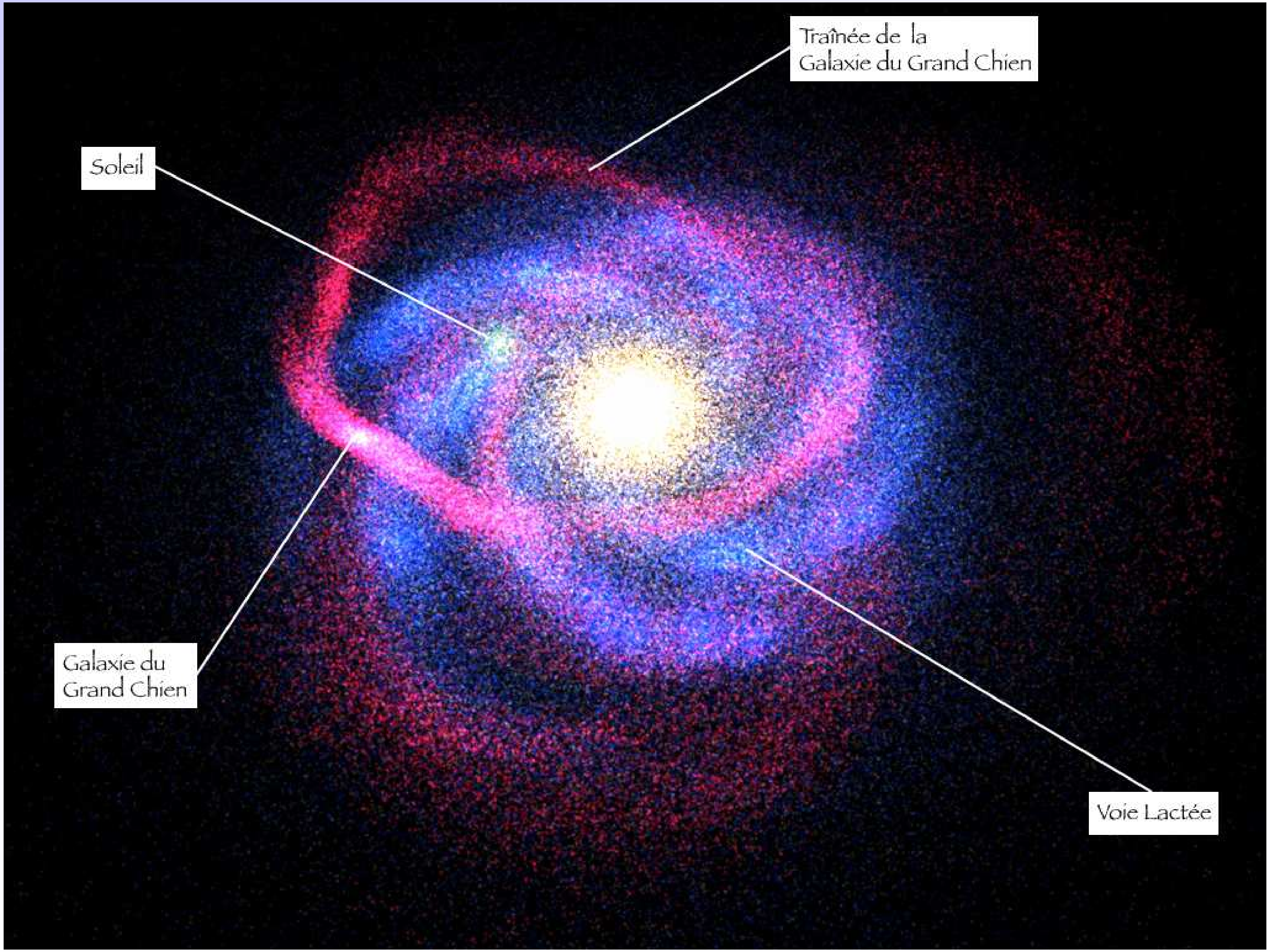
