathbf{56}, 4^+]$ multiplet is similar to that of $[\mathbf{56}, 2^+]$ as analyzed in Ref. [19], where the mass spectrum is studied in the $1/N_c$ expansion up to and including $\mathcal{O}(1/N_c)$ effects. The mass operator must be rotationally invariant, parity and time reversal even. The isospin breaking is neglected. The SU(3) symmetry breaking is implemented to $\mathcal{O}(\varepsilon)$,

where $\varepsilon \sim 0.3$ gives a measure of this breaking. As the $[\mathbf{56}, 4^+]$ baryons are described by a symmetric representation of SU(6), it is not necessary to distinguish between excited and core quarks for the construction of a basis of mass operators, as explained in Ref. [19]. Then the mass operator of the $[\mathbf{56}, 4^+]$ multiplet has the following structure

$$M = \sum_i c_i O_i + \sum_i b_i \bar{B}_i \tag{8}$$

given in terms of the linearly independent operators O_i and $\bar{B}_i$, similar to that of the $[\mathbf{56}, 2^+]$ multiplet. Here O_i $(i = 1, 2, 3)$ are rotational invariants and SU(3)-flavor singlets [12], $\bar{B}_1$ is the strangeness quark number operator with negative sign, and the operators $\bar{B}_i$ $(i = 2, 3)$ are also rotational invariants but contain the SU(6) flavor-spin generators G_{i8} as well. The operators $\bar{B}_i$ $(i = 1, 2, 3)$ provide SU(3) breaking and are defined to have vanishing matrix elements for nonstrange baryons. The relation (8) contains the effective coefficients c_i and b_i as parameters. They represent reduced matrix elements that encode the QCD dynamics. The above operators and the values of the corresponding coefficients obtained from fitting the experimentally known masses (see next section) are given in Table I.

We recall that a generic n-body operator has the structure

$$O^{(n)} = \frac{1}{N_c^{n-1}} O_\ell O_{SF} \tag{9}$$

where the factors O_ℓ and O_{SF} can be expressed in terms of products of generators ℓ_i $(i = 1, 2, 3)$ of the group O(3), and of the spin-flavor group SU(6) S_i, T_a and G_{ia} $(i = 1, 2, 3;$ $a = 1, \ldots, 8)$. Because an n-body operator requires that at least $(n-1)$ gluons be exchanged between n quarks, an overall factor of $1/N_c^{n-1}$ appears. Matrix elements of some operators can carry a nontrivial N_c dependence due to coherence effects [5]: for example, G_{ia} $(a = 1, 2, 3)$ and T_8 have matrix elements of $\mathcal{O}(N_c)$. This explains the dependence on N_c of the operators listed in Table I.

The matrix elements of O_1, O_2 and O_3 are trivial to calculate. They are given in Table II for the octet and the decuplet states belonging to the $[\mathbf{56}, 4^+]$ multiplet. The $\bar{B}_1$ matrix elements are also trivial. To calculate the $\bar{B}_2$ matrix elements we use the expression

$$G_{i8} = G^{i8} = \frac{1}{2\sqrt{3}}(S^i - 3S_s^i), \tag{10}$$

where S^i and S_s^i are the components of the total spin and of the total strange-quark spin

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respectively [10]. We rewrite this expression as

$$\bar{B}_2 = -\frac{\sqrt{3}}{2N_c}\vec{l}\cdot\vec{S}_s \tag{11}$$

with the decomposition

$$\vec{l}\cdot\vec{S}_s = l_0 S_{s0} + \frac{1}{2}\left(l_+ S_{s-} + l_- S_{s+}\right), \tag{12}$$

which we apply on the wave functions (2)–(7) in order to obtain the diagonal and off-diagonal matrix elements. For $\bar{B}_3$, one can use the following relation [18]

$$S_i G_{i8} = \frac{1}{4\sqrt{3}}[3I(I+1) - S(S+1) - \frac{3}{4}N_s(N_s+2)] \tag{13}$$

