

# THE NUCLEON-NUCLEON INTERACTION IN A CHIRAL CONSTITUENT QUARK MODEL

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## Abstract

We study the short-range nucleon-nucleon interaction in a chiral constituent quark model by diagonalizing a Hamiltonian comprising a linear confinement and a Goldstone boson exchange interaction between quarks. The six-quark harmonic oscillator basis contains up to two excitation quanta. We show that the highly dominant configuration is  $|s^4p^2[42]_O[51]_{FS}\rangle$  due to its specific flavour-spin symmetry. Using the Born-Oppenheimer approximation we find a strong effective repulsion at zero separation between nucleons in both  ${}^3S_1$  and  ${}^1S_0$  channels. The symmetry structure of the highly dominant configuration implies the existence of a node in the S-wave relative motion wave function at short distances. The amplitude of the oscillation of the wave function at short range will be however strongly suppressed. We discuss the mechanism leading to the effective short-range repulsion within the chiral constituent quark model as compared to that related with the one-gluon exchange interaction.

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## I. INTRODUCTION

An interest in the constituent quark model (CQM) has recently been revitalized [1] after recognizing the fact that the constituent (dynamical) mass of the light quarks appears as a direct consequence of the spontaneous chiral symmetry breaking (SCSB) [2,3] and is related with the light quark condensates  $\langle \bar{q}q \rangle$  of the QCD vacuum. This feature becomes explicit in any microscopical approach to SCSB in QCD, e.g. in the instanton gas (liquid) model [4]. The mechanism of the dynamical mass generation in the Nambu-Goldstone mode of chiral symmetry is very transparent within the  $\sigma$ -model [5] or Nambu and Jona-Lasinio model [6]. Another consequence of the chiral symmetry in the Nambu-Goldstone mode is the appearance of an octet of Goldstone bosons ( $\pi, K, \eta$  mesons). It was suggested in [1] that beyond the scale of SCSB, nonstrange and strange baryons should be viewed as systems of three constituent quarks which interact via the exchange of Goldstone bosons and are subject to confinement. This type of interaction between the constituent quarks provides a very satisfactory description of the low-lying nonstrange and strange baryon spectra [1,7,8] including the correct ordering of the levels with positive and negative parity in all parts of the considered spectrum.

So far, all studies of the short-range NN interaction within the constituent quark model were based on the one-gluon exchange interaction (OGE) between quarks. They explained the short-range repulsion in the NN system as due to the colour-magnetic part of OGE combined with quark interchanges between 3q clusters. (For reviews and earlier references see [9–11]). There are also models which attribute the short-range repulsion in the NN system to the colour-electric part of OGE [12].

In order to provide the necessary long- and intermediate-range attraction in the baryon-baryon system, hybrid models were suggested [13–15], where in addition to OGE, the quarks belonging to different 3q clusters interact via pseudoscalar and scalar meson exchange. In these hybrid models the short-range repulsion in the NN system is still attributed to OGE between the constituent quarks.

It has been shown, however [1,16], that the hyperfine splittings as well as the correct ordering of positive and negative parity states in spectra of baryons with u,d,s quarks are produced in fact not by the colour-magnetic part of OGE, but by the short-range part of the Goldstone boson exchange (GBE) interaction. This short-range part of GBE has just opposite sign as compared to the Yukawa tail of the GBE interaction and is much stronger at short interquark separations. There is practically no room for OGE in light baryon spectroscopy and any appreciable amount of colour-magnetic interaction, in addition to GBE, destroys the spectrum [8]. If so, the question arises which interquark interaction is responsible for the short-range NN repulsion. The goal of this paper is to show that the same short-range part of GBE, which produces good baryon spectra, also induces a short-range repulsion in the NN system.

The present study is rather exploratory. We calculate an effective NN interaction at zero separation distance only. We also want to stress that all main ingredients of the NN interaction, such as the long- and middle-range attraction and the short-range repulsion are implied by the chiral constituent quark model. Indeed, the long- and middle-range attraction automatically appear in the present framework due to the long-range Yukawa tail of the pion-exchange interaction between quarks belonging to different nucleons and due to  $2\pi$  (or sigma) exchanges. Thus, the only important open question is whether or not the chiral constituent quark model is able to produce a short-range repulsion in the NN system.

For this purpose, we diagonalize the Hamiltonian of Ref. [7] in a six-quark harmonic oscillator basis up to two excitations quanta. Using the Born-Oppenheimer (adiabatic) approximation, we obtain an effective internucleon potential at zero separation between nucleons from the difference between the lowest eigenvalue and two times the nucleon mass calculated in the same model. We find a strong effective repulsion between nucleons in both  $^3S_1$  and  $^1S_0$  channels of a height of 800-1300 MeV. This repulsion implies a strong suppression of the NN wave function in the nucleon overlap region as compared to the wave function of the well separated nucleons.

Due to the specific flavour-spin symmetry of the GBE interaction, we also find

that the highly dominant  $6q$  configuration at zero-separation between nucleons is  $|s^4p^2[42]_O[51]_{FS} \rangle$ . As a consequence the  $6q$  region (i.e. the nucleon overlap region) cannot be adequately represented by the one-channel resonating group method (RGM) ansatz  $\hat{A}\{N(1, 2, 3)N(4, 5, 6)\chi(\vec{r})\}$  which is commonly used at present for the short-range NN interaction with the OGE interaction.

The symmetry structure  $[42]_O[51]_{FS}$  of the lowest configuration will induce an additional effective repulsion at short range related to the ‘‘Pauli forbidden state’’ in this case. This latter effective repulsion is not related to the energy of the lowest configuration as compared to two-nucleon threshold and thus cannot be obtained within the Born-Oppenheimer approximation procedure. We notice, however, that the structure of the six-quark wave function in the nucleon overlap region is very different from the one associated with the soft or hard core  $NN$  potentials.

This paper is organized as follows. In section 2, in a qualitative analysis at the Casimir operator level, we show that the short-range GBE interaction generates a repulsion between nucleons in both  ${}^3S_1$  and  ${}^1S_0$  channels. We also suggest there that the configuration with the  $[51]_{FS}$  flavour-spin symmetry should be the dominant one. Section 3 describes the Hamiltonian. Section 4 contains results of the diagonalization of the  $6q$  Hamiltonian and of the NN effective interaction at zero separation between nucleons. The structure of the short-range wave function is also discussed in this section. In section 5, we show why the single-channel RGM ansatz is not adequate in the present case. In section 6, we present a summary of our study.

