Baryon resonances in large N_c QCD

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Abstract

The baryon spectra are discussed in the context of the $1/N_c$ expansion approach, with emphasis on mixed symmetric states. The contributions of the spin dependent terms as a function of the excitation energy are shown explicitly. At large energies these contributions are expected to vanish.

1 Introduction

At energies corresponding to a length scale of the order of the hadron size the standard perturbative QCD cannot be applied, because the coupling constant is too large. In the nonperturbative regime one can use the so-called $1/N_c$ expansion approach which is based on the 32 years old idea of 't Hooft [1], who suggested a perturbative expansion of QCD in terms of the parameter $1/N_c$ where N_c is the number of colors. The double line diagrammatic method proposed by 't Hooft has been implemented by Witten [2] to describe hadrons by using convenient power counting rules for Feynman diagrams. According to Witten's intuitive picture, a baryon containing N_c quarks is seen as a bound state in an average self-consistent potential of a Hartree type and the corrections to the Hartree approximation are of order $1/N_c$ which means that in the $N_c \to \infty$ limit the Hartree approximation is exact. Ground state baryons correspond to the ground state of the average potential.

Ten years after 't Hooft's work, Gervais and Sakita [3] and independently Dashen and Manohar in 1993 [4] discovered that QCD has an exact contracted $\mathrm{SU}(2N_f)_c$ symmetry when $N_c \to \infty$, N_f being the number of flavors. The contracted algebra generators acting in the spin-flavour space X^{ia} are related to the $\mathrm{SU}(2N_f)$ generators G^{ia} in the limit $N_c \to \infty$ by

$$X^{ia} = \lim_{N_c \to \infty} \frac{G^{ia}}{N_c}. \tag{1}$$

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For ground state baryons the SU($2N_f$) symmetry is broken by corrections proportional to $1/N_c$. Applications to ground state QCD baryons ($N_c = 3$) were considered since 1993-1994. Presently the $1/N_c$ expansion provides a systematic method to analyze baryon properties such as masses, magnetic moments, axial currents, etc.

The $1/N_c$ expansion method has been extended to excited states since 1997 in the spirit of the Hartree approximation developed by Witten [8]. It was shown that the SU($2N_f$) breaking occurs at order N_c^0, at variance with the ground state. This conflict generated a conceptual problem, presently under investigation.

Here we are concerned with baryon spectra only. If the SU(N_f) symmetry is exact, the baryon mass operator is a linear combination of terms

$$M = \sum_i c_i O_i, \tag{2}$$

with the operators O_i having the general form

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{3}$$

where $O_\ell^{(k)}$ is a k-rank tensor in SO(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2), but invariant in SU(N_f). The latter is expressed in terms of SU(N_f) generators. For the ground state one has $k = 0$. The first factor gives the order $\mathcal{O}(1/N_c)$ of the operator in the series expansion and reflects Witten's power counting rules. The lower index i represents a specific combination of generators, see examples below. In the linear combination, Eq. (2), each term of type (3) is multiplied by an unknown coefficient c_i which is a reduced matrix element. All these coefficients encode the QCD dynamics and are obtained from a fit to the existing data. It is important to find regularities in their behaviour, as shown below. Additional terms are needed if SU(N_f) is broken, as it is the case for $N_f = 3$ [15].

2 The ground state

A considerable amount of work has been devoted to ground state baryons summarized in several review papers as, for example, [5, 6, 7]. The ground state is described by the symmetric representation $[N_c]$. For $N_c = 3$ this becomes [3] or [**56**] in an SU(6) dimensional notation. Let us consider below the simple case of two flavours, *i.e.* SU(4). Its algebra is

$$\begin{aligned} [S_i, S_j] &= i\varepsilon_{ijk} S_k, \qquad [T_a, T_b] = i\varepsilon_{abc} T_c, \\ [G_{ia}, G_{jb}] &= \tfrac{i}{4}\delta_{ij}\varepsilon_{abc} T_c + \tfrac{i}{2}\delta_{ab}\varepsilon_{ijk} S_k. \end{aligned} \tag{4}$$

As SU(4) is a group of rank 3 it has three invariants: S^2, I^2 and G^2. *i.e.* three operators of type (3). But for the ground state one can take $I^2 = S^2$ in SU(4). Moreover due to the operator identity [6]

$$\{J^i, J^i\} + \{I^a, I^a\} + 4\{G^{ia}, G^{ia}\} = \frac{3}{2} N_c (N_c + 4) \tag{5}$$

the invariant G^2 can be expressed in terms of S^2 and I^2. So, one is left with one linearly independent operator, which we choose to be S^2. Accordingly, the mass formula takes the following simple form

$$M = m_0 N_c + m_2 \frac{1}{N_c} S^2 + m_4 \frac{1}{N_c^3} (S^2)^2 + \ldots + m_{N_c-1} \frac{1}{N_c^{N_c-2}} (S^2)^{N_c-3}. \qquad (6)$$

This describes a tower of large N_c baryon states with $S = 1/2, ..., N_c/2$, which collapses into a degenerate state when $N_c \to \infty$. One can see that the splitting starts at order $1/N_c$ when SU($2N_f$) is broken. The coefficients $c_i = m_i$ must be fitted from the data.