in agreement with [10]. Here I is the isospin, S is the total spin and N_s the number of strange quarks. Both the diagonal and off-diagonal matrix elements of $\bar{B}_i$ are exhibited in Table III. Note that only $\bar{B}_2$ has non-vanishing off-diagonal matrix elements. Their role is very important in the state mixing, as discussed in the next section. We found that the diagonal matrix elements of O_2, O_3, $\bar{B}_2$ and $\bar{B}_3$ of strange baryons satisfy the following relation

$$\frac{\bar{B}_2}{\bar{B}_3} = \frac{O_2}{O_3}, \tag{14}$$

for any state, irrespective of the value of J in both the octet and the decuplet. This can be used as a check of the analytic expressions in Table III. Such a relation also holds for the multiplet $[\mathbf{56}, 2^+]$ studied in Ref. [19] and might possibly be a feature of all $[\mathbf{56}, \ell+]$ multiplets. In spite of the relation (14) which holds for the diagonal matrix elements, the operators O_i and $\bar{B}_i$ are linearly independent, as it can be easily proved. As a proof, the off-diagonal matrix elements of $\bar{B}_2$ are entirely different from those of $\bar{B}_3$.

IV. STATE MIXING

As mentioned above, only the operator $\bar{B}_2$ has non-vanishing off-diagonal matrix elements, so $\bar{B}_2$ is the only one which induces mixing between the octet and decuplet states of $[\mathbf{56}, 4^+]$ with the same quantum numbers, as a consequence of the SU(3)-flavor breaking. Thus this mixing affects the octet and the decuplet Σ and Ξ states. As there are four off-diagonal matrix elements (Table III), there are also four mixing angles, namely, θ_J^Σ and θ_J^Ξ, each with $J = 7/2$ and $9/2$. In terms of these mixing angles, the physical Σ_J and Σ_J' states are defined

by the following basis states

$$|\Sigma_J\rangle \; = \; |\Sigma_J^{(8)}\rangle \cos\theta_J^\Sigma + |\Sigma_J^{(10)}\rangle \sin\theta_J^\Sigma, \tag{15}$$

$$|\Sigma_J'\rangle \; = \; -|\Sigma_J^{(8)}\rangle \sin\theta_J^\Sigma + |\Sigma_J^{(10)}\rangle \cos\theta_J^\Sigma, \tag{16}$$

and similar relations hold for Ξ. The masses of the physical states become

$$M(\Sigma_J) \; = \; M(\Sigma_J^{(8)}) + b_2\langle\Sigma_J^{(8)}|\bar{B}_2|\Sigma_J^{(10)}\rangle \tan\theta_J^\Sigma, \tag{17}$$

$$M(\Sigma_J') \; = \; M(\Sigma_J^{(10)}) - b_2\langle\Sigma_J^{(8)}|\bar{B}_2|\Sigma_J^{(10)}\rangle \tan\theta_J^\Sigma, \tag{18}$$

where $M(\Sigma_J^{(8)})$ and $M(\Sigma_J^{(10)})$ are the diagonal matrix of the mass operator (8), here equal to $c_1O_1 + c_2O_2 + c_3O_3 + b_1\bar{B}_1$, for Σ states and similarly for Ξ states (see Table V). If replaced in the mass operator (8), the relations (17) and (18) and their counterparts for Ξ, introduce four new parameters which should be included in the fit. Actually the procedure of Ref. [19] was simplified to fit the coefficients c_i and b_i directly to the physical masses and then to calculate the mixing angle from

$$\theta_J = \frac{1}{2}\arcsin\left(2\,\frac{b_2\langle\Sigma_J^{(8)}|\bar{B}_2|\Sigma_J^{(10)}\rangle}{M(\Sigma_J) - M(\Sigma_J')}\right). \tag{19}$$

for Σ_J states and analogously for Ξ states.

As we shall see below, due to the scarcity of data in the 2-3 GeV mass region, even such a simplified procedure is not possible at present in the $[\mathbf{56}, 4^+]$ multiplet.