## II. A QUALITATIVE ANALYSIS AT THE CASIMIR OPERATOR LEVEL

In order to have a preliminary qualitative insight it is convenient first to consider a schematic model which neglects the radial dependence of the GBE interaction. In this model the short-range part of the GBE interaction between the constituent quarks is approximated by [1]

$$V_\chi = -C_\chi \sum_{i < j} \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (1)$$

where  $\lambda^F$  with an implied summation over F (F=1,2,...,8) and  $\vec{\sigma}$  are the quark flavour Gell-Mann and spin matrices respectively. The minus sign of the interaction (1) is related to the sign of the short-range part of the pseudoscalar meson-exchange interaction (which is opposite to that of the Yukawa tail), crucial for the hyperfine splittings in baryon spectroscopy. The constant  $C_\chi$  can be determined from the  $\Delta - N$  splitting. For that purpose one only needs the spin (S), flavour (F) and flavour-spin (FS) symmetries of the  $N$  and  $\Delta$  states, identified by the corresponding partitions [f] associated with the groups  $SU(2)_S, SU(3)_F$  and  $SU(6)_{FS}$  :

$$|N \rangle = |s^3[3]_{FS}[21]_F[21]_S \rangle, \quad (2)$$

$$|\Delta \rangle = |s^3[3]_{FS}[3]_F[3]_S \rangle. \quad (3)$$

Then the matrix elements of the interaction (1) are [1] :

$$\langle N | V_\chi | N \rangle = -14C_\chi, \quad (4)$$

$$\langle \Delta | V_\chi | \Delta \rangle = -4C_\chi. \quad (5)$$

Hence  $E_\Delta - E_N = 10C_\chi$ , which gives  $C_\chi = 29.3\text{MeV}$ , if one uses the experimental value of 293 MeV for the  $\Delta - N$  splitting.

To see the effect of the interaction (1) in the six-quark system, the most convenient is to use the coupling scheme called FS, where the spatial  $[f]_O$  and colour  $[f]_C$  parts are coupled together to  $[f]_{OC}$ , and then to the  $SU(6)_{FS}$  flavour-spin part of the wave function in order to provide a totally antisymmetric wave function in the OCFS space [18]. The antisymmetry condition requires  $[f]_{FS} = [\tilde{f}]_{OC}$ , where  $[\tilde{f}]$  is the conjugate of  $[f]$ .

The colour-singlet 6q state is  $[222]_C$ . Assuming that N has a  $[3]_O$  spatial symmetry, there are two possible states  $[6]_O$  and  $[42]_O$  compatible with the S-wave relative motion in the NN system [17]. The flavour and spin symmetries are  $[42]_F$  and  $[33]_S$  for  $^1S_0$  and  $[33]_F$  and  $[42]_S$  for  $^3S_1$  channels. Applying the inner product rules of the symmetric group for

both the  $[f]_O \times [f]_C$  and  $[f]_F \times [f]_S$  products one arrives at the following 6q antisymmetric states associated with the  ${}^3S_1$  and  ${}^1S_0$  channels [18,19] :  $|[6]_O[33]_{FS} \rangle$ ,  $|[42]_O[33]_{FS} \rangle$ ,  $|[42]_O[51]_{FS} \rangle$ ,  $|[42]_O[411]_{FS} \rangle$ ,  $|[42]_O[321]_{FS} \rangle$ ,  $|[42]_O[2211]_{FS} \rangle$ .

Then the expectation values of the GBE interaction (1) for these states can be easily calculated in terms of the Casimir operators eigenvalues for the groups  $SU(6)_{FS}$ ,  $SU(3)_F$  and  $SU(2)_S$  using the formula given in Appendix A. The corresponding matrix elements are given in Table 1, from where one can see that, energetically, the most favourable configuration is  $[51]_{FS}$ . This is a direct consequence of the general rule that at short range and with fixed spin and flavour, the more ‘‘symmetric’’ a given FS Young diagram is, the more negative is the expectation value of (1) [1]. The difference in the potential energy between the configuration  $[51]_{FS}$  and  $[33]_{FS}$  or  $[411]_{FS}$  is of the order :

$$\begin{aligned} & \langle [33]_{FS} | V_\chi | [33]_{FS} \rangle - \langle [51]_{FS} | V_\chi | [51]_{FS} \rangle = \\ & \langle [411]_{FS} | V_\chi | [411]_{FS} \rangle - \langle [51]_{FS} | V_\chi | [51]_{FS} \rangle = 24C_\chi \end{aligned} \quad (6)$$

and using  $C_\chi$  given above one obtains approximately 703 MeV for both the SI = 10 and 01 sectors.

In a harmonic oscillator basis containing up to  $2\hbar\omega$  excitation quanta, there are two different 6q states corresponding to the  $[6]_O$  spatial symmetry with removed center of mass motion. One of them,  $|s^6[6]_O \rangle$ , belongs to the  $N = 0$  shell, where  $N$  is the number of excitation quanta in the system, and the other,  $\sqrt{\frac{5}{6}}|s^5 2s[6]_O \rangle - \sqrt{\frac{1}{6}}|s^4 p^2[6]_O \rangle$ , belongs to the  $N = 2$  shell. There is only one state with  $[42]_O$  symmetry, the  $|s^4 p^2[42]_O \rangle$  state belonging to the  $N=2$  shell. While here and below we use notations of the shell model it is always assumed that the center of mass motion is removed.

The kinetic energy KE for the  $|s^4 p^2[42]_O \rangle$  state is larger than the one for the  $|s^6[6]_O \rangle$  state by  $KE_{N=2} - KE_{N=0} = \hbar\omega$ . Taking  $\hbar\omega \simeq 250$  MeV [1], and denoting the kinetic energy operator by  $H_0$ , we obtain :

$$\langle s^6[33]_{FS} | H_0 + V_\chi | s^6[33]_{FS} \rangle - \langle s^4 p^2[51]_{FS} | H_0 + V_\chi | s^4 p^2[51]_{FS} \rangle \simeq 453 \text{ MeV} \quad (7)$$

which shows that  $[51]_{FS}$  is far below the other states of Table 1. For simplicity, here we

have neglected a small difference in the confinement potential energy between the above configurations.

This qualitative analysis suggests that in a more quantitative study, where the radial dependence of the GBE interaction is taken into account, the state  $|s^4p^2[42]_O[51]_{FS} \rangle$  will be highly dominant and, due to the important lowering of this state by the GBE interaction with respect to the other states, the mixing angles with these states will be small. That this is indeed the case, it will be proved in the section 4 below.

Table 1 and the discussion above indicate that the following configurations should be taken into account for the diagonalization of the realistic Hamiltonian in section 4:

$$\begin{aligned}
|1 \rangle &= |s^6[6]_O[33]_{FS} \rangle \\
|2 \rangle &= |s^4p^2[42]_O[33]_{FS} \rangle \\
|3 \rangle &= |s^4p^2[42]_O[51]_{FS} \rangle \\
|4 \rangle &= |s^4p^2[42]_O[411]_{FS} \rangle \\
|5 \rangle &= |(\sqrt{\frac{5}{6}}s^52s - \sqrt{\frac{1}{6}}s^4p^2)[6]_O[33]_{FS} \rangle
\end{aligned} \tag{8}$$

A strong dominance of the configuration  $|3 \rangle$  also implies that the one-channel approximation  $\hat{A}\{NN\chi(\vec{r})\}$  is highly inadequate for the short-range NN system. This problem will be discussed in section 5.

