ANALYSIS OF SHIP IMPACT ON LOCK GATES - SEINE-ESCAUT EST WATERWAY

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Abstract: This paper presents a ship impact analysis on lock gates. The case study is the downstream lock gates of the one of the four new locks planned within the framework of the "Seine-Escaut Est (SEE)" project in the Walloon Region of Belgium. The design and optimization process of the gate was realized with the LBR5 lock gate optimization software considering the cost and weight aspects of the structure. This method led to an optimized solution of the lock gate on which further studies were concentrated. After a brief overview of the state of art of ship impact analysis on lock gates, it was decided to perform a quasi-static analysis by finite elements. The non linear finite elements software FINELG was used to conduct a non linear numerical analysis of the effect of a boat impact on the gate. Several analyses were performed that highlighted the influence of the stiffener dimensions and the influence of the impact zone on the gate structural behavior submitted to impact. Finally, two different behaviors have been distinguished, a ductile one and a fragile one. The results showed the importance of the development of a global plastic mechanism, thanks to a ductile behavior, with the purpose of dissipating a large amount of energy.

Keywords: lock gate, structure optimisation, ship impact, crashworthiness analysis

1. Introduction

1.1. The "Seine-Escaut Est" Waterway Upgrading

"Seine-Escaut Est (SEE)" is an ambitious project with the purpose to connect the river basin of the Seine to the European waterway network towards Northern Europe and Central and Eastern Europe, to the Black Sea (Fig. 1). This connection affects a zone of first importance for Europe: this zone represents less than 4% of the surface of Europe-25, but it includes 12.6% of its population and concentrates 17% of its GDP (Fig. 2). Besides, the project connects the large seaports from the Havre, Antwerp and Rotterdam, which concentrate 60% of the maritime flows of Western Europe [2].

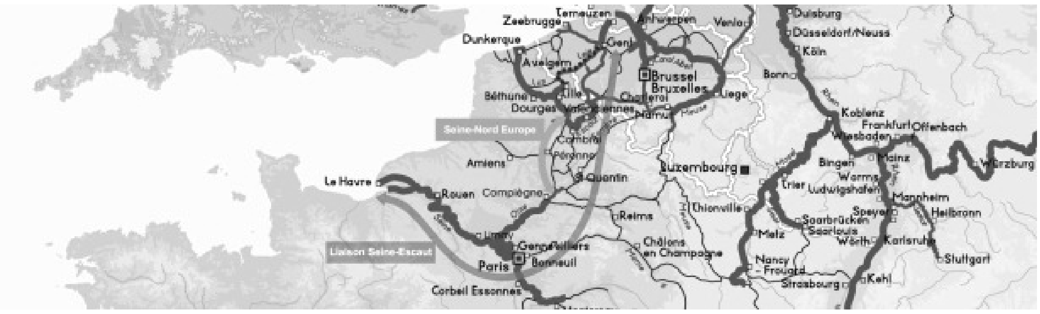


Fig. 1 - Seine-Escaut Est (SEE) connexion

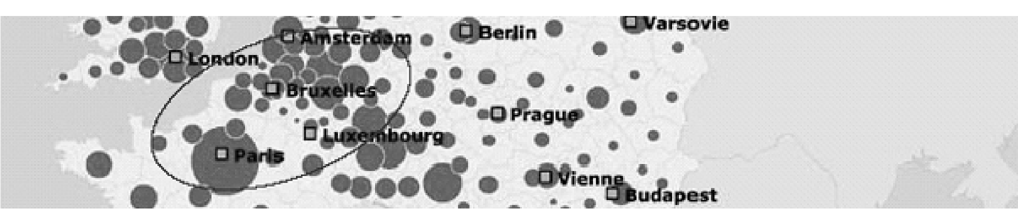


Fig. 2 - GDP by region [2]

The Walloon Region of Belgium plans several works to enlarge some hydraulic structures of its network so that it will be able to receive the new traffic generated by the “Seine Nord Europe” project, and to keep in this way its strategic position within the European waterway network. As part of these developments, four new locks of class V_a in Europe were planned on the section connecting the Schelde (Escaut) river basin and the Meuse river. The lock gate studied in this paper is based on the configuration of the downstream gates of these locks.

1.2. Characteristics of the Gate

At the stage of the basic preliminary design, realized collectively by University of Liege, Hydroconsult and the Service Public of Wallonia (SPW), it was decided to use suspended gates moved transversally to the lock. It was planned to use four identical downstream gates to take advantage of the standardization. The gates are 13.70 m length and 13.60 m height, whereas the gate width remains to be determined.

A lock gate is constituted by one or several plate elements called “panels”, which provide the function of watertightness, as well as a series of linear “beam” type elements, which support the waterproof panels and provide the strength of the gate. Beam elements include stiffeners, frames and girders. Assembling the panels with the beam elements gives what is called a stiffened panel. The design and optimization of a lock gate is the process by which a certain number of parameters are determined and optimized, as the thicknesses of the plates and the position and dimensions of the beam elements. For the considered study, a S235 steel grade was considered (235 MPA as yield stress) and an allowable stress of 175 MPa.

In addition, other design choices must be performed, in particular the question of the gate width as well as the possibility of using waterproof compartments (ballast tanks) to lighten the gate.

2. Design and Optimization of the Gate

2.1. Design and Optimization Process

The lock gate optimization was realized using the LBR5 software developed by Professor P. Rigo [13]. Four different models of downstream gate were analyzed: two models with additional ballast tanks and two different gate widths (Fig. 3), and two models without ballast tank also for two different gate widths (Fig. 4). The aim was to optimize each model and then to compare the optimized solutions to keep the most interesting design. The optimization of the models was realized by performing a multicriteria optimization; the criteria being the weight and the cost of the gate structure. To make it possible, the Pareto curve was calculated for each model using five optimized solutions of the same model but changing the ratio between the criteria. The Pareto curve is the curve giving the zone of the design space (adimensional cost versus adimensional weight) where the feasible solutions are located (Fig. 5).

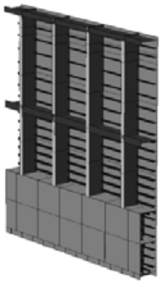


Fig. 3 - Gate with lower ballast tanks

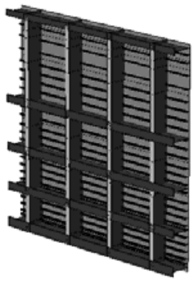


Fig. 4 - Without ballast tank

For each model, a same optimization process using several successive stages was followed, leading to an optimized feasible solution of the model. The considered load case was the exceptional hydrostatic load case for which the downstream section of the canal is empty, so that the maximum hydrostatic pressure is applied on the upstream side. The optimization was based on an elastic structural analysis. The risks of instability (buckling) of the stiffened elements (stiffeners, frames and girders) were taken into account by the definition of adequate slenderness ratio and the assessment of the ultimate capacity of the beam-column components. The risks of plate buckling were considered using the PLTBEN algorithm integrated into the LBR5 software [plate bending of O. Hugues (1988)].

Based on the Pareto curve of the gate model, it is possible to determine the optimum solution for this model in agreement with the criteria of selection of the decision-makers. The same process applied to the four models gives four optimum solutions differing by their initial choices of design (gate width, ballast tanks). The comparison of these four solutions based on the cost and weight of their structure is a key element to perform the best design choice. Nevertheless, only a more extensive study taking into account all the impacts of the initial design, at the same time on the gate structure and on the other parts of the lock, could ensure to make the best choice. Such an analysis is out of the scope of the present study.

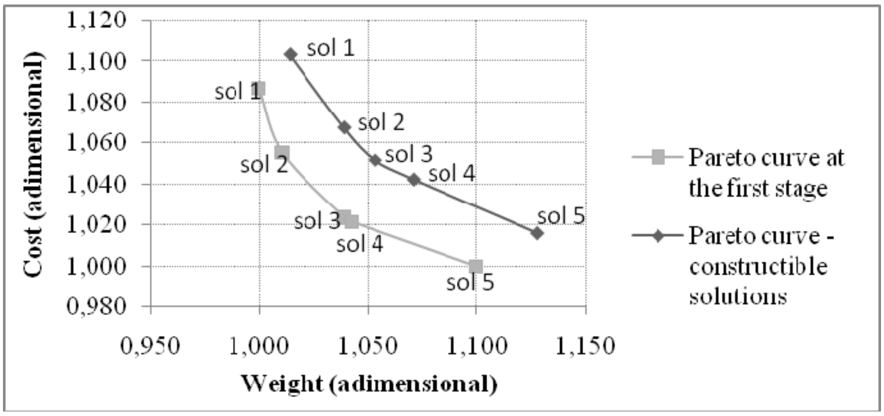


Fig. 5 - Pareto model 1 – 1st optimization and optimization of constructible solutions

As presented on Fig. 5, the feasible solutions are slightly more expensive and heavier than the solutions obtained after the first optimization run. Indeed, the first dimensions supplied by the software are rounded off and standardized to obtain at the end a feasible solution, easily constructible.

2.2. Optimum Solution

The design and optimization process led to an optimum solution for the lock gate consisting in a 1.0 m width gate (minimal value in order to place a footbridge) without ballast tanks. The ballast tanks generate a significant additional cost for a rather small benefit in terms of weight on the mechanisms. The optimum solution is presented at Fig. 6. The analysis of this solution was realized under all the hydrostatic load cases susceptible to act in service or exceptional situation. These analyses were made in both directions of loading (taking into account the two possible orientations of the gate) to consider successively the risks of instability of the plate and the reinforcement elements. It was verified that the stresses in the different elements of the stiffened panels remain below 175 MPa and that no instability phenomenon appeared. The total weight of the gate is 51.4 t and the production cost of the primary structure was estimated to 56,202 €.

Fig. 6 - Optimum solution

2.3. Finite Elements odel

A finite elements model of the optimum solution was realized using FINELG, a non linear finite elements modeling software [15]. The aim was to realize a non linear numerical analysis of the gate submitted to ship impacts. The symmetry of the structure was considered. Thus half a gate was modeled using 4,187 shell elements, 850 beam elements and 222 elements for linear constraints (Fig. 7). The mesh size is of 30 cm x 30 cm in the zones of low stress and 15 cm x 15 cm in the zones directly subjected to the ship impact and to higher stresses.

First of all, the model was tested by realizing a linear analysis with the hydrostatic load case studied in the previous stage with the LBR5 software. It allowed to validate the finite elements model and, on the other hand, to discuss the comparison between the LBR5 software and a finite elements analysis with FINELG. Very good concordance of the results given by

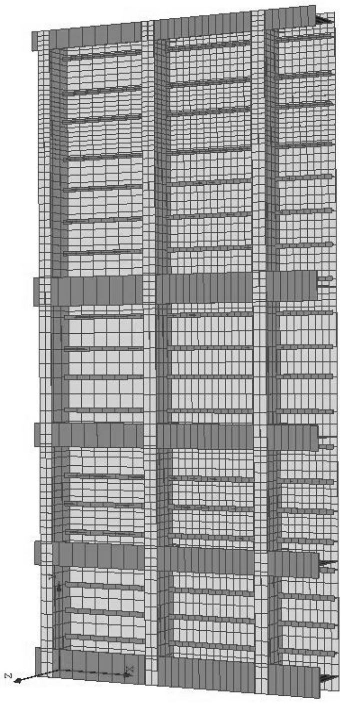


Fig. 7 - FEM of the gate

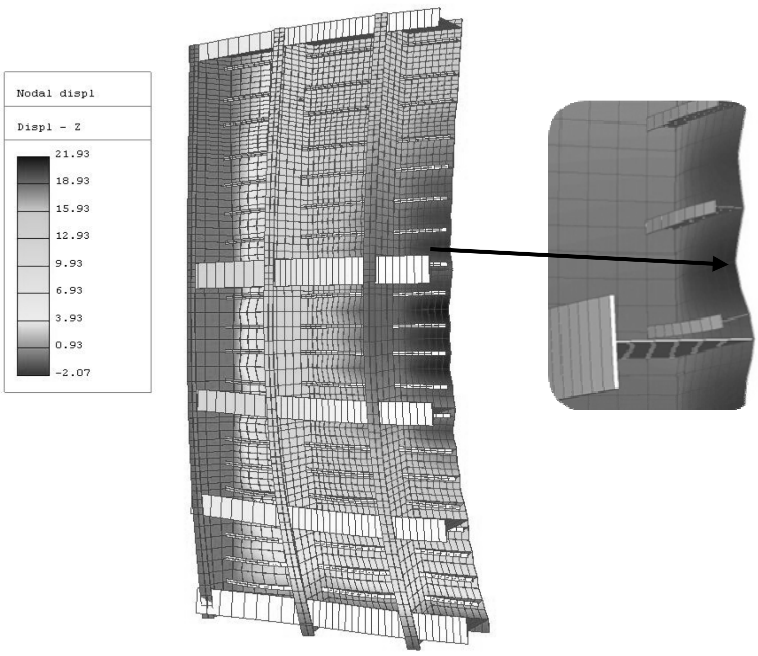


Fig. 8 - Service load deflection and local effect (x80)

both software was obtained except for the maximum deflections: the maximum deflection given by the finite elements linear analysis was 20% superior to the value given by the LBR5 analysis. The reason is that LBR5 does not consider the local bending of the plate between two frames and two stiffeners. So, the plate deflections given by LBR5 and FINELG along the girders are equal (no local effect) whereas they can differ by 20% in the middle of an unstiffened plate, the deflections given by FINELG being the biggest. This difference is due to the local plate bending (Fig. 8).

3. State of Art of the Ship Impact Analysis

3.1. Specifications

Design recommendations for the ship impact load cases are not clearly determined in the currently used practice codes. Some decisive decisions are left to the designers. First of all, the "vessel impact" must be defined in terms of ship weight and speed. This has to be defined by the client and to be consistent with the project. Then, a decision must be taken on whether the gate must have sufficient impact strength or protective measures must be designed in order to prevent the ship from impacting the gate. Both solutions have to be compared on an economic basis. In Germany, the downstream side is generally equipped with a protection system (on the chamber side). This protection system cannot be very stiff because it must avoid destruction of the ship, since a sinking ship will result in a long downtime period for the lock [10].

If the gate has to serve as a ship stopping device, an analysis of the ship impact must be performed on the gate structure. Different kinds of possible ship impact analysis are presented below with their principal characteristics. The analysis to perform depends notably on the importance of the project and the time and money that can be spent on the study.

3.2. Empirical Approach

These methods are based on empirical data and practice experience. They offer a very simple way to evaluate an order of magnitude of the impact strength of a lock gate but their simplicity does not allow a correct representation of an impact phenomenon. They can only be used as a rule of thumb. Always more detailed analysis must be performed (see here after).

3.3. Analytical-rational Approach

Analytical models can represent simple cases of impact with a good accuracy. Some hypothesis have to be made on the strain state of the gate but numerical studies with uniform gate structures have highlighted the ways of dissipating energy which should be considered, i.e. a local deformation of structure elements in the vicinity of the impact and a global bending around plastic hinge lines [7]. The principal assumption in analytical analysis is that the totality of the energy brought by the ship is dissipated by the gate. Various numerical studies have validated this assumption [6, 7].

Those models do not take into account instability phenomena that could appear during the rotation of the plastic hinge lines, considering the global plastic failure mechanism. Such instabilities could reduce the structure capacity for energy dissipation. However, such models, if they are correctly applied, can be seen as very effective and time-saver for gate structure with plane geometry.

3.4. FEM, Quasi-static Analysis

Finite elements methods can be used to analyze the ship impact. As dynamic effects are usually not significant for lock gate, a quasi-static analysis can be seen as sufficient to model the impact. One possible method is to consider the bow of the ship as perfectly stiff and so to apply a quasi-static load on the gate structure until equalization of the strain energy of the gate with the initial kinetic energy brought by the ship. The major approximations of this method are that the totality of the energy is dissipated by the gate; there is no dynamic effect and no evolution of the contact between the bow and the gate. Nevertheless, this kind of analysis gives good results when a dynamic analysis can't be performed.

3.5. FEM, Dynamic Analysis

Such analysis allows modeling the deformable bow of the ship so that giving an initial position and speed, the contact between the bow and the gate can be considered. Moreover, dynamic effects are taken into account. This is unfortunately a highly time consuming method –main disadvantage. Using this method for few study cases can offer reference results to validate assumptions made in analytical models or quasi-static finite elements methods.

4. Ship Impact Analysis

4.1. Hypothesis

In this study, it was decided to perform a quasi-static analysis by finite elements method to analyze the effect of the ship impact on the previously designed lock gate. It was assumed that the totality of the energy brought by the ship was dissipated by the gate as strain energy. A quasi-static equivalent load was defined to perform the analysis, neglecting the dynamic effects. According to this approach, it was possible to link the initial kinetic energy of the ship to a given strain state of the gate as follows:

$$\begin{cases} W_E = E_{kinetic,initial} \\ E_{dissipated} = F_{impact} \times \delta \end{cases} \tag{1}$$

$$W_E = E_{dissipated} \Rightarrow E_{kinetic,initial} \leftrightarrow \delta \tag{2}$$

The constitutive law of the steel is an elastic – perfectly plastic law. The impact was applied by increasing a uniform force on a perfectly rigid element which represents the ship bow.

Three different scenarios of impact were studied to allow discussing on the influence of the hydrostatic loads and the impact zone (Fig. 9).

1. The ship impacts the gate in its upper part (at upstream water level: U.W.L.), but the hydrostatic loads are neglected. This first analysis allows the characterization of the ship impact effects.
2. The ship impacts the gate in its upper part (at upstream water level) while the hydrostatic service loads are already applied to the gate. This analysis models the case of the ship entering in the lock from upstream and hitting the downstream gate.
3. The ship impacts the gate in its lower part (at downstream water level: D.W.L.). There is no hydrostatic load to take into account since the water-levels are identical on both sides of the gate.

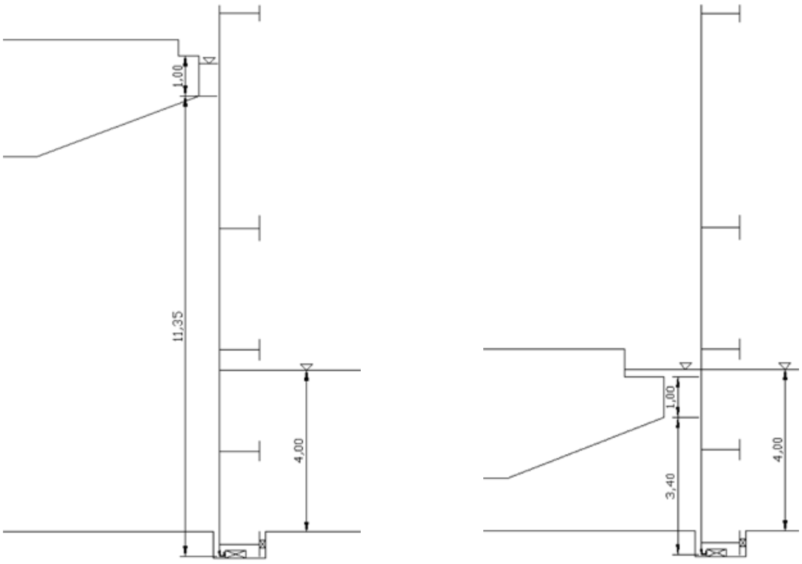


Fig. 9 - Impact at upstream water level and downstream water level

The first analysis is the analysis of the impact at upstream water level (U.W.L.), without any hydrostatic load, considering the gate structure designed and optimized in part 0 using the LBR5 software. The finite elements model of the gate was defined in part 0.

3.2. U.W.L. Impact with the Initially Optimized Structure

The effect of an upstream water level impact on the gate structure elastically designed with LBR5 was investigated in this part. For this structure, the slenderness ratio of the stiffened panels respects Hugues' criteria for T-elements [5]. Hugues' criteria fit with the Eurocode class 3 [1]: they guarantee that the section is able to develop its elastic bending moment before collapsing through buckling, but not its fully plastic bending moment. This slenderness is perfectly adapted for structures working in the elastic field but, on the other hand, they are not enabled to take advantage of the plastic field.

The results of the non linear numerical ship impact analysis conducted with FINELG are given on Fig. 10. The evolution of the impact force is given as a function of the indentation. The observed behavior is fragile: the collapse appeared suddenly, while the structure stiffness was still significant. Consequently, the capacity for energy dissipation was weak. The point representing the impact effect of a 2,400 t ship at 0.25 m/s (initial kinetic energy of 75 kJ) is plotted on the curve of Fig. 10.

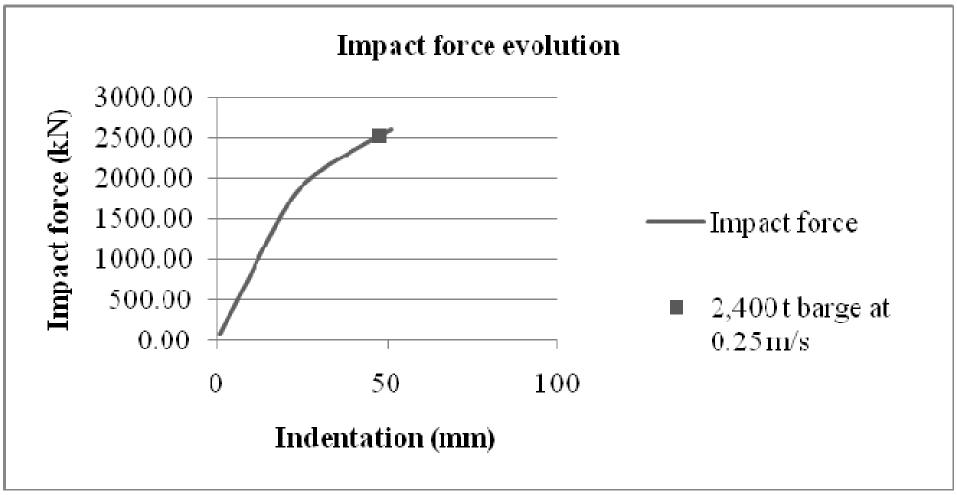


Fig. 10 - Impact force evolution for an upstream water level impact on the initially designed structure

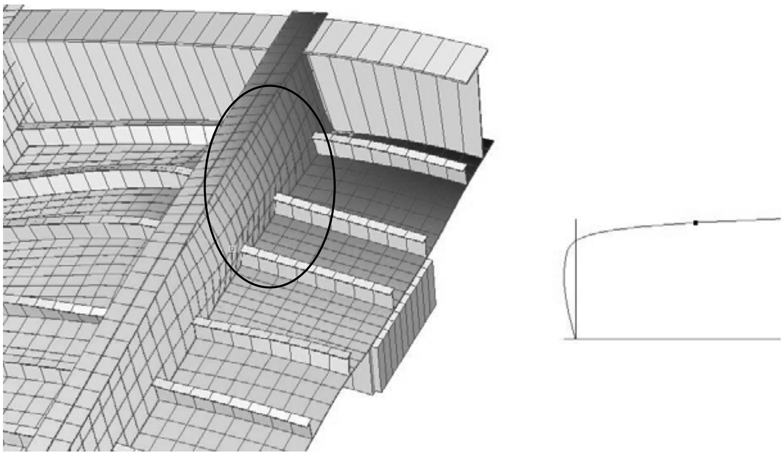


Fig. 11 - Buckling of the central frame and curve load-displacement of one node of this frame

The analysis of the strain level at collapse stage allows a better understanding of the structure behavior. The buckling of the central frame is clearly visible. This buckling led to a sudden, fragile collapse.

Fig. 11 shows this buckling phenomenon. Therefore, the structure was not able to develop plasticity; its capacity for energy dissipation was indeed extremely weak.

As a result of this first analysis, it was decided to reinforce the structure to provide a better behavior in case of impact. The aim was to avoid buckling phenomenon in the main reinforcement elements, which would prevent the structure from developing a yielding behavior and would induce prematurely collapse of the gate. Thus, it was decided to increase the thickness-height ratios of the sections of the primary reinforcement elements (frames and girders) to use class-1 sections according to the Eurocode classification. As a reminder, a class-1 section is a section able to develop its fully plastic bending moment and to sufficiently rotate to allow the structure to form a global plastic mechanism. The next analyses were all performed with such reinforced structure.

3.3. U.W.L. Impact with the Reinforced Structure

The frames and the girders of the gate were reinforced to be class-1 elements. Their web thickness was increased from 10 mm to 20 mm and their flange thickness from 17 mm to 25 mm. The total weight of the structure has gone up from 51.4 t to 68.7 t (+34%). The results of the non linear numerical analysis of the impact on this structure are given on Fig. 12, next to the curve of the initially optimized structure. Different impact levels are marked on these curves.

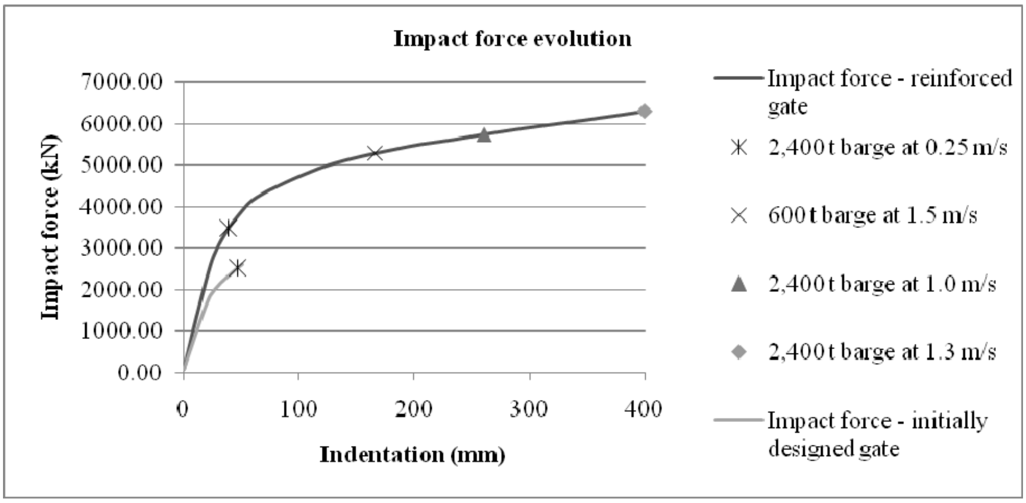


Fig. 12 - Impact force evolution for an upstream water level impact

As it can be observed, the global behavior of the structure for the same impact scenario is fundamentally different after increasing the gate stiffness. The response of the class-1 higher stiffened structure was ductile; its capacity for energy dissipation was very significant (the kinetic energy of a 2,400 t barge at 1.3 m/s is 2,028 kJ). The choice of such a higher rigid structure allowed reaching a much more favorable behavior in case of ship impact.

The analysis of the strain state of the collapsed structure and the plastic hinges highlights the formation of a global plastic failure mechanism (Fig. 13). This plastic failure mechanism strongly contrasts with the fragile failure of the initial structure, in which there is almost no yielding developed before the ultimate stage (collapse).

The analysis of the global plastic failure mechanism shows that the loss of stiffness of the structure, visible on the Fig. 12 curve is due to the successive plastic hinges in the girders. In order to provide the structure ductility, and so a good capacity for impact absorption, it is important to ensure a good ductility of these main girders. Fig. 14 shows the points on the load-displacement curve where successive plastic hinges appear in the girders. These points correspond to the loss of stiffness of the gate. Finally, Fig. 15 shows the global plastic failure mechanism developed by the gate, including two plastic hinges lines along the gate height.

Locally, it is interesting to notice that the maximum strain reaches 6.4% at the failure stage. In other words, when the gate structure was submitted to the impact of a 2,400 t barge at 1.3 m/s, the maximum strain did not exceed 6.4%. In the minor collision analysis performed by Mc Dermott [8], the critical rupture strain for mild steel material in side collision is evaluated from the tensile ductility, so that $\varepsilon_c \approx 10\%$. It means that the gate structure could develop its plastic failure mechanism and absorb the energy corresponding to the impact of a 2,400 t barge at 1.3 m/s, without apparition of local failure.

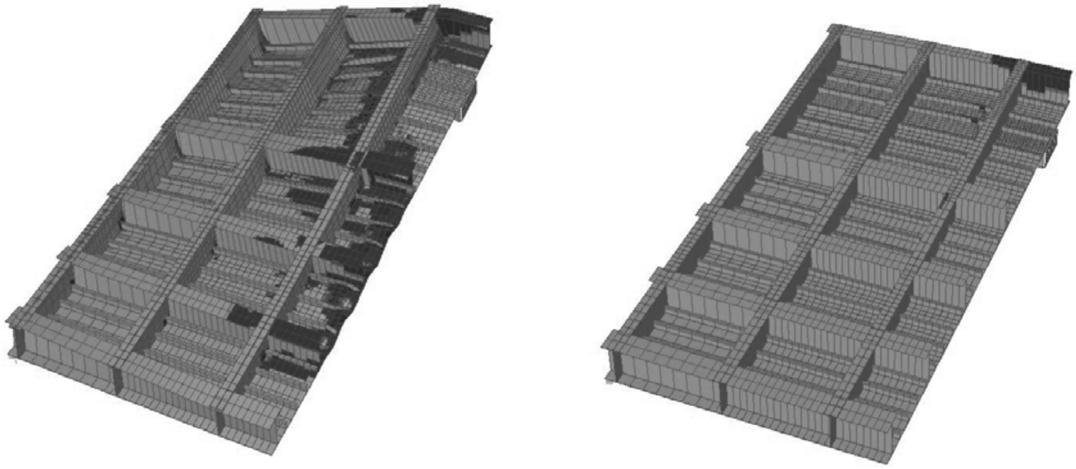
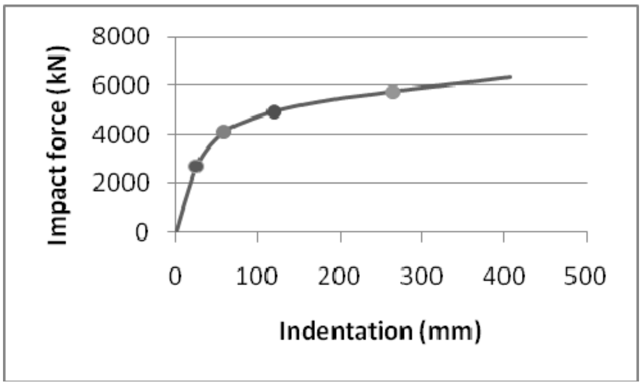


Fig. 13 - Yielding at the collapse stage – reinforced structure (left) and initially designed structure (right)



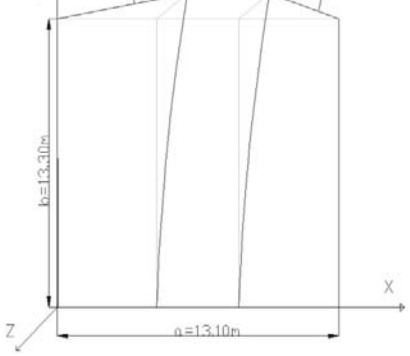


Fig. 14 - Plastic hinges occur in the girders

Fig. 15 - Global plastic failure mechanism

3.3. Taking into Account the Hydrostatic Loads

This analysis concerns an upstream water level impact combined with the hydrostatic load applied on the gate. First, the hydrostatic load was applied and then, keeping the water pressure constant, the impact was applied. On Fig. 16, the evolution of the impact force with and without hydrostatic load is presented. Note that the displacement given on the curve ("Impact force with hydrostatic loading", Fig. 16) is the displacement only due to ship impact. It is different from the total displacement, resulting of the sum of the hydrostatic load and the impact.

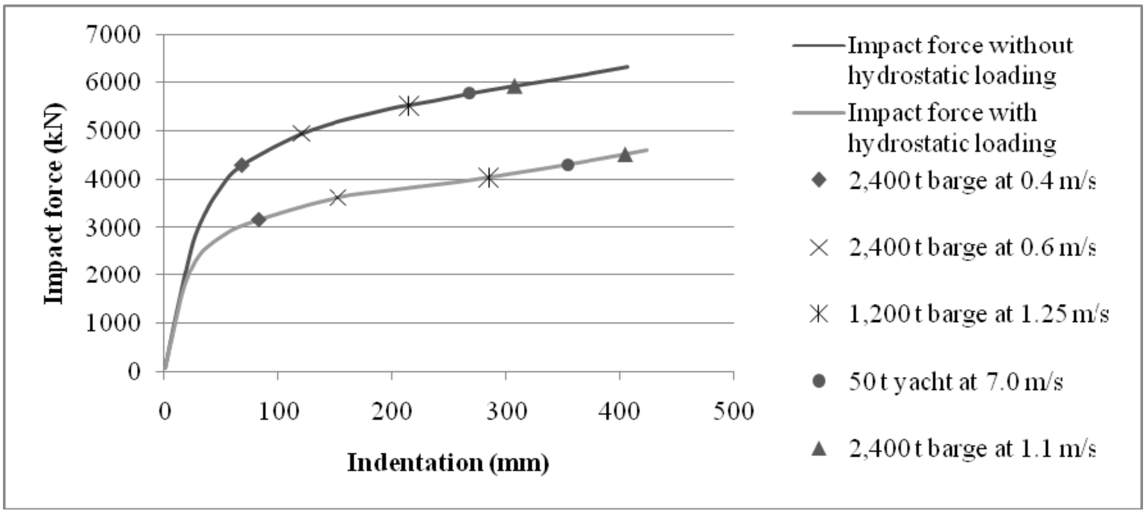


Fig. 16 - Impact force evolution for an U.W.L. impact, with and without hydrostatic load

The global behavior of the gate was identical but the structure was more deformable when it was previously submitted to the hydrostatic load. Indeed, since the water pressure was applied, the gate was already submitted to a stress field. Then, when the ship impacted the gate, plasticity appeared faster in the gate elements. Consequently, for a same impact load, the indentation was more significant with hydrostatic load. Besides, yielding was increased. On the contrary, the impact force was reduced.

As a conclusion, the global behavior of the gate is unchanged whether the hydrostatic load is applied or not. On the other hand, neglecting this load during the impact analysis leads to underestimate the deformation and underestimate the yielding of the structure. So, a method which first considers the hydrostatic load, and then adds the deformations due to the only ship impact, underestimates the deformation level and the yielding of the structure. But this approach is safe from the point of view of the impact force. So, such a method is conservative to assess, for instance, the maximum reaction susceptible to act on supports.

3.4. Impact on the Downstream Side of the Gate

The next analysis deals with the case of a downstream side impact. In this case, there is no situation where a hydrostatic load could be added to the impact effect and increase this effect. The study of this case allows analyzing the influence of the impact zone. Here, the ship hits the gate in a highly more stiffened zone than in the previous cases. Fig. 17 shows the impact force evolution in the case of a downstream side impact, next to the correspondent curves for an upstream water level impact.

The gate structure was more fragile for a downstream side impact compared with an upstream side impact. The impact force increased much more quickly and reached significantly more important values while the indentation remained small. Finally, the collapse arose suddenly for an impact of energy in the order of 450 kJ.

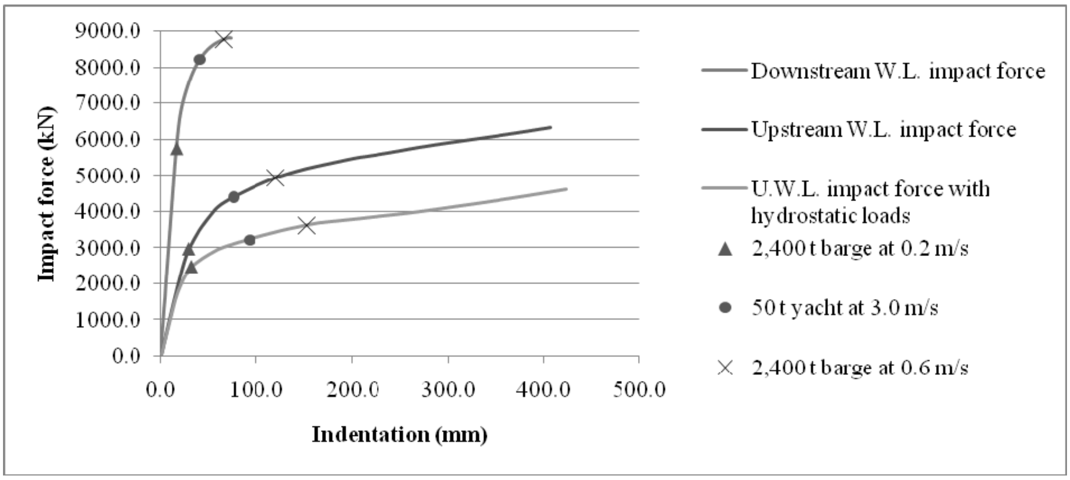


Fig. 17 - Impact force evolution for different impact cases

The strain pattern in the gate at the collapse stage shows that there were strain concentrations in the impact zone, mainly in the frame in contact with the barge bow (Fig. 18). This strain peak was due to the small ratio between the transverse and longitudinal stiffness in this zone, which prevented the propagation of yielding and thus the development of a global plastic failure mechanism. It was harmful to the structure ductility and thus to its energy dissipation capacity. Finally, the collapse arose by frame buckling at the level of the plastic hinge, because the rotation of this frame became too big. When this collapse occurred, the indentation was still small because the plastic deformations had not propagated much.

Then, the analysis of the downstream side impact clearly shows the importance of a study on the stiffness ratios in the potential impact zones. For such gate, the transverse elements (frames) stiffness should be higher compared with the longitudinal elements (girders) stiffness, in order to guarantee a good propagation of the plastic deformations and, ideally, the development of a global plastic failure mechanism.

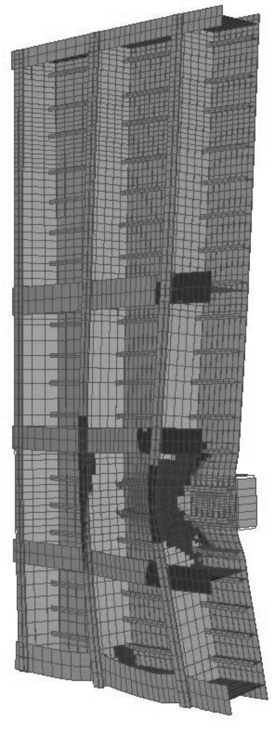


Fig. 18 - Yielding at the collapse stage, downstream side impact

3.5. Results of the Impact Analysis

The results of the ship impact analysis are summarized below. Fig. 19 gives the relationship between the impact speed and the indentation for a 2,400 t barge, function of the impact zone. Table 1 gives the effect of a determined impact for the different studied cases.

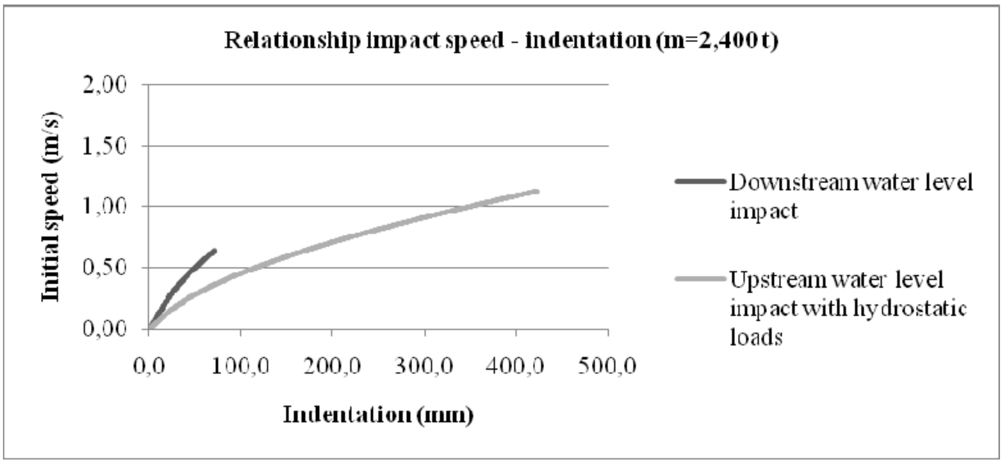


Fig. 19 - Relationship between the impact speed and the indentation for a 2,400 t barge, in the cases of downstream and upstream side impacts

Table 1 - Synthesis of the effects of a 1,200 t barge impact at 0.8 m/s, function of the impact zone and the hydrostatic loads

Impact of a 1,200 t barge at 0.8 m/s (384 kJ)	**U.W.L. without hydrostatic loads**	**U.W.L. with hydrostatic loads**	**D.W.L.**
Impact force	4,845 kN	3,550 kN	8,706 kN
Indentation (only due to the impact)	11.1 cm	13.9 cm	5.9 cm
Number of plastic hinges in frames and girders	2 girders	3 girders	1 frame

The main point learned from this analysis of a gate impact was the interest of developing a global plastic failure mechanism to provide the structure a good carrying capacity for energy dissipation. As it was shown a condition to allow this behavior was to use class-1 cross sections for the main reinforcement elements (frames and girders). In addition, it was necessary to guarantee adequate stiffness ratios in the impact zone, the structure response being fundamentally different depending on whether the impact happened at downstream or upstream side. For an upstream side impact, the behavior was ductile, which allowed absorbing significant impacts. For a downstream side impact, the behavior was fragile, which reduced the impact strength.

4. Conclusion

A design and optimization process of a lock gate using the LBR5 software has been applied to the study case of the one of the four new locks projected by the Walloon Region of Belgium (SPW) within the framework of the “Seine-Escaut Est” project. This work led to an optimized solution in terms of cost and weight for the lock gate structure, on which a ship impact analysis has then been realized.

In this considered example, the aim was to design a gate able to resist the ship impact by itself. To provide the gate a good capacity for dissipating energy, it is necessary to provide a ductile behavior for the gate structure, i.e. to avoid instability phenomenon of its primary reinforcement elements and to allow the gate forming a global plastic failure mechanism. Ductility of the elements can be achieved by using EN class-1 cross sections. Ductility of the structure requires a good propagation of yielding, which can be achieved by an adequate design of the stiffness ratios in the potential impact zones.

The elastic design and optimization of the lock gates considering hydrostatic load cases, as performed in the first part of this work, is a current practice. However, the gate impact analysis may force the designer to fulfill some other constraints in order to provide a ductile behavior for the gate structure, guaranteeing it good impact strength. Future studies should focus on the determination of these constraints. The objective is finally to derive from complex numerical analysis simple design guides to recommend to practitioners dealing with the problem of ship impact. In this case, these constraints should be included from the design and optimization stage. For instance, the present analysis has brought the author to increase the dimensions (cross section) of the frames and the girders of the optimum solution to obtain class-1 cross sections, which has been proved to provide ductility to the gate elements. Consequently for design purpose the main recommendation is to implement in the optimization software a new constraint that consists in using only class-1 cross sections for the frames and the girders. It would permit to obtain optimized solutions considering impact strength. Then, this solution should be compared in term of cost with the elastic optimum solution coupled with protective measures against ship impact.

References

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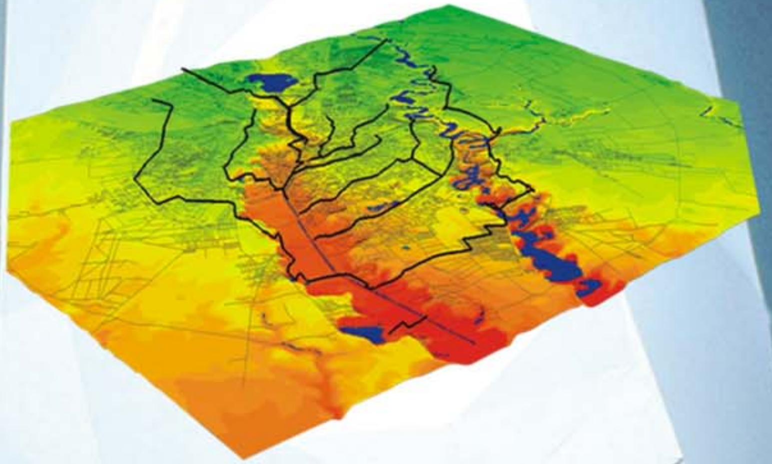

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