V. MASS RELATIONS

In the isospin symmetric limit, there are twenty four independent masses, as presented in the first column of Table V. Our operator basis contains six operators, so there are eighteen mass relations that hold irrespective of the values of the coefficients c_i and b_i. These relations can be easily calculated from the definition (8) and are presented in Table IV.

One can identify the Gell-Mann–Okubo (GMO) mass formula for each octet (two such relations) and the equal spacing relations (EQS) for each decuplet (eight such relations). There are eight relations left, that involve states belonging to different SU(3) multiplets as well as to different values of J. Presently one cannot test the accuracy of these relations due to lack of data. But they may be used in making some predictions. The theoretical masses satisfy the Gell-Mann–Okubo mass formula for octets and the Equal Spacing Rule for decuplets, providing another useful test of the $1/N_c$ expansion.

VI.   THE $[\mathbf{56}, 2^+]$ REVISITED

We have checked the analytic work of Ref. [19] and refitted the masses of the octets and decuplets with a slightly different input. This is related to a different assignement of the $\Delta_{5/2}$ resonances.

The analysis performed in Ref. [19] is based on the standard identification of resonances due to the pioneering work of Isgur and Karl [25]. In that work the spectrum of positive parity resonances is calculated from a Hamiltonian containing a harmonic oscillator confinement and a hyperfine interaction of one-gluon exchange type. The mixing angles in the $\Delta_{5/2^+}$ sector turn out to be

State	Mass (MeV)	Mixing angles	
$^4\Delta[\mathbf{56}, 2^+]\frac{5}{2}^+$	1940	0.94	0.38
$^4\Delta[\mathbf{70}, 2^+]\frac{5}{2}^+$	1975	-0.38	0.94

which shows that the lowest resonance at 1940 MeV is dominantly a $[\mathbf{56}, 2^+]$ state. As a consequence, the lowest observed F_{35} $\Delta(1905)$ resonance is interpreted as a member of the $[\mathbf{56}, 2^+]$ multiplet.

In a more realistic description, based on a linear confinement [26], the structure of the $\Delta_{5/2^+}$ sector appeared to be different. The result was

State	Mass (MeV)	Mixing angles	
$^4\Delta[\mathbf{56}, 2^+]\frac{5}{2}^+$	1962	0.408	0.913
$^4\Delta[\mathbf{70}, 2^+]\frac{5}{2}^+$	1985	0.913	-0.408

which means that in this case the higher resonance of mass 1985 MeV is dominantly $[\mathbf{56}, 2^+]$. Accordingly, here we interpret the higher experimentally observed resonance F_{35} $\Delta(2000)$ as belonging to the $[\mathbf{56}, 2^+]$ multiplet instead of the lower one. Thus we take as experimental input the mass 1976 ± 237 MeV, determined from the full listings of the PDG [20] in the same manner as for the one- and two-star resonances of the $[\mathbf{56}, 4^+]$ multiplet (see below). The χ^2_{dof} obtained is 0.58, as compared to $\chi^2_{\mathrm{dof}} = 0.7$ of Ref. [19]. Although $\Delta(2000)$ is a two-star resonance only, the incentive of making the above choice was that the calculated

pion decay widths of the $\Delta_{5/2^+}$ sector were better reproduced [27] with the mixing angles of the model [26] than with those of the standard model of [25]. It is well known that decay widths are useful to test mixing angles. Moreover, it would be more natural that the resonances $\Delta_{1/2}$ and $\Delta_{5/2}$ would have different masses, contrary to the assumption of Ref. [19] where these masses are identical.

VII.   FIT AND DISCUSSION

The fit of the masses derived from Eq. (8) and the available empirical values used in the fit, together with the corresponding resonance status in the Particle Data Group [20] are listed in Table V. The values of the coefficients c_i and b_1 obtained from the fit are presented in Table I, as already mentioned. For the four and three-star resonances we used the empirical masses given in the summary table. For the others, namely the one-star resonance $\Delta(2390)$ and the two-star resonance $\Delta(2300)$ we adopted the following procedure. We considered as "experimental" mass the average of all masses quoted in the full listings. The experimental error to the mass was defined as the quadrature of two uncorrelated errors, one being the average error obtained from the same references in the full listings and the other was the difference between the average mass relative to the farthest off observed mass. The masses and errors thus obtained are indicated in the before last column of Table V.