Now we want to give a rough estimate of the interaction potential of the NN system at zero separation distance between nucleons. We calculate this potential in the Born-Oppenheimer (or adiabatic) approximation defined as :

$$V_{NN}(R) = \langle H \rangle_R - \langle H \rangle_\infty \tag{9}$$

where  $\vec{R}$  is a collective coordinate which is the separation distance between the two  $s^3$  nucleons,  $\langle H \rangle_R$  is the lowest expectation value of the Hamiltonian describing the 6q system at fixed  $R$  and  $\langle H \rangle_\infty = 2m_N$  for the NN problem, i.e. the energy of two well separated nucleons. As above we ignore the small difference between the confinement energy

of  $\langle H \rangle_{R=0}$  and  $\langle H \rangle_{\infty}$ . That this difference is small follows from the  $\lambda_i^c \cdot \lambda_j^c$  structure of the confining interaction and from the identity :

$$\langle [222]_c | \sum_{i<j}^6 \lambda_i^c \cdot \lambda_j^c | [222]_c \rangle = 2 \langle [111]_c | \sum_{i<j}^3 \lambda_i^c \cdot \lambda_j^c | [111]_c \rangle . \quad (10)$$

If the space parts  $[6]_O$  and  $[3]_O$  contain the same single particle state, for example an s-state, then the difference is identically zero.

It has been shown by Harvey [18] that when the separation  $R$  between two  $s^3$  nucleons approaches 0, then only two types of 6q configurations survive:  $|s^6[6]_O \rangle$  and  $|s^4p^2[42]_O \rangle$ . Thus in order to extract an effective NN potential at zero separation between nucleons in adiabatic Born-Oppenheimer approximation one has to diagonalize the Hamiltonian in the basis  $|1 \rangle - |4 \rangle$ . In actual calculations in section 4 we extend the basis adding the configuration  $|5 \rangle$ , which practically does not change much the result. For the rough estimate below we take only the lowest configuration  $|3 \rangle$ . One then obtains

$$\begin{aligned} \langle s^4p^2[42]_O[51]_{FS} | H_0 + V_\chi | s^4p^2[42]_O[51]_{FS} \rangle - 2 \langle N | H_0 + V_\chi | N \rangle = \\ \left\{ \begin{array}{l} (-100/3 + 28)C_\chi + 7/4\hbar\omega = 280 \text{ MeV, if } SI = 10 \\ (-32 + 28)C_\chi + 7/4\hbar\omega = 320 \text{ MeV, if } SI = 01 \end{array} \right. \quad (11) \end{aligned}$$

The rough estimate (11) suggests that there is an effective repulsion of approximately equal magnitude in the NN system in the nucleon overlap region in both  ${}^3S_1$  and  ${}^1S_0$  channels. In a more quantitative calculation in Section 4 we find that the height of the effective core is much larger, in particular 830 MeV for  ${}^3S_1$  and about 1.3 GeV for  ${}^1S_0$ .

At this stage it is useful to compare the nature of the short-range repulsion generated by the GBE interaction to that produced by the OGE interaction.

In the constituent quark models based on OGE the situation is more complex. Table 1 helps in summarizing the situation there. In this table we also give the expectation value of the simplified chromo-magnetic interaction

$$V_{cm} = -C_{cm} \sum_{i<j} \lambda_i^c \cdot \lambda_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j \quad (12)$$



in units of the constant  $C_{cm}$  (the constant  $C_{cm}$  can also be determined from the  $\Delta - N$  splitting to be  $C_{cm} \simeq 293/16$  MeV).

The expectation values of (12) can be easily obtained in the CS scheme with the help of Casimir operator formula in Appendix A and can be transformed to FS scheme by using the unitary transformations from the CS scheme to the FS scheme given in Appendix B.

The colour-magnetic interaction pulls the configuration  $|s^4p^2[42]_O[42]_{CS} \rangle$  down to become approximately degenerate with  $|s^6[6]_O[222]_{CS} \rangle$  which is pulled up. In a more detailed calculation with explicit radial dependence of the colour-magnetic interaction as well as with a Coulomb term the configuration  $|s^6[6]_O \rangle$  is still the lowest one [9,20]. (With the model (12) the  $\hbar\omega$  should be about 500 MeV). Thus in the Born-Oppenheimer approximation we can roughly estimate an effective interaction with OGE model through the difference

$$\begin{aligned} & \langle s^6[6]_O[222]_{CS} | H_0 + V_{cm} | s^6[6]_O[222]_{CS} \rangle - 2 \langle N | H_0 + V_{cm} | N \rangle \\ &= \begin{cases} \frac{56}{3}C_{cm} + 3/4\hbar\omega = 717MeV & \text{if } SI = 10 \\ 24C_{cm} + 3/4\hbar\omega = 815MeV & \text{if } SI = 01 \end{cases} \end{aligned} \quad (13)$$

We conclude that both the GBE and OGE models imply effective repulsion at short range of approximately same magnitude.

### III. THE HAMILTONIAN

In this section we present the GBE model [1,7] used in the diagonalization of six-quark Hamiltonian in the basis (8). The Hamiltonian reads

$$H = 6m + \sum_i \frac{\vec{p}_i^2}{2m} - \frac{(\sum_i \vec{p}_i)^2}{12m} + \sum_{i < j} V_{conf}(r_{ij}) + \sum_{i < j} V_\chi(r_{ij}) \quad (14)$$

where  $m$  is the constituent quark mass;  $r_{ij} = |\vec{r}_i - \vec{r}_j|$  is the interquark distance.

The confining interaction is

$$V_{conf}(r_{ij}) = -\frac{3}{8}\lambda_i^c \cdot \lambda_j^c C r_{ij} \quad (15)$$

where  $\lambda_i^c$  are the  $SU(3)$ -colour matrices and  $C$  is a parameter given below.

The spin-spin component of the GBE interaction between the constituent quarks  $i$  and  $j$  reads:

$$V_\chi(\vec{r}_{ij}) = \left\{ \sum_{F=1}^3 V_\pi(\vec{r}_{ij}) \lambda_i^F \lambda_j^F + \sum_{F=4}^7 V_K(\vec{r}_{ij}) \lambda_i^F \lambda_j^F + V_\eta(\vec{r}_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (16)$$

where  $\lambda^F$ ,  $F = 1, \dots, 8$  are flavour Gell-Mann matrices and  $\lambda^0 = \sqrt{2/3} \mathbf{1}$ , where  $\mathbf{1}$  is the  $3 \times 3$  unit matrix. Thus the interaction (16) includes  $\pi$ ,  $K$ ,  $\eta$  and  $\eta'$  exchanges. While the  $\pi$ ,  $K$ ,  $\eta$  mesons are (pseudo)Goldstone bosons of the spontaneously broken  $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$  chiral symmetry, the  $\eta'$  (flavour singlet) is a priori not a Goldstone boson due to the axial  $U(1)_A$  anomaly. In the large  $N_C$  limit the axial anomaly disappears, however, and the  $\eta'$  becomes the ninth Goldstone boson of the spontaneously broken  $U(3)_L \times U(3)_R \rightarrow U(3)_V$  chiral symmetry [21]. Thus in the real world with  $N_C = 3$  the  $\eta'$  should also be taken into account, but with parameters essentially different from  $\pi$ ,  $K$ ,  $\eta$  exchanges due to  $1/N_C$  corrections. For the system of u and d quarks only the  $K$ -exchange does not contribute.

In the simplest case, when both the constituent quarks and mesons are point-like particles and the boson field satisfies the linear Klein-Gordon equation, one has the following spatial dependence for the meson-exchange potentials [1] :

$$V_\gamma(\vec{r}_{ij}) = \frac{g_\gamma^2}{4\pi} \frac{1}{3} \frac{1}{4m^2} \left\{ \mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right\}, \quad (\gamma = \pi, K, \eta, \eta') \quad (17)$$

where  $\mu_\gamma$  are the meson masses and  $g_\gamma^2/4\pi$  are the quark-meson coupling constants given below.

Eq. (17) contains both the traditional long-range Yukawa potential as well as a  $\delta$ -function term. It is the latter that is of crucial importance for baryon spectroscopy and short-range  $NN$  interaction since it has a proper sign to provide the correct hyperfine splittings in baryons and is becoming highly dominant at short range. Since one deals with structured particles (both the constituent quarks and pseudoscalar mesons) of finite extension, one must

smear out the  $\delta$ -function in (17). In Ref. [7] a smooth Gaussian term has been employed instead of the  $\delta$ -function

$$4\pi\delta(\vec{r}_{ij}) \Rightarrow \frac{4}{\sqrt{\pi}}\alpha^3 \exp(-\alpha^2(r - r_0)^2). \quad (18)$$

where  $\alpha$  and  $r_0$  are adjustable parameters.

The parameters of the Hamiltonian (14) are [7]:

$$\frac{g_{\pi q}^2}{4\pi} = \frac{g_{\eta q}^2}{4\pi} = 0.67; \quad \frac{g_{\eta' q}^2}{4\pi} = 1.206$$

$$r_0 = 0.43 \text{ fm}, \quad \alpha = 2.91 \text{ fm}^{-1}, \quad C = 0.474 \text{ fm}^{-2}, \quad m = 340 \text{ MeV}.$$

$$\mu_\pi = 139 \text{ MeV}, \quad \mu_\eta = 547 \text{ MeV}, \quad \mu_{\eta'} = 958 \text{ MeV}. \quad (19)$$

The Hamiltonian (14) with the parameters (19) provides a very satisfactory description of the low-lying  $N$  and  $\Delta$  spectra in a fully dynamical nonrelativistic 3-body calculation [7].

At present we are limited to use a  $|s^3\rangle$  harmonic oscillator wave function for the nucleon in the  $NN$  problem. The parametrization (19) is especially convenient for this purpose since it allows to use the  $|s^3\rangle$  as a variational ansatz. Otherwise the structure of  $N$  should be more complicated. Indeed,  $\langle N|H|N\rangle$  takes a minimal value of 969.6 MeV at a harmonic oscillator parameter value of  $\beta = 0.437 \text{ fm}$  [22], i.e. only 30 MeV above the actual value in the dynamical 3-body calculation. In this way one satisfies one of the most important constraint for the microscopical study of the  $NN$  interaction : the nucleon stability condition [9]

$$\frac{\partial}{\partial b} \langle N|H|N\rangle = 0. \quad (20)$$

The other condition, the qualitatively correct  $\Delta - N$  splitting, is also satisfied [22].

We keep in mind, however, that a nonrelativistic description of baryons cannot be completely adequate. Within the semirelativistic description of baryons [8] the parameters extracted from the fit to baryon masses become considerably different and even the representation of the short-range part of GBE (18) has a different form. Within a semirelativistic

description the simple  $s^3$  wave function for the nucleon is not adequate anymore. All this suggests that the description of the nucleon based on the parameters (19) and an  $s^3$  wave function is only effective. Since in this paper we study only qualitative effects, related to the spin-flavour structure and sign of the short-range part of GBE interaction, we consider the present nonrelativistic parametrization as a reasonable framework.

We diagonalize the Hamiltonian (14) in the basis (8). All the necessary matrix elements are calculated with the help of the fractional parentage technique. Some important details can be found in Appendices C and D.

#### IV. RESULTS AND DISCUSSION

In Tables II and III we present our results obtained from the diagonalization of the Hamiltonian (14) in the basis (8). According to the definition of the effective potential within the Born-Oppenheimer approximation (9) at zero separation between nucleons all energies presented in the Tables II and III are given relative to two-nucleon threshold, i.e. the quantity  $2 < N|H|N > = 1939$  MeV has always been subtracted. In the second column we present the diagonal matrix elements for all the states listed in the first column. In the third column we present all the eigenvalues obtained from the diagonalization of a  $5 \times 5$  matrix. In the fourth column the amplitudes of all components of the ground state are given. In agreement with Sec. 2, one can see that the expectation value of the configuration  $|s^4p^2[42]_O[51]_{FS} >$  given in column 2 is much lower than all the other ones, and in particular it is about 1.5 GeV below the expectation value of the configuration  $|s^6[6]_O[33]_{FS} >$ . The substantial lowering of the configuration  $|s^4p^2[42]_O[51]_{FS} >$  relative to the other ones implies that this configuration is by far the most important component in the ground state eigenvector. The last column shows that the probability of this configuration is 93% both for  $SI = 10$  and  $SI = 01$ . As a consequence, the lowest eigenvalue is only about 100 MeV lower than the expectation value of the configuration  $|3 >$ .

The main outcome is that  $V_{NN}(R = 0)$  is highly repulsive in both  $^3S_1$  and  $^1S_0$  channels,

the height being 0.830 GeV in the former case and 1.356 GeV in the latter one.

In order to see that it is the GBE interaction which is responsible for the short range repulsion, it is very instructive to remove  $V_\chi$  from the Hamiltonian (14), compute the “nucleon mass” in this case, which turns out to be  $m_N = 1.633$  GeV at the harmonic oscillator parameter  $\beta = 0.917$  fm and diagonalize such a Hamiltonian again in the basis (8). In this case the most important configuration is  $|s^6[6]_O[33]_{FS} \rangle$ . Subtracting from the lowest eigenvalue the “two-nucleon energy”  $2m_N = 2 \times 1.633$  GeV one obtains  $V_{NN}^{NO\ GBE}(R = 0) = -0.197$  GeV. This soft attraction comes from the unphysical colour Van der Waals forces related to the pairwise confinement. The Van der Waals forces would not appear if the basis was restricted to the  $|s^6 \rangle$  state only. If the spatially excited 3q clusters from the  $s^4p^2$  configurations were removed the Van der Waals forces would disappear and we would arrive at  $V_{NN}^{NO\ GBE}(R = 0) = 0$ . Thus it is the GBE interaction which brings about 1 GeV repulsion, consistent with the previous discussion.

The effective repulsion obtained above implies a strong suppression of the  $L = 0$  relative motion wave function in the nucleon overlap region, as compared to the wave function of two well separated nucleons.

There is another important mechanism producing additional effective repulsion in the  $NN$  system, which is related to the symmetry structure of the lowest configuration but not related to its energy relative to the  $NN$  threshold. This “extra” repulsion, related to the “Pauli forbidden state” [23], persists if any of the configurations from the  $|s^4p^2 \rangle$  shell becomes highly dominant [17]. Indeed, the  $NN$  phase shift calculated with a pure  $[51]_{FS}$  state, which is projected “by hands” (not dynamically) from the full  $NN$  state in a toy model [9], shows a behaviour typical for repulsive potentials. As a result the S-wave  $NN$  relative motion wave function has an almost energy independent node [24]. A similar situation occurs in  ${}^4He - {}^4He$  scattering [25]. The only difference between this nuclear case and the  $NN$  system is that while in the former case a configuration  $s^8$  is indeed forbidden by the Pauli principle in eight-nucleon system, the configuration  $s^6$  is allowed in a six-quark system, but is highly suppressed by dynamics, as it was discussed above. In the OGE model

this effect is absent because none of the  $[42]_O$  states is dominant [9,20,27]. The existence of a strong effective repulsion, related to the energy balance in the adiabatic approximation, as in our case, suggests, however, that the amplitude of the oscillating  $NN$  wave function at short distance will be strongly suppressed.