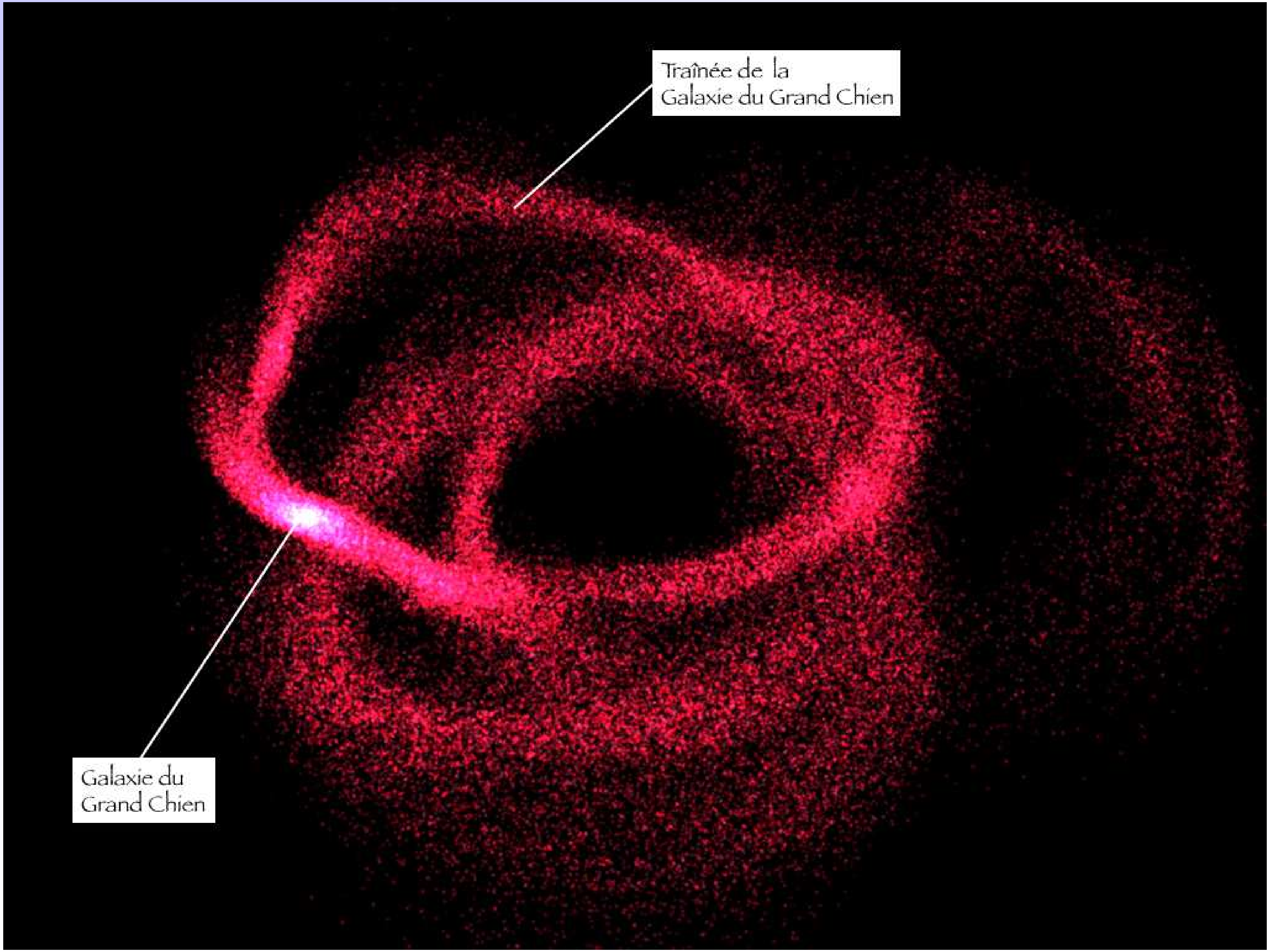