3 The excited states

One expects 't Hooft's suggestion [1] to hold in all QCD regimes. Accordingly, the applicability of the $1/N_c$ expansion method to excited states is a subject of current investigation. The experimental facts indicate a small breaking of SU(6) which make the $1/N_c$ studies of excited states entirely plausible. The general form of a mass operator is given by Eq. (2) with O_i defined as in Eq. (3). For simplicity, here we discuss nonstrange baryons where SU(4) symmetry is exact.

Excited baryons can be divided into SU(6) multiplets, as in the constituent quark model. If an excited baryon belongs to the [**56**]-plet the mass problem can be treated similarly to the ground state in the flavour-spin degrees of freedom, but one has to take into account the presence of an orbital excitation in the space part of the wave function [9, 10]. If the baryon belongs to the mixed symmetric representation [21], or [**70**] in SU(6) notation, the treatment becomes much more complicated. In particular, the resonances up to 2 GeV belong to [$\mathbf{70}, 1^-$], [$\mathbf{70}, 0^+$] or [$\mathbf{70}, 2^+$] multiplets.

There is one standard way to study the [**70**]-plets which is related to the Hartree approximation [8]. This consists in reducing the description of an excited baryon to that of an excited quark coupled to a symmetric core, see *e.g.* [11, 12, 13, 15]. In that case the core can be treated in a way similar to that of the ground state. In this method each SU($2N_f$) $\times$ O(3) generator is splitted into two terms

$$S^i = s^i + S^i_c; \quad T^a = t^a + T^a_c; \quad G^{ia} = g^{ia} + G^{ia}_c, \quad \ell^i = \ell^i_q + \ell^i_c, \qquad (7)$$

where s^i, t^a, g^{ia} and ℓ^i_q are the excited quark operators and S^i_c, T^a_c, G^{ia}_c and ℓ^i_c the corresponding core operators. As an example, we discuss the latest in date results, for nonstrange baryons belonging to the [$\mathbf{70}, \ell^+$] multiplets with $\ell = 0$ and 2. The list of the dominant operators up to order $1/N_c$ is given in Table 1 together with the values of the coefficients c_i obtained from the data. It is customary to drop corrections of order $1/N_c^2$. In this list, the first is the trivial operator of order $\mathcal{O}(N_c)$. The second is the 1-body part of the spin-orbit operator of order $\mathcal{O}(1)$ which acts on the excited quark. The third is a composite 2-body operator formally of order $\mathcal{O}(1)$ as well. It involves the tensor operator

$$\ell^{(2)ij}_q = \frac{1}{2}\left\{\ell^i_q, \ell^j_q\right\} - \frac{1}{3}\delta_{i,-j}\vec{\ell}_q \cdot \vec{\ell}_q \ , \qquad (8)$$

Operator	Fitted coef. (MeV)			
$O_1 = N_c \mathbb{1}$	$c_1 =$	555	$\pm$	11
$O_2 = \ell_q^i s^i$	$c_2 =$	47	$\pm$	100
$O_3 = \frac{3}{N_c} \ell_q^{(2)ij} g^{ia} G_c^{ja}$	$c_3 =$	-191	$\pm$	132
$O_4 = \frac{1}{N_c}(S_c^i S_c^i + s^i S_c^i)$	$c_4 =$	261	$\pm$	47

Table 1: List of operators and the coefficients resulting from the fit of nonstrange baryon masses assumed to belong to the $[\mathbf{70}, 0^+]$- and $[\mathbf{70}, 2^+]$-plets. The fit gave $\chi^2_{\mathrm{dof}} \simeq 0.83$ [13].

acting on the excited quark and the SU(6) generators g^{ia} acting on the excited quark and G_c^{ja} acting on the core. The latter is a coherent operator which introduces an extra power N_c so that the order of O_3 is $\mathcal{O}(1)$.

In this procedure, there are two major drawbacks, related to each other. The first is that the number of linearly independent operators constructed from the generators given in the right-hand side of Eqs. (7) increases tremendously so that the number of coefficients to be determined becomes much larger than the experimental data available. Consequently, in selecting the most dominant operators one has to make an arbitrary choice, as we did in Table 1, similarly to previous literature. The second drawback is related to the truncation of the available basis vector space. Among the basis vectors of the irreducible representation $[N_c - 1, 1]$ of S_{N_c}, in this procedure only the vector corresponding to the normal Young tableau is kept, the reason being to decouple the system into a symmetric core and an excited quark. In a normal Young tableau this can be easily done by removing the N_c-th particle from the second row. The terms represented by the other possible Young tableaux, needed to construct a symmetric orbital-flavour-spin state are neglected, *i.e.* antisymmetry is ignored. As a result the procedure brings in terms of order N_c^0, which is in conflict with the $1/N_c$ expansion for the ground state.