To illustrate this discussion we project the lowest eigenvector in Table II onto the  $NN$  and  $\Delta\Delta$  channels. The projection onto any baryon-baryon channel  $B_1B_2$  is defined as follows [13,26]

$$\Psi_{B_1B_2}(\vec{r}) = \sqrt{\frac{6!}{3!3!2}} \langle B_1(1, 2, 3)B_2(4, 5, 6)|\Psi(1, 2, \dots, 6) \rangle, \quad (21)$$

where  $\Psi(1, 2, \dots, 6)$  is a fully antisymmetric 6q wave function, which in the present case is represented by the eigenvector in Table II, and  $B_1(1, 2, 3)$  and  $B_2(4, 5, 6)$  are intrinsic baryon wave functions.

In order to calculate (21) we need a “3 + 3” expansion of each state in the basis (8). The corresponding “3 + 3” decomposition of each state can be found in [26] in the  $CS$  coupling scheme. To use it here one needs the unitary matrix from the  $CS$  basis to the  $FS$  one. This matrix can be found in Appendix B.

In Fig. 1 we show the projections (21) onto the  $NN$  and  $\Delta\Delta$  channels in the  ${}^3S_1$  partial  $NN$  wave at short range. In fact, such projections can be shown for other channels too as e.g.  $NN^*$ ,  $N^*N^*$ ,... some of them being not small. Note that our six-quark wave function, calculated at short range only, was normalized to 1. Hence, we cannot show the suppression of the  $NN$  projection in the nucleon overlap region as compared to the wave function of the well separated nucleons, discussed above. This can only be seen from  $\Psi_{NN}(\vec{r})$  obtained in dynamical calculations including not only the short-range 6q configurations, like in the present paper, but also the basis states representing the middle- and long distances in the  $NN$  system.

In Fig. 1 one observes a nodal behaviour of both  $\Psi_{NN}(\vec{r})$  and  $\Psi_{\Delta\Delta}(\vec{r})$  at short range. Also  $\Psi_{\Delta\Delta}(\vec{r})$  is essentially larger at short range than  $\Psi_{NN}(\vec{r})$ . At large distances only  $\Psi_{NN}(\vec{r})$  will survive. This nodal behaviour is related to the fact that the configuration  $|3 \rangle$  is highly

dominant. In the case of any configuration  $s^4p^2$  or  $s^52s$  from the  $N=2$  shell, the relative motion of two  $s^3$  clusters (e.g.  $NN$  and  $\Delta\Delta$ ) is described by a nodal wave function.

Now we want to discuss the question which type of  $NN$  potential would be equivalent to the short-range picture obtained above. If one considers the effect of the short-range dynamics on the  $NN$  phase shifts, in a limited energy interval the phase shifts in both  $^3S_1$  and  $^1S_0$  partial waves can be simulated by strong repulsive core potentials or by “deep attractive potentials with forbidden states” [24]. The latter potentials are in fact supersymmetric partners of the former ones [28].

If, on the other hand, one considers the effect of the short-range dynamics on the structure of the wave function at short range, it is difficult to construct a potential which would be adequate. For example, a repulsive core potential produces a wave function which is indeed suppressed at short range, but does not have any nodal structure. If one takes, instead, a “deep attractive potential with forbidden state”, one obtains a nodal behaviour, but the wave function is not suppressed at short range (i.e. the amplitude left to the node is a very large one). As a direct consequence, the latter potential produces a very rich high-momentum component, which is in contradiction with the deuteron electromagnetic form factors [29]. The high-momentum component, implied by a “very soft node”, like in our case, will be much smaller and closer to that one obtained from the potentials with strong repulsive core.

We also see large projections onto other  $B_1B_2$  channels (exemplified by the  $\Delta\Delta$  channel in Fig. 1). These components cannot be taken into account in any simple  $NN$  potential, in principle. Thus, if we are interested in effects, related to the short-range  $NN$  system, there is no way, other than to consider the full 6q wave function in this region.

## V. WHY THE SINGLE CHANNEL RESONATING GROUP METHOD ANSATZ IS NOT ADEQUATE ?

In this section we show that the currently used one-channel resonating group method (RGM) ansatz for the two-nucleon wave function is not adequate in a study of the short-range  $NN$  interaction with the chiral quark model.

In the one-channel RGM approximation the 6q wave function has the form

$$\begin{aligned}\psi &= \hat{A}\{N(1, 2, 3)N(4, 5, 6)\chi(\vec{r})\}, \\ \hat{A} &= \frac{1}{\sqrt{10}}(1 - 9\hat{P}_{36}), \\ \vec{r} &= \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3} - \frac{\vec{r}_4 + \vec{r}_5 + \vec{r}_6}{3}.\end{aligned}\tag{22}$$

This is reasonable in the case where the short-range quark dynamics is described in terms of the OGE interaction. In this case the addition of new channels, orthogonal to (22), does not change considerably the full wave function in the nucleon overlap region. This is not the case for the chiral constituent quark model, where the short-range quark dynamics is due to the GBE interaction. To have a better insight why (22) is a poor approximation in the present case, we begin with the explanation why (22) is reasonable for OGE model [20].

To this end, it is very convenient to use the six-quark shell model basis for the NN function in the nucleon overlap region [13,20,30]. Such a basis is much more flexible than (22). Diagonalizing a Hamiltonian comprising OGE and a confining interaction in the harmonic oscillator basis up to two excitation quanta, one can obtain the 6q wave function in the form [20]

$$\psi = C_0|s^6\rangle + \sum_{\alpha} C_{\alpha}|\alpha\rangle,\tag{23}$$

where  $\alpha$  lists all possible configurations in the  $N=2$  shell :  $[6]_O[222]_{CS}$ ,  $[42]_O[42]_{CS}$ ,  $[42]_O[321]_{CS}$ ,  $[42]_O[222]_{CS}$ ,  $[42]_O[3111]_{CS}$ ,  $[42]_O[21111]_{CS}$ . With the OGE interaction, the CS coupling scheme based on the chain  $SU(6)_{CS} \supset SU(3)_C \times SU(2)_S$  is more convenient. It has been found that there are a few most important configurations -  $|s^6[6]_O[222]_{CS}\rangle$



,  $|s^4p^2[42]_O[42]_{CS}\rangle$ ,  $|s^4p^2[42]_O[321]_{CS}\rangle$ ,  $|(\sqrt{5/6}s^52s - \sqrt{1/6}s^4p^2)[6]_O[222]_{CS}\rangle$  - with sizeable amplitudes  $C_\alpha$  [20,26,30].

Now, let us expand the RGM wave function (22) in the shell model basis. For that purpose, the trial function  $\chi(\vec{r})$  in (22) should be expanded in a harmonic oscillator basis too

$$\chi_{L=0}(\vec{r}) = \sum_{N=0,2,4,\dots} \langle \chi_{L=0} | \phi_{NS} \rangle \phi_{NS}(\vec{r}), \quad (24)$$

where  $\phi_{NS}(\vec{r})$  is a harmonic oscillator state with N quanta and  $L = 0$ . Thus in the ansatz (22) the variational coefficients based on the expansion (24) are  $\langle \chi_{L=0} | \phi_{NS} \rangle$ . The last step is to use the expressions (21) and (22) of Ref. [26] for  $\hat{A}\{N(1,2,3)N(4,5,6)\phi_{0S}(\vec{r})\}$ ,  $\hat{A}\{N(1,2,3)N(4,5,6)\phi_{2S}(\vec{r})\}$ , written in the shell model basis. These are transformations from one basis to another and do not depend on the 6q dynamics. If it turns out that for a given Hamiltonian the variational coefficients  $C_\alpha$  in (23) are close to the algebraical ones  $\langle \alpha | \hat{A}\{N(1,2,3)N(4,5,6)\phi_{2S}(\vec{r})\} \rangle$ , then one can conclude that (22) is a good approximation for the variational solution (23). If not, the variational ansatz (22) is poor and other channels, not equivalent to (22), should be added (e.g.  $\hat{A}\{N^*N\chi^*(\vec{r})\}$ ,  $\hat{A}\{N^*N^*\chi^{**}(\vec{r})\}, \dots$ ). For the OGE model it is found that indeed the variational coefficients  $C_\alpha$  in (23) are very close to the algebraical ones [20] (see also [26]).

Let us now turn to the analysis of the results of Sec. 4 based on the GBE interaction. Using the unitary transformation from the CS to FS scheme, given in Appendix B, one can rewrite Eqs.(21) and (22) of Ref. [26] as :

$$\hat{A}\{N(1,2,3)N(4,5,6)\phi_{0s}(\vec{r})\}_{SI=10} = \sqrt{\frac{10}{9}} |s^6[6]_O[33]_{FS}\rangle, \quad (25)$$

$$\begin{aligned} \hat{A}\{N(1,2,3)N(4,5,6)\phi_{2s}(\vec{r})\}_{SI=10} &= \frac{3\sqrt{2}}{9} |(\sqrt{\frac{5}{6}}s^52s - \sqrt{\frac{1}{6}}s^4p^2)[6]_O[33]_{FS}\rangle \\ &\quad - \frac{4\sqrt{2}}{9} |s^4p^2[42]_O[33]_{FS}\rangle - \frac{4\sqrt{2}}{9} |s^4p^2[42]_O[51]_{FS}\rangle. \end{aligned} \quad (26)$$

From the expression (26) we see that the relative amplitudes of the states  $|5\rangle$ ,  $|2\rangle$  and  $|3\rangle$  are in the ratio

$$|5 \rangle : |2 \rangle : |3 \rangle = 3 : -4 : -4 \quad (27)$$

and the amplitude of the state  $|4 \rangle$  is zero. The diagonalization of the Hamiltonian made in the previous section gives

$$|5 \rangle : |2 \rangle : |3 \rangle : |4 \rangle \simeq 0.06 : 0.08 : -0.96 : 0.20, \quad (28)$$

Therefore the ansatz (22) is completely inadequate in the nucleon overlap region and the incorporation of additional channels is required in RGM calculations.

## VI. SUMMARY

In the present paper we have calculated an adiabatic  $NN$  potential at zero separation between nucleons in the framework of a chiral constituent quark model, where the constituent quarks interact via pseudoscalar meson exchange. Diagonalizing a Hamiltonian in a basis consisting of the most important 6q configurations in the nucleon overlap region, we have found a very strong effective repulsion of the order of 1 GeV in both  ${}^3S_1$  and  ${}^1S_0$   $NN$  partial waves. Due to the specific flavour-spin symmetry of the Goldstone boson exchange interaction the configuration  $|s^4p^2[42]_O[51]_{FS} \rangle$  becomes highly dominant at short range. As a consequence, the projection of the full 6q wave function onto the  $NN$  channel should have a node at short range in both  ${}^3S_1$  and  ${}^1S_0$  partial waves. The amplitude of the oscillation left to the node should be strongly suppressed as compared to the wave function of two well separated nucleons.

We have also found that due to the strong dominance of the configuration  $|s^4p^2[42]_O[51]_{FS} \rangle$  the commonly used one-channel RGM ansatz is a very poor approximation to the 6q wave function in the nucleon overlap region.

Thus, within the chiral constituent quark model one has all the necessary ingredients to understand microscopically the  $NN$  interaction. There appears strong effective short-range repulsion from the same part of Goldstone boson exchange which also produces hyperfine splittings in baryon spectroscopy. The long- and middle-range attraction in the  $NN$  system

is automatically implied by the Yukawa part of pion exchange and two-pion (or  $\sigma$ ) exchanges between quarks belonging to different nucleons. With this first encouraging result, it might be worthwhile to perform a more elaborate calculation of  $NN$  system and other baryon-baryon systems within the present framework.

### APPENDIX A:

The expectation value of the operators (1) and (12), displayed in Table I, are calculated with the following formulae:

$$\langle \sum_{i<j} \lambda_i \cdot \lambda_j \vec{\sigma}_i \cdot \vec{\sigma}_j \rangle = 4C_2^{SU(6)} - 2C_2^{SU(3)} - \frac{4}{3}C_2^{SU(2)} - 8N \quad (\text{A1})$$

where  $N$  is the number of particles, here  $N=6$ , and  $C_2^{SU(n)}$  is the Casimir operator eigenvalues of  $SU(n)$  which can be derived from the expression :

$$C_2^{SU(n)} = \frac{1}{2}[f'_1(f'_1 + n - 1) + f'_2(f'_2 + n - 3) + f'_3(f'_3 + n - 5) + f'_4(f'_4 + n - 7) + \dots + f'_{n-1}(f'_{n-1} - n + 3)] - \frac{1}{2n} \left( \sum_{i=1}^{n-1} f'_i \right)^2 \quad (\text{A2})$$

where  $f'_i = f_i - f_n$ , for an irreducible representation given by the partition  $[f_1, f_2, \dots, f_n]$ .

### APPENDIX B:

This appendix reproduces transformations, derived elsewhere, from the CS coupling scheme to the FS coupling scheme, or vice versa, related to the orbital symmetries  $[6]_O$  and  $[42]_O$ , appearing in the basis vectors (8).

For the  $[6]_O$  symmetry one obviously has :

$$[6]_O[33]_{FS} = [6]_O[222]_{CS} \quad (\text{B1})$$

either for IS=01 or 10.

For the  $[42]_O$  symmetry, sector IS=01, the Table IV reproduces Table 7 of Ref. [31] with a phase change in columns 3 and 5, required by consistency with Ref. [26].

In this Table, the column headings are

$$\begin{aligned}
\psi_1^{CS} &= [42]_O[42]_{CS} \\
\psi_2^{CS} &= [42]_O[321]_{CS} \\
\psi_3^{CS} &= [42]_O[3111]_{CS} \\
\psi_4^{CS} &= [42]_O[222]_{CS} \\
\psi_5^{CS} &= [42]_O[21111]_{CS}
\end{aligned} \tag{B2}$$

For the  $[42]_O$  symmetry, sector IS=10, we reproduce in Table V the corresponding Table from Ref. [32] by interchanging rows with columns and then reorder the rows. In this case, the notation is

$$\begin{aligned}
\bar{\psi}_1^{CS} &= [42]_O[411]_{CS} \\
\bar{\psi}_2^{CS} &= [42]_O[33]_{CS} \\
\bar{\psi}_3^{CS} &= [42]_O[2211]_{CS}^1 \\
\bar{\psi}_4^{CS} &= [42]_O[2211]_{CS}^2 \\
\bar{\psi}_5^{CS} &= [42]_O[1^6]_{CS}
\end{aligned} \tag{B3}$$

The upper indices 1 and 2 take into account the two representations  $[2211]$  appearing in the inner product  $[222] \times [33]$ .

### APPENDIX C:

The calculation of the matrix elements of the Hamiltonian (14) is based on the fractional parentage (cfp) technique described in Ref. [18]. For details, see also Ref. [19], chapter 10. In dealing with  $n$  particles the matrix elements of a symmetric two-body operator between totally (symmetric or) antisymmetric states  $\psi_n$  and  $\psi'_n$  reads

$$\langle \psi_n | \sum_{i < j} V_{ij} | \psi'_n \rangle = \frac{n(n-1)}{2} \langle \psi_n | V_{n-1,n} | \psi'_n \rangle \tag{C1}$$

The matrix elements of  $V_{n-1,n}$  are calculated by expanding  $\psi_n$  and  $\psi'_n$  in terms of products of antisymmetric states of the first  $n-2$  particles  $\psi_{n-2}$  and of the last pair  $\phi_2$

$$\psi_n = \sum_{\alpha\beta} P_{\alpha\beta} \psi_{n-2}(\alpha) \phi_2(\beta) \quad (\text{C2})$$

with  $\alpha, \beta$  denoting the possible structures of  $\psi_{n-2}$  and  $\phi_2$  and  $P_{\alpha\beta}$  the products of cfp coefficients in the orbital, spin-flavour and colour space states. In practical calculations, the colour space cfp coefficients are not required. The orbital cfp are taken from Ref. [33], Tables 1 and 2 by using the replacement  $r^4 l^2 \rightarrow s^4 p^2$  and  $r^5 l \rightarrow s^5 p$ . The trivial ones are equal to one. The flavour-spin cfp for IS=01 are identical to the  $\bar{K}$ -matrices of Table 1 Ref. [31] with  $[42]_S[33]_F$  in the column headings. For IS=10 they are the same as for IS=01 but the column headings is  $[42]_F[33]_S$  instead of  $[42]_S[33]_F$  as above, and this is due to the commutativity of inner products of  $S_n$  (see for example Ref. [19]). The cfp used in the OC coupling are from Ref. [33] Table 3, for  $[42]_O \times [222]_C \rightarrow [3111]_{OC}$  and Table 5 of Ref. [31] for  $[42]_O \times [222]_C \rightarrow [222]_{OC}$  and  $[42]_O \times [222]_C \rightarrow [21111]_{OC}$ .

In this way, after decoupling all degrees of freedom one can integrate out in the colour, spin and flavour space. The net outcome of this algebra is that any six-body matrix element becomes a linear combination of two-body orbital matrix elements,  $\langle V_\pi \rangle$ ,  $\langle V_\eta \rangle$  and  $\langle V_{\eta'} \rangle$ . The coefficients of  $\langle V_\pi \rangle$  are the same for IS=01 and 10, but the coefficients of  $\langle V_\eta \rangle$  are usually different. In both cases the coefficients of  $\langle V_{\eta'} \rangle$  are two times those of  $\langle V_\eta \rangle$ . We found that the two-body GBE matrix elements satisfy the relations  $\langle V_\pi \rangle \simeq \langle V_\eta \rangle$  and  $\langle V_{\eta'} \rangle \simeq 2 \langle V_\pi \rangle$ . As an example, in Table VI we show the matrix elements obtained for IS=01. Except for  $\langle ss|V|pp \rangle$  and  $\langle s2s|V|pp \rangle$  they are all negative, i.e. carry the sign of Eq. (1).

In a harmonic oscillator basis the confinement potential matrix elements can be performed analytically. As an illustration, in Appendix D, we reproduce the results for all configurations required in these calculations.

Finally, the kinetic energy matrix elements can be calculated as above, by writing the relative kinetic energy operator as a two-body operator

$$T = \sum_i \frac{p_i^2}{2m} - \frac{1}{12} (\sum_i \vec{p}_i)^2 = \sum_{i<j} T_{ij} \quad (\text{C3})$$

with

$$T_{ij} = \frac{1}{12m}(p_i^2 + p_j^2) - \frac{1}{6m}\vec{p}_i \cdot \vec{p}_j \quad (\text{C4})$$

Alternatively we can use an universal formula for the kinetic energy of harmonic oscillator states

$$K.E. = \frac{1}{2}[N + \frac{3}{2}(n-1)]\hbar\omega + \frac{3}{4}\hbar\omega \quad (\text{C5})$$

where N is the number of quanta and n the number of particles. The last term is the kinetic energy of the center of mass.

### APPENDIX D:

We work with the following single particle harmonic oscillator states :

$$|s\rangle = \pi^{-3/4}\beta^{-3/2}\exp(-r^2/2\beta^2) \quad (\text{D1})$$

$$|p\rangle_m = 8^{1/2}3^{-1/2}\pi^{-1/4}\beta^{-5/2}r\exp(-r^2/2\beta^2)Y_{1m} \quad (\text{D2})$$

$$|2s\rangle = 2^{1/2}3^{-1/2}\pi^{-3/4}\beta^{-3/2}\left(\frac{3}{2} - \frac{r^2}{\beta^2}\right)\exp(-r^2/2\beta^2) \quad (\text{D3})$$

In this basis the two-body matrix elements of the confining potential  $V^c = Cr$  of Eq.(15) are

$$\langle ss|V^c|ss\rangle = \sqrt{\frac{2}{\pi}}2C\beta \quad (\text{D4})$$

$$\langle sp|V^c|sp\rangle = \sqrt{\frac{2}{\pi}}\frac{7C\beta}{3} \quad (\text{D5})$$

$$\langle sp|V^c|ps\rangle = -\sqrt{\frac{2}{\pi}}\frac{C\beta}{3} \quad (\text{D6})$$

$$\langle ss|V^c|(pp)_{L=0}\rangle = -\sqrt{3}\langle sp|V^c|ps\rangle \quad (\text{D7})$$

$$\langle (pp)_{L=0}|V^c|(pp)_{L=0}\rangle = \sqrt{\frac{2}{\pi}}\frac{5C\beta}{2} \quad (\text{D8})$$

$$\langle s2s|V^c|s2s\rangle = \sqrt{\frac{2}{\pi}}\frac{31C\beta}{12} \quad (\text{D9})$$

$$\langle ss|V^c|s2s\rangle = -\sqrt{\frac{1}{3\pi}}C\beta \quad (\text{D10})$$

$$\langle s2s|V^c|(pp)_{L=0}\rangle = -\frac{1}{\sqrt{\pi}}\frac{C\beta}{2} \quad (\text{D11})$$

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TABLES

TABLE I. Expectation values of the operators defined by Eqs. (1) and (12) for all compatible symmetries  $[f]_O[f]_{FS}$  in the IS=(01) and (10) sector.  $\langle V_\chi \rangle$  is in units of  $C_\chi$  and  $\langle V_{cm} \rangle$  in units of  $C_{cm}$ .

$[f]_O[f]_{FS}$	I=0 S=1		I=1 S=0	
	$\langle V_\chi \rangle$	$\langle V_{cm} \rangle$	$\langle V_\chi \rangle$	$\langle V_{cm} \rangle$
$[6]_O[33]_{FS}$	-28/3	8/3	-8	8
$[42]_O[33]_{FS}$	-28/3	-26/9	-8	-4/3
$[42]_O[51]_{FS}$	-100/3	16/9	-32	16/9
$[42]_O[411]_{FS}$	-28/3	20/9	-8	44/9
$[42]_O[321]_{FS}$	8/3	-164/45	4	232/45
$[42]_O[2211]_{FS}$	68/3	-62/15	60	42/5

TABLE II. Results of the diagonalization of the Hamiltonian (14) for IS=(01). Column 1 - the basis states, column 2 - diagonal matrix elements (GeV), column 3 - eigenvalues (GeV) for a  $5 \times 5$  matrix, column 4 - components of the lowest state. The results are for  $\beta = 0.437$  fm. In columns 2 and 3, the quantity  $2m_N = 1.939$  GeV is subtracted.

state	$1 \times 1$	$5 \times 5$	lowest state components
$ s^6[6]_o[33]_{FS} \rangle$	2.346	0.830	-0.14031
$ s^4p^2[42]_o[33]_{FS} \rangle$	2.824	1.323	0.07747
$ s^4p^2[42]_o[51]_{FS} \rangle$	0.942	2.693	-0.96476
$ s^4p^2[42]_o[411]_{FS} \rangle$	2.949	3.049	0.20063
$ (\sqrt{5/6}s^52s - \sqrt{1/6}s^4p^2[6]_o[33]_{FS} \rangle$	3.011	4.169	0.05747

TABLE III. Same as Table 2 but for IS=(10).

state	$1 \times 1$	$5 \times 5$	lowest state components
$ s^6[6]_o[33]_{FS} \rangle$	2.990	1.356	-0.12195
$ s^4p^2[42]_o[33]_{FS} \rangle$	3.326	1.895	0.08825
$ s^4p^2[42]_o[51]_{FS} \rangle$	1.486	3.178	-0.96345
$ s^4p^2[42]_o[411]_{FS} \rangle$	3.543	3.652	-0.21644
$ (\sqrt{5/6}s^52s - \sqrt{1/6}s^4p^2[6]_o[33]_{FS} \rangle$	3.513	4.777	0.04756

 TABLE IV. The unitary transformation between the CS and FS basis vectors of orbital symmetry  $[42]_o$ , isospin I=0, and spin S=1.

	$\psi_1^{CS}$	$\psi_2^{CS}$	$\psi_3^{CS}$	$\psi_4^{CS}$	$\psi_5^{CS}$
$[42]_o[33]_{FS}$	$\frac{9\sqrt{5}}{36}$	$-\frac{8\sqrt{5}}{36}$	$\frac{5\sqrt{2}}{36}$	$\frac{11}{36}$	$\frac{20}{36}$
$[42]_o[51]_{FS}$	$\frac{9\sqrt{5}}{45}$	$-\frac{8\sqrt{5}}{45}$	$\frac{5\sqrt{2}}{45}$	$-\frac{25}{45}$	$-\frac{25}{45}$
$[42]_o[411]_{FS}$	$\frac{9\sqrt{10}}{180}$	$-\frac{8\sqrt{10}}{180}$	$-\frac{170}{180}$	$-\frac{25\sqrt{2}}{180}$	$\frac{20\sqrt{2}}{180}$
$[42]_o[2211]_{FS}$	$\frac{11}{20}$	$\frac{8}{20}$	$-\frac{\sqrt{10}}{20}$	$\frac{5\sqrt{5}}{20}$	$-\frac{4\sqrt{5}}{20}$
$[42]_o[321]_{FS}$	$-\frac{18}{45}$	$-\frac{29}{45}$	$-\frac{2\sqrt{10}}{45}$	$\frac{10\sqrt{5}}{45}$	$-\frac{8\sqrt{5}}{45}$

TABLE V. Same as Table IV but for S=0, I=1.

	$\bar{\psi}_1^{CS}$	$\bar{\psi}_2^{CS}$	$\bar{\psi}_3^{CS}$	$\bar{\psi}_4^{CS}$	$\bar{\psi}_5^{CS}$
$[42]_o[33]_{FS}$	$\sqrt{\frac{25}{72}}$	$-\sqrt{\frac{25}{144}}$	$-\sqrt{\frac{49}{144}}$	$-\sqrt{\frac{1}{36}}$	$-\sqrt{\frac{1}{9}}$
$[42]_o[51]_{FS}$	$\sqrt{\frac{2}{9}}$	$-\sqrt{\frac{1}{9}}$	$\sqrt{\frac{1}{9}}$	$\sqrt{\frac{4}{9}}$	$\sqrt{\frac{1}{9}}$
$[42]_o[411]_{FS}$	$\sqrt{\frac{1}{36}}$	$\sqrt{\frac{25}{72}}$	$-\sqrt{\frac{25}{72}}$	$\sqrt{\frac{1}{18}}$	$\sqrt{\frac{2}{9}}$
$[42]_o[2211]_{FS}$	$-\sqrt{\frac{9}{40}}$	$-\sqrt{\frac{1}{80}}$	$-\sqrt{\frac{9}{80}}$	$\sqrt{\frac{9}{20}}$	$-\sqrt{\frac{1}{5}}$
$[42]_o[321]_{FS}$	$-\sqrt{\frac{8}{45}}$	$-\sqrt{\frac{16}{45}}$	$-\sqrt{\frac{4}{45}}$	$-\sqrt{\frac{1}{45}}$	$\sqrt{\frac{16}{45}}$

TABLE VI. All one-meson exchange two-body matrix elements (GeV) for the sector IS=01 evaluated at  $\beta = 0.437$  fm. The remaining one is  $\langle sp|V_F|ps \rangle = -\langle ss|V_F|(pp)_{L=0} \rangle / \sqrt{3}$ .

Two-body matrix elements	$F = \pi$	$F = \eta$	$F = \eta'$
$\langle ss V_F ss \rangle$	-0.108357	-0.104520	-0.189153
$\langle ss V_F (pp)_{L=0} \rangle$	0.043762	0.042597	0.076173
$\langle sp V_F sp \rangle$	-0.083091	-0.079926	-0.145175
$\langle (pp)_{L=0} V_F (pp)_{L=0} \rangle$	-0.081160	-0.078594	-0.142205
$\langle s2s V_F s2s \rangle$	-0.069492	-0.066963	-0.121701
$\langle ss V_F s2s \rangle$	-0.030945	-0.030121	-0.053863
$\langle s2s V_F (pp)_{L=0} \rangle$	0.033309	0.031753	0.057499

## Figure Captions

### Fig. 1

Projections of the lowest eigenvector in Table II onto  $NN$  and  $\Delta\Delta$  channels (in arbitrary units).