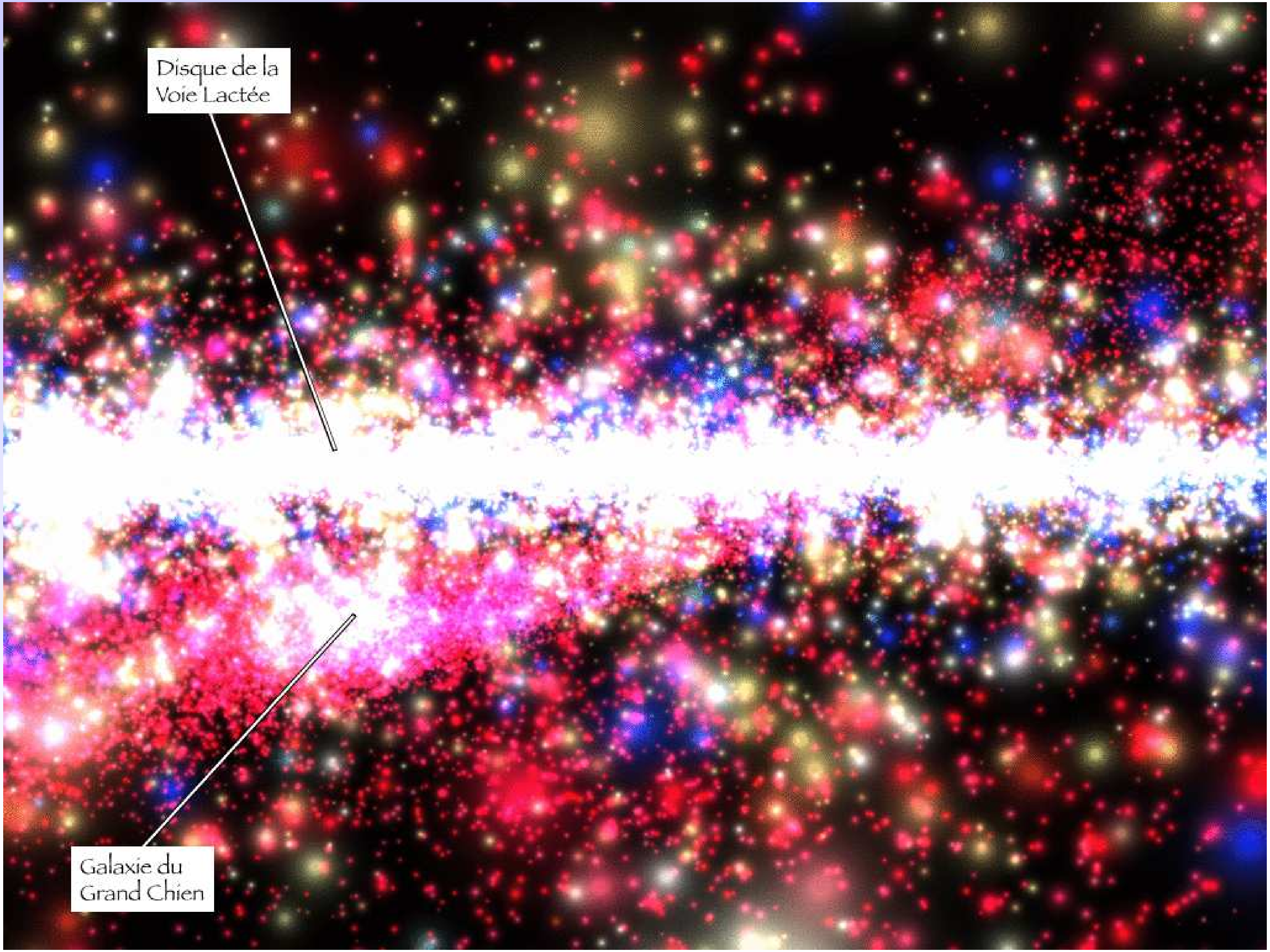
