Using time-dependent reference profiles for an instability analysis of an evaporating binary liquid layer

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Summary

This study treats an evaporating horizontal binary-liquid layer (aqueous solution of 10 \% wt ethanol) in contact with air with an imposed transfer distance. Solutal and thermal Rayleigh-Bénard-Marangoni instabilities are taken into account together with the Soret effect. The critical times with corresponding liquid thicknesses are calculated, showing that a critical liquid thickness can be found under which no instability can occur.

Keywords: Bénard-Rayleigh-Marangoni convection, transient profile, stability analysis, evaporating binary liquid

1. Introduction

This work concerns a study about the evaporation of a horizontal binary-liquid layer in contact with air. The liquid in consideration is an aqueous solution of ethanol. The evaporation causes concentration changes of ethanol at the liquid-gas interface accompanied with a cooling. These two phenomena can trigger both solutal and thermal Rayleigh and Marangoni instabilities in the liquid, which are to be studied in this work. For this purpose, the evaporation-induced instability is analysed taking into account horizontally uniform reference profiles. In previous work [1], the instability process, using quasi-stationary reference profiles, has been studied thoroughly. Making comparisons between the thermal, solutal and Soret Rayleigh and Marangoni effects, it appeared that the solutal Marangoni effect is by far the most important in stability mechanism. Extremely small critical liquid layer thicknesses were observed (the liquid layer thickness was used as a control parameter). This means that a liquid layer with a realistic thickness (say, 1 mm) would become unstable almost immediately after its contact with air, much before the transient diffusional boundary layers that develop from the free surface reach the bottom of the liquid. This makes considering a time-dependent reference diffusive solution of much interest for an instability analysis.

2. Model and outline

The physical system is a horizontal binary-liquid (10 wt\% ethanol in water) layer of thickness $d_l$ evaporating into an inert gas from a free flat interface on which a local phase equilibrium is assumed. The gas layer, of thickness $d_g$, consists of a mixture of an inert gas and the vapors of the solute (ethanol) and the solvent (water). At a distance $d_g$ above the interface (the transfer distance in the gas) a fixed vapor concentration is assumed, which in the present paper will be taken equal to zero, as well as a fixed temperature (300 K) and pressure (1 bar) of the environment. Air absorption in the liquid is neglected. The equations in the bulk of the liquid and gas phases and the boundary conditions except for one (zero-flux condition instead of the fixed-concentration condition) are the same as presented in [1], where also more details, a full mathematical formulation of the problem and the values of physico-chemical properties of the system can be consulted.

In this work, fully transient reference profiles are used for the instability analysis. The frozen-time approach and the normal-mode representation allow us to obtain for each time $t$ an eigenvalue problem for the complex growth rates $\sigma$ as function of the wavenumber $k$ and other parameters of the problem. In order to solve for
this eigenvalue problem, the Tau-Chebyshev decomposition method is used. The marginal condition is then found by taking $\sigma = 0$ (it turns out that all the eigenvalues $\sigma$ are real, which means that the instability is monotonic). Having already defined the liquid composition (10 wt% of ethanol in water) and the ambient conditions (an initial temperature and that of the environment $T_b$, atmospheric pressure $p_T = 1$ bar), the main control parameters that are kept are $d_l$ and $d_g$ (in the same way, $H \equiv (d_l + d_g)/d_l$). Other key parameters are $t$ and $k$. In this work, a parametric study is first presented at fixed values of $H$, obtaining a critical value of $d_l$ below which all $\sigma < 0$ at all times $t > 0$ and for all $k$. Second, for $d_l$ higher than this critical value, it is the purpose to calculate the instants of $t$ at which the system passes through the marginal condition ($\sigma = 0$) at a certain $k$, that is the critical times.

3. Results and conclusion

Fig. 1 shows an example of the results, presenting at the left side of the figure reference profiles of the mass fraction in the liquid and at the left side the critical time $t_c$ as a function of the liquid thickness $d_l$, both for $H = 11$.

![Figure 1](image)

Figure 1: Reference profiles of the mass fraction in the liquid corresponding to $t=0.00315$, 0.0126, 0.0315, 0.126, 0.315 and 1.26 (left) and the critical time $t_c$ as a function of the liquid thickness $d_l$ (right), both for $H = 11$ and $c_b = 0.1$.

The limit point close to $d_l \approx 18.5\mu m$ in Fig. 1 (right) is approximately characterized with the boundary layer reaching the bottom of the liquid, which is the most unstable configuration, knowing that this point signifies a critical liquid thickness below which no instability occurs. The upper branch of Fig. 1 (right) presents the limit above which the system regains its stability if the perturbation would not have changed the mass fraction profiles.

Acknowledgements

The authors gratefully acknowledge the financial support of the European Space Agency and of the Belgian Science Policy through the PRODEX-CIMEX and PRODEX-BOILING projects. This research was also partially supported by the ARCHIMEDES project funded by the “Communauté Française de Belgique“. PC gratefully acknowledges financial support of the Fonds de la Recherche Scientifique (FNRS).

References