Comparative analysis of large N_c QCD and quark model approaches to baryons*

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We show that a remarkable compatibility exists between the results of a potential model with constituent quarks and the $1/N_c$ expansion mass formula for strange and nonstrange baryon resonances. Such compatibility brings support to the basic assumptions of relativistic quark models and sheds light on the physical content of the model-independent large N_c mass formula. Good agreement between both approaches is also found for heavy baryons, made of one heavy and two light quarks, in the ground state band.

PACS numbers: 11.15.Pg, 12.39.Ki, 12.39.Pn, 14.20.-c

1. Baryons in large N_c QCD

1.1. Light baryons

In large N_c QCD, the gauge group is SU(N_c) and a baryon is made of N_c quarks. The $1/N_c$ expansion is based on the discovery that, in the limit $N_c \to \infty$, QCD possesses an exact contracted SU($2N_f$) symmetry where N_f is the number of flavors. This symmetry is approximate for finite N_c

* Excited QCD 2009, February 8-14, Zakopane, Poland
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so that corrections have to be added in powers of $1/N_c$. When the SU(N_f) symmetry is exact the mass operator M has the general form

$$M = \sum_i c_i O_i. \tag{1}$$

The coefficients c_i encode the QCD dynamics and have to be determined from a fit to available data, while the operators O_i are SU($2N_f$) $\otimes$ SO(3) scalars of the form

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}. \tag{2}$$

Here $O_\ell^{(k)}$ is a k-rank tensor in SO(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2)-spin, but invariant in SU(N_f)-flavor. n represents the minimum of gluon exchanges to generate the operator. In practical applications, terms of order $1/N_c^2$ are neglected.

One obviously has to set $N_f = 2$ for light nonstrange baryons. As an example, the ground state mass formula reads in this case $M = c_1 N_c \mathbf{1} + c_4 S^2/N_c + \mathcal{O}\left(N_c^{-3}\right)$. Other terms like spin-orbit and isospin-dependent contributions appear in excited bands [1]. For light strange baryons ($N_f = 3$), a general mass term of the form

$$n_s\, \Delta M_s = \sum_{i=1} d_i B_i, \tag{3}$$

has to be added to equation (1) to account for SU(3)-flavor symmetry breaking. In the left hand side n_s is the number of strange quarks and ΔM_s is the mass shift of every strange quark. The operators B_i break SU(3)-flavor symmetry and the coefficients d_i have to be fitted in a global fit of nonstrange and strange baryons.

The classification scheme used in the $1/N_c$ expansion for baryon resonances is based on the standard SU(6) classification as in a constituent quark model. Baryons are grouped into excitation bands $N = 0, 1, 2, \ldots$, each band containing at least one SU(6) multiplet. The band number N is the total number of excitation quanta in a harmonic oscillator picture. Note that the coefficients c_i and d_i depend on N.

1.2. Heavy quarks

The approximate spin-flavor symmetry of baryons containing two light and one heavy quark is SU(6)$\times$ SU(2)$_c$ $\times$ SU(2)$_b$, *i.e.* there is a separate spin symmetry for each heavy flavor. For these baryons, an $1/m_Q$ expansion can be combined with the $1/N_c$ expansion, m_Q being the heavy quark mass.

In the case of an exact SU(3)-flavor symmetry, the mass operator reads

$$M = m_Q \mathbf{1} + c_0\, N_c\, \mathbf{1} + \frac{c_2}{N_c}\, J_{qq}^2 + \frac{c_0^{'}}{2m_Q}\mathbf{1} + \frac{c_2^{'}}{2m_Q N_c^2} J_{qq}^2 + 2\frac{c_2^{''}}{N_c m_Q}\vec{J}_{qq}\cdot\vec{J}_Q, \quad (4)$$

where $\vec{J}_{qq}$ ($\vec{J}_Q$) is the total spin of the light quark pair (of the heavy quark). The unknown coefficients have to be fitted to experimental data, but physical and dimensional arguments suggest to introduce a typical QCD energy scale Λ and the relations

$$c_0 = \Lambda, \quad c_2 \sim \Lambda, \quad c_0^{'} \sim c_2^{'} \sim c_2^{''} \sim \Lambda^2. \quad (5)$$

The inclusion of SU(3)-flavor breaking leads to an expansion of the mass operator in the SU(3)-violating parameter $\epsilon \sim (m_s - m) \sim 0.2-0.3$, where m is the average mass of the u, d quarks and where m_s is the strange quark mass. Its value is measured in units of the chiral symmetry breaking scale parameter $\Lambda_\chi \sim 1$ GeV.