Due to the lack of experimental data in the strange sector it was not possible to include all the operators $\bar{B}_i$ in the fit in order to obtain some reliable predictions. As the breaking of SU(3) is dominated by $\bar{B}_1$ we included only this operator in Eq. (8) and neglected the contribution of the operators $\bar{B}_2$ and $\bar{B}_3$. At a later stage, when more data will hopefully be available, all analytical work performed here could be used to improve the fit. That is why Table I contains results for c_i ($i = 1$, 2 and 3) and b_1 only. The χ^2_{dof} of the fit is 0.26, where the number of degrees of freedom (dof) is equal to one (five data and four coefficients).

The first column of Table V contains the 56 states (each state having a $2I+1$ multiplicity from assuming an exact SU(2)-isospin symmetry) [32]. The columns two to five show the partial contribution of each operator included in the fit, multiplied by the corresponding coefficient c_i or b_1. The column six gives the total mass according to Eq. (8). The errors shown in the predictions result from the errors on the coefficients c_i and b_1 given in Table I. As there are only five experimental data available, nineteen of these masses are predictions.

The breaking of SU(3)-flavor due to the operator $\bar{B}_1$ is 110 MeV as compared to 200 MeV produced in the $[\mathbf{56}, 2^+]$ multiplet.

The main question is, of course, how reliable is this fit. The answer can be summarized as follows:

- The main part of the mass is provided by the spin-flavor singlet operator O_1, which is $\mathcal{O}(N_c)$.

- The spin-orbit contribution given by $c_2 O_2$ is small. This fact reinforces the practice used in constituent quark models where the spin-orbit contribution is usually neglected. Our result is consistent with the expectation that the spin-orbit term vanishes at large excitation energies [24].

- The breaking of the $SU(6)$ symmetry keeping the flavor symmetry exact is mainly due to the spin-spin operator O_3. This hyperfine interaction produces a splitting between octet and decuplet states of approximately 130 MeV which is smaller than that obtained in the $[\mathbf{56}, 2^+]$ case [19], which gives 240 MeV.

- The contribution of $\bar{B}_1$ per unit of strangeness, 110 MeV, is also smaller here than in the $[\mathbf{56}, 2^+]$ multiplet [19], where it takes a value of about 200 MeV. That may be quite natural, as one expects a shrinking of the spectrum with the excitation energy.

- As it was not possible to include the contribution of $\bar{B}_2$ and $\bar{B}_3$ in our fit, a degeneracy appears between Λ and Σ.

VIII.   CONCLUSIONS

We have studied the spectrum of highly excited resonances in the 2-3 GeV mass region by describing them as belonging to the $[\mathbf{56}, 4^+]$ multiplet. This is the first study of such excited states based on the $1/N_c$ expansion of QCD. A better description should include multiplet mixing, following the lines developed, for example, in Ref. [30].

We support previous assertions that better experimental values for highly excited non-strange baryons as well as more data for the Σ^* and Ξ^* baryons are needed in order to understand the role of the operator $\bar{B}_2$ within a multiplet and for the octet-decuplet mixing.

With better data the analytic work performed here will help to make reliable predictions in the large N_c limit formalism.