A solution to this problem has been found in Ref. [16], where the separation into a symmetric core and an excited quark is avoided through the calculation of the matrix elements of the SU(4) generators by using a generalized Wigner-Eckart theorem [17]. In this way the antisymmetry is properly taken into account. The result is that the $1/N_c$ expansion starts at order $1/N_c$, as for the ground state.

Based on group theory arguments it is expected that the mass splitting starts at order $1/N_c$, as a general rule, irrespective of the angular momentum and parity of the state and also of the number of flavours, provided SU(N_f) is an exact symmetry.

Despite the drawbacks of the splitting method, the application of the $1/N_c$ expansion method gave useful results, as a first approximation. They predicted the behaviour of the coefficients c_i in the mass formula as a function of the excitation energy. This is illustrated in Fig. 1. This figure suggests that the spin-orbit and the spin-spin terms vanish with the excitation energy, bringing a strong support to constituent quark models and that the spin-spin term is dominant among all the other spin dependent terms. Note that in a quark

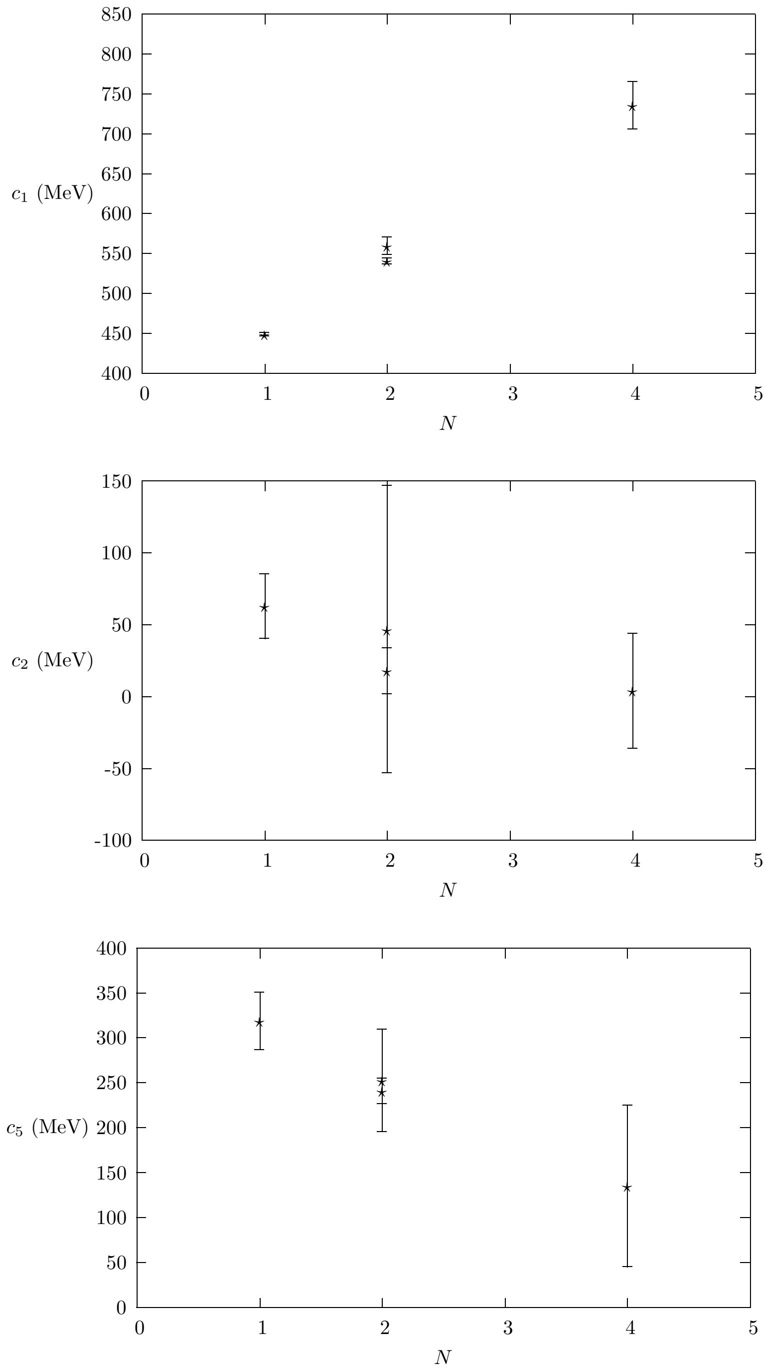

Figure 1: The coefficients c_i vs N from various sources: for $N = 1$ from Ref. [12] for $N = 2$ from Refs. [9] (lower values) and [13] (upper values), for $N = 4$ from Ref. [10].

model picture, the coefficient c_1 would correspond to the additional contribution of a free mass term, the kinetic energy and the confinement. It is not thus surprising that it raises with the excitation energy.

4 Conclusion

The $1/N_c$ expansion method provides a powerful theoretical tool to analyze the spin-flavour symmetry of baryons and explains the succes of models based on spin-flavor symmetry.

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