2. Quark model

A baryon, viewed as a bound state of three quarks, can be described in a first approximation by the spinless Salpeter Hamiltonian

$$H = \sum_{i=1}^{3}\left[\sqrt{\vec{p}_i^{\,2} + m_i^2} + \sigma|\vec{x}_i - \vec{R}|\right], \quad (6)$$

where m_i is the current quark mass and where σ is the string tension. The confinement potential is a Y-junction in which the Toricelli point is replaced by the center of mass $\vec{R}$. It is also necessary to include some perturbative corrections, namely one-gluon exchange and quark self-energy mass terms, respectively

$$\Delta M_{\text{oge}} = -\frac{2}{3}\sum_{i<j=1}^{3}\left\langle \frac{\alpha_{s,ij}}{|\vec{x}_i - \vec{x}_j|}\right\rangle, \quad \Delta M_{\text{qse}} = -\frac{fa}{2\pi}\sum_{i=1}^{3}\frac{\eta(m_i/\delta)}{\mu_i}, \quad (7)$$

where $\alpha_{s,ij}$ is the strong coupling constant between the quarks i and j and $\mu_i = \left\langle\sqrt{\vec{p}_i^{\,2} + m_i^2}\right\rangle$ is the kinetic energy of the quark i. The factors $3 \le f \le 4$ and $(1.0 \le \delta \le 1.3)$ GeV have been computed in lattice QCD. $\eta(x)$ is analytically known and can accurately be fitted by $\eta(x) \approx 1 - \beta x^2$ with $\beta = 2.85$ for $0 \le x \le 0.3$ and by γ/x^2 with $\gamma = 0.79$ for $1.0 \le x \le 6.0$.

Within our model, we have $m_u = m_d = 0$. In this case, using the auxiliary field technique, analytical mass formulas can be obtained for both light qqq and heavy qqQ baryons at order $\mathcal{O}(m_s^2)$ and $\mathcal{O}(1/m_Q)$. For light baryons one has $M_{qqq} = M_0 + n_s\,\Delta M_{0s}$ ($n_s = 0,1,2,3$) with [2, 3]

$$M_0 \;=\; 6\mu_0 - \frac{2\pi\sigma\alpha_0}{6\sqrt{3}\mu_0} - \frac{f\sigma}{4\mu_0 k_0}, \quad \Delta M_{0s} = \frac{m_s^2}{\mu_0}\Theta(N). \tag{8}$$

We refer the reader to [3] for the explicit expression of $\Theta(N)$. In these equations, $\mu_0 = \sqrt{\pi\sigma(N+3)/18}$, and $\alpha_0 = \alpha_{s,qq}$. k is a corrective factor equal to $k_0 = 0.952$ ($k_1 = 0.930$) for qqq (qqQ) baryons, resulting from the replacement of the Toricelli point by the center of mass. Moreover, N is the baryon band number in a harmonic oscillator picture, just as the one which is used in large N_c QCD as in section 1. This allows a direct comparison between both approaches.

For heavy baryons one has $M_{qqQ} = m_Q + M_1 + n_s\,\Delta M_{1s} + \Delta M_Q$ ($n_s = 0,1,2$), with [4]

$$\begin{aligned} M_1 \;&=\; 4\mu_1 - \frac{2}{3}\left(\alpha_0\sqrt{\frac{k_1\pi\sigma}{18k_0}} + 2\alpha_1\sqrt{\frac{k_1\pi\sigma}{3k_0(N+3)}}\right) - \frac{f\sigma}{6k_0\mu_1}, \\ \Delta M_{1s} \;&=\; \frac{m_s^2}{\mu_1}\bar{\Theta}(N), \quad \Delta M_Q = \frac{k_1\pi\sigma}{12k_0 m_Q}\,K(N). \end{aligned} \tag{9}$$

The interested reader will find the explicit expressions of $\bar{\Theta}(N)$ and $K(N)$ in Ref. [4]. Moreover, $\mu_1 = \sqrt{k_1\pi\sigma(N+3)/12k_0}$ and $\alpha_1 = \alpha_{s,qQ} = 0.7\alpha_0$. The band number N corresponds this time to the relative quantum of excitation of the heavy quark and the light quark pair. The heavy quark–light diquark picture is favored since the quark pair tends to remain in its ground state [4].

3. Comparison of both approaches

3.1. Light baryons

The coefficients c_i appearing in the $1/N_c$ mass operator can be obtained from a fit to experimental data and compared with the quark model results. The dominant term $c_1\,N_c$ in the mass formula (1) contains the spin- and strangeness-independent mass contributions, which in a quark model language represents the confinement and the kinetic energy. So, for $N_c = 3$, we expect

$$c_1^2 = M_0^2/9. \tag{10}$$

Figure 1 (left panel) shows a comparison between the values of c_1^2 obtained in the $1/N_c$ expansion method and those derived from Eq. (8) for various values of N. It appears that the results of large N_c QCD are entirely compatible with the formula (10) for standard values of the parameters.