APPENDIX A: FLAVOR-SPIN STATES

Here we reproduce the definitions of the flavor and spin states used in the calculations of the matrix elements of $\bar{B}_2$ using the standard notations [31]. In the $[\mathbf{56}, 4^+]$ multiplet the spin-flavor part of the baryon wave function is symmetric (see relation (1)). Consider first the octet. One can write

$$|SS_z; 8, Y, II_z\rangle_{\text{Sym}} = \frac{1}{\sqrt{2}} \left(\chi^\rho \phi^\rho + \chi^\lambda \phi^\lambda \right) \tag{A1}$$

with the flavor states ϕ^ρ and ϕ^λ defined in Table VI. The states χ^ρ and χ^λ represent of spin $S = 1/2$ and permutation symmetry ρ and λ respectively, can be obtained from $\phi^\rho_{p(n)}$ and $\phi^\lambda_{p(n)}$ by making the replacement

$$u \rightarrow \uparrow, \; d \rightarrow \downarrow$$

where $\uparrow$ and $\downarrow$ are spin $1/2$ single-particle states of projection $S_z = +1/2$ and $S_z = -1/2$, respectively. The three particle states of mixed symmetry and $S = 1/2$, $S_z = +1/2$ are

$$\chi^\lambda_+ = -\frac{1}{\sqrt{6}} \left(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow \right) \tag{A2}$$

$$\chi^\rho_+ = \frac{1}{\sqrt{2}} \left(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right) \tag{A3}$$

and those having $S = 1/2$, $S_z = -1/2$ are

$$\chi^\lambda_- = \frac{1}{\sqrt{6}} \left(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2 \downarrow\downarrow\uparrow \right) \tag{A4}$$

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$$\chi_-^\rho = \frac{1}{\sqrt{2}} \left(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow\right). \tag{A5}$$

For the decuplet, we have

$$|SS_z; 10, Y, II_z\rangle_{\text{Sym}} = \chi\phi^S \tag{A6}$$

with ϕ^S defined in Table VII. χ must be an $S = 3/2$ state. With the previous notation, the $\chi_{\frac{3}{2}m}$ states take the form

$$\chi_{\frac{3}{2}\frac{3}{2}} = \uparrow\uparrow\uparrow \tag{A7}$$

$$\chi_{\frac{3}{2}\frac{1}{2}} = \frac{1}{\sqrt{3}} \left(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow\right) \tag{A8}$$

$$\chi_{\frac{3}{2}-\frac{1}{2}} = \frac{1}{\sqrt{3}} \left(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow\right) \tag{A9}$$

$$\chi_{\frac{3}{2}-\frac{3}{2}} = \downarrow\downarrow\downarrow \tag{A10}$$

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Operator	Fitted coef. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 = 736 \pm 30$
$O_2 = \dfrac{1}{N_c} l_i S_i$	$c_2 = \quad 4 \pm 40$
$O_3 = \dfrac{1}{N_c} S_i S_i$	$c_3 = 135 \pm 90$
$\bar{B}_1 = -\mathcal{S}$	$b_1 = 110 \pm 67$
$\bar{B}_2 = \dfrac{1}{N_c} l_i G_{i8} - \dfrac{1}{2\sqrt{3}} O_2$	
$\bar{B}_3 = \dfrac{1}{N_c} S_i G_{i8} - \dfrac{1}{2\sqrt{3}} O_3$	

TABLE I: Operators of Eq. (8) and coefficients resulting from the fit with $\chi^2_{\text{dof}} \simeq 0.26$.

	O_1	O_2	O_3
$^2 8_{7/2}$	N_c	$-\dfrac{5}{2N_c}$	$\dfrac{3}{4N_c}$
$^2 8_{9/2}$	N_c	$\dfrac{2}{N_c}$	$\dfrac{3}{4N_c}$
$^4 10_{5/2}$	N_c	$-\dfrac{15}{2N_c}$	$\dfrac{15}{4N_c}$
$^4 10_{7/2}$	N_c	$-\dfrac{4}{N_c}$	$\dfrac{15}{4N_c}$
$^4 10_{9/2}$	N_c	$\dfrac{1}{2N_c}$	$\dfrac{15}{4N_c}$
$^4 10_{11/2}$	N_c	$\dfrac{6}{N_c}$	$\dfrac{15}{4N_c}$

TABLE II: Matrix elements of SU(3) singlet operators.

	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$
N_J	0	0	0
Λ_J	1	$\dfrac{\sqrt{3}\,a_J}{2N_c}$	$-\dfrac{3\sqrt{3}}{8N_c}$
Σ_J	1	$-\dfrac{\sqrt{3}\,a_J}{6N_c}$	$\dfrac{\sqrt{3}}{8N_c}$
Ξ_J	2	$\dfrac{2\sqrt{3}\,a_J}{3N_c}$	$-\dfrac{\sqrt{3}}{2N_c}$
Δ_J	0	0	0
Σ_J	1	$\dfrac{\sqrt{3}\,b_J}{2N_c}$	$-\dfrac{5\sqrt{3}}{8N_c}$
Ξ_J	2	$\dfrac{\sqrt{3}\,b_J}{N_c}$	$-\dfrac{5\sqrt{3}}{4N_c}$
Ω_J	3	$\dfrac{3\sqrt{3}\,b_J}{2N_c}$	$-\dfrac{15\sqrt{3}}{8N_c}$
$\Sigma^8_{7/2}-\Sigma^{10}_{7/2}$	0	$-\dfrac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Sigma^8_{9/2}-\Sigma^{10}_{9/2}$	0	$-\dfrac{\sqrt{11}}{\sqrt{3}N_c}$	0
$\Xi^8_{7/2}-\Xi^{10}_{7/2}$	0	$-\dfrac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Xi^8_{9/2}-\Xi^{10}_{9/2}$	0	$-\dfrac{\sqrt{11}}{\sqrt{3}N_c}$	0

TABLE III: Matrix elements of SU(3) breaking operators, with $a_J = 5/2, -2$ for $J = 7/2, 9/2$ respectively and $b_J = 5/2, 4/3, -1/6, -2$ for $J = 5/2, 7/2, 9/2, 11/2$, respectively.

(1)	$9(\Delta_{7/2} - \Delta_{5/2}) \ = \ 7(N_{9/2} - N_{7/2})$
(2)	$9(\Delta_{9/2} - \Delta_{5/2}) \ = \ 16(N_{9/2} - N_{7/2})$
(3)	$9(\Delta_{11/2} - \Delta_{9/2}) \ = \ 11(N_{9/2} - N_{7/2})$
(4)	$8(\Lambda_{7/2} - N_{7/2}) + 14(N_{9/2} - \Lambda_{9/2}) \ = \ 3(\Lambda_{9/2} - \Sigma_{9/2}) + 6(\Delta_{11/2} - \Sigma_{11/2})$
(5)	$\Lambda_{9/2} - \Lambda_{7/2} + 3(\Sigma_{9/2} - \Sigma_{7/2}) \ = \ 4(N_{9/2} - N_{7/2})$
(6)	$\Lambda_{9/2} - \Lambda_{7/2} + \Sigma_{9/2} - \Sigma_{7/2} \ = \ 2(\Sigma'_{9/2} - \Sigma'_{7/2})$
(7)	$11 \ \Sigma'_{7/2} + 9 \ \Sigma_{11/2} \ = \ 20 \ \Sigma'_{9/2}$
(8)	$20 \ \Sigma_{5/2} + 7 \ \Sigma_{11/2} \ = \ 27 \ \Sigma'_{7/2}$
(GMO)	$2(N + \Xi) \ = \ 3 \ \Lambda + \Sigma$
(EQS)	$\Sigma - \Delta \ = \ \Xi - \Sigma = \Omega - \Xi$

TABLE IV: The 18 independent mass relations including the GMO relations for the octets and the EQS relations for the decuplets.

	1/N_c expansion results					
	Partial contribution (MeV)				Total (MeV)	Empirical	Name, status
	c_1O_1	c_2O_2	c_3O_3	$b_1\bar{B}_1$		(MeV)	
$N_{7/2}$	2209	-3	34	0	2240 ± 97		
$\Lambda_{7/2}$				110	2350 ± 118		
$\Sigma_{7/2}$				110	2350 ± 118		
$\Xi_{7/2}$				220	2460 ± 166		
$N_{9/2}$	2209	2	34	0	2245 ± 95	2245 ± 65	N(2220)****
$\Lambda_{9/2}$				110	2355 ± 116	2355 ± 15	Λ(2350)***
$\Sigma_{9/2}$				110	2355 ± 116		
$\Xi_{9/2}$				220	2465 ± 164		
$\Delta_{5/2}$	2209	-9	168	0	2368 ± 175		
$\Sigma_{5/2}$				110	2478 ± 187		
$\Xi_{5/2}$				220	2588 ± 220		
$\Omega_{5/2}$				330	2698 ± 266		
$\Delta_{7/2}$	2209	-5	168	0	2372 ± 153	2387 ± 88	Δ(2390)*
$\Sigma'_{7/2}$				110	2482 ± 167		
$\Xi'_{7/2}$				220	2592 ± 203		
$\Omega_{7/2}$				330	2702 ± 252		
$\Delta_{9/2}$	2209	1	168	0	2378 ± 144	2318 ± 132	Δ(2300)**
$\Sigma'_{9/2}$				110	2488 ± 159		
$\Xi'_{9/2}$				220	2598 ± 197		
$\Omega_{9/2}$				330	2708 ± 247		
$\Delta_{11/2}$	2209	7	168	0	2385 ± 164	2400 ± 100	Δ(2420)****
$\Sigma_{11/2}$				110	2495 ± 177		
$\Xi_{11/2}$				220	2605 ± 212		
$\Omega_{11/2}$				330	2715 ± 260		

TABLE V: The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion as compared with the empirically known masses.

Baryon	ϕ^λ	ϕ^ρ
p	$-\frac{1}{\sqrt{6}}\left(udu + duu - 2uud\right)$	$\frac{1}{\sqrt{2}}\left(udu - duu\right)$
n	$\frac{1}{\sqrt{6}}\left(udd + dud - 2ddu\right)$	$\frac{1}{\sqrt{2}}\left(udd - dud\right)$
Σ^+	$\frac{1}{\sqrt{6}}\left(usu + suu - 2uus\right)$	$-\frac{1}{\sqrt{2}}\left(usu - suu\right)$
Σ^0	$-\frac{1}{\sqrt{12}}\left(2uds + 2dus - sdu - sud - usd - dsu\right)$	$-\frac{1}{2}\left(usd + dsu - sdu - sud\right)$
Σ^-	$\frac{1}{\sqrt{6}}\left(dsd + sdd - 2dds\right)$	$-\frac{1}{\sqrt{2}}\left(dsd - sdd\right)$
Λ^0	$\frac{1}{2}\left(sud - sdu + usd - dsu\right)$	$\frac{1}{\sqrt{12}}\left(2uds - 2dus + sdu - sud + usd - dsu\right)$
Ξ^0	$-\frac{1}{\sqrt{6}}\left(uss + sus - 2ssu\right)$	$-\frac{1}{\sqrt{2}}\left(uss - sus\right)$
Ξ^-	$-\frac{1}{\sqrt{6}}\left(dss + sds - 2ssd\right)$	$-\frac{1}{\sqrt{2}}\left(dss - sds\right)$

TABLE VI: Mixed symmetry flavor states of three quarks for the baryon octet.

Baryon	ϕ^S
Δ^{++}	uuu
Δ^{+}	$\frac{1}{\sqrt{3}}(uud + udu + duu)$
Δ^{0}	$\frac{1}{\sqrt{3}}(udd + dud + ddu)$
Δ^{-}	ddd
Σ^{+}	$\frac{1}{\sqrt{3}}(uus + usu + suu)$
Σ^{0}	$\frac{1}{\sqrt{6}}(uds + dus + usd + sud + sdu + dsu)$
Σ^{-}	$\frac{1}{\sqrt{3}}(sdd + dsd + dds)$
Ξ^{0}	$\frac{1}{\sqrt{3}}(uss + sus + ssu)$
Ξ^{-}	$\frac{1}{\sqrt{3}}(dss + sds + ssd)$
Ω^{-}	sss

TABLE VII: Symmetric flavor states of three quarks for the baryon decuplet.