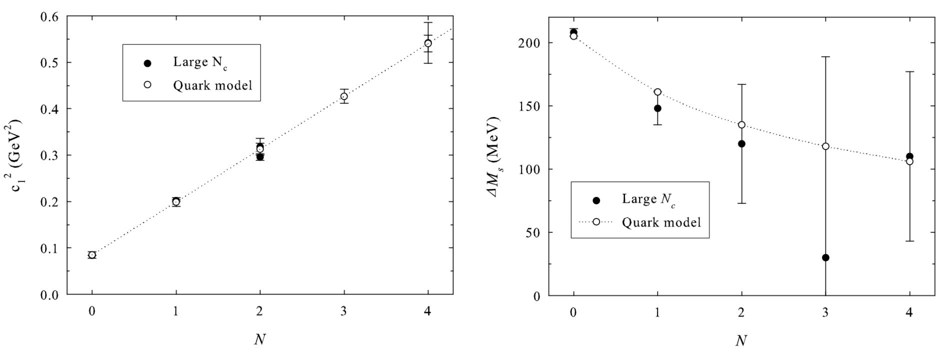

Fig. 1. Plot of c_1^2 (left) and ΔM_s (right) versus the band number N. The values computed in the $1/N_c$ expansion (full circles) from a fit to experimental data are compared with the quark model results with $\sigma = 0.163$ GeV^2, $\alpha_0 = 0.4$, $f = 3.5$, and $m_s = 0.243$ GeV (empty circles and dotted line are given to guide the eyes).

The spin-dependent corrections between quarks i and j should be of order $O(1/\mu_i\mu_j)$. Therefore we expect both c_2 and c_4 to be proportional to $(N+3)^{-1}$: Such a behavior is consistent with the large N_c results, where it is also observed that the spin-spin contribution (c_4) is much larger than the spin-orbit contribution (c_2) [2].

The mass shift due to strange quarks is given in the quark model by ΔM_{0s}. A comparison of this term with its large N_c counterpart is given in Fig. 1 (right panel), where we can see that the quark model predictions are always located within the error bars of the large N_c results. In both approaches, one predicts a mass correction term due to SU(3)-flavor breaking which decreases with N.

3.2. Heavy baryons

The heavy quark masses m_c and m_b can be independently fitted to the experimental data in both the quark model and the $1/N_c$ frameworks [4]. In large N_c QCD one obtains $m_c = 1315$ MeV and $m_b = 4642$ MeV, while the quark model mass formula (9) is compatible with the experimental data provided that $m_c = 1252$ MeV and $m_b = 4612$ MeV (the other parameters have been fitted to light baryons). Both approaches lead to quark masses that differ by less than 5%.

The other parameter involved in the large N_c mass formula is Λ, which in the ground state band can be identified to the mass formula (9) as follows: $\Lambda = c_0 = \frac{1}{3}\, M_1|_{N=0}$. According to the large N_c fits one has $c_0 = \Lambda \simeq 0.324$ GeV while the quark model gives 0.333 GeV, which means a very good agreement for the QCD scale Λ. The terms of order $1/m_Q$ lead to the identity $c_0^{'} = 2m_Q \, \Delta M_Q|_{N=0}$. The large N_c parameter $\Lambda = 0.324$ GeV gives $c_0^{'} \sim \Lambda^2 = 0.096$ GeV^2 and the quark model gives 0.091 GeV^2, which is again a good agreement. Finally, the SU(3)-flavor breaking term is proportional to $\epsilon\Lambda_\chi \sim m_s$. One should have $\epsilon\Lambda_\chi = \frac{2}{\sqrt{3}} \, \Delta M_{1s}|_{N=0}$ by definition of $\epsilon\Lambda_\chi$; indeed the large N_c value $\epsilon\Lambda_\chi = 0.206$ GeV and the quark model estimate 0.170 GeV also compare satisfactorily. We point out that, except for m_c and m_b, all the model parameters are determined from theoretical arguments combined with phenomenology, or are fitted on light baryon masses. The comparison of our results with the $1/N_c$ expansion coefficients c_0, $c_0^{'}$ and $\epsilon\Lambda_\chi$ are independent of the m_Q values. So we can say that this analysis is parameter free.

An evaluation of the coefficients c_2, c_2', and c_2'' through the computation of the spin-dependent effects is out of the scope of the present spin-independent formalism. But at the dominant order, the ratio c_2''/c_2 should be similar to $\mu_1 = 356$ MeV, which is roughly in agreement with equation (5) stating that $c_2''/c_2 \sim \Lambda$.

4. Conclusion

We have established a connection between the quark model and the combined $1/N_c$, $1/m_Q$ expansion both for light baryons and for heavy baryons containing a heavy quark. Our results bring reliable QCD-based support in favor of the constituent quark model assumptions and lead to a better insight into the coefficients c_i encoding the QCD dynamics in the $1/N_c$ mass operator. As an outlook, we mention that a combined quark model – $1/N_c$ expansion could lead to predictions for excited heavy baryons masses or even for ground state masses of baryons with two heavy quarks. This work is in progress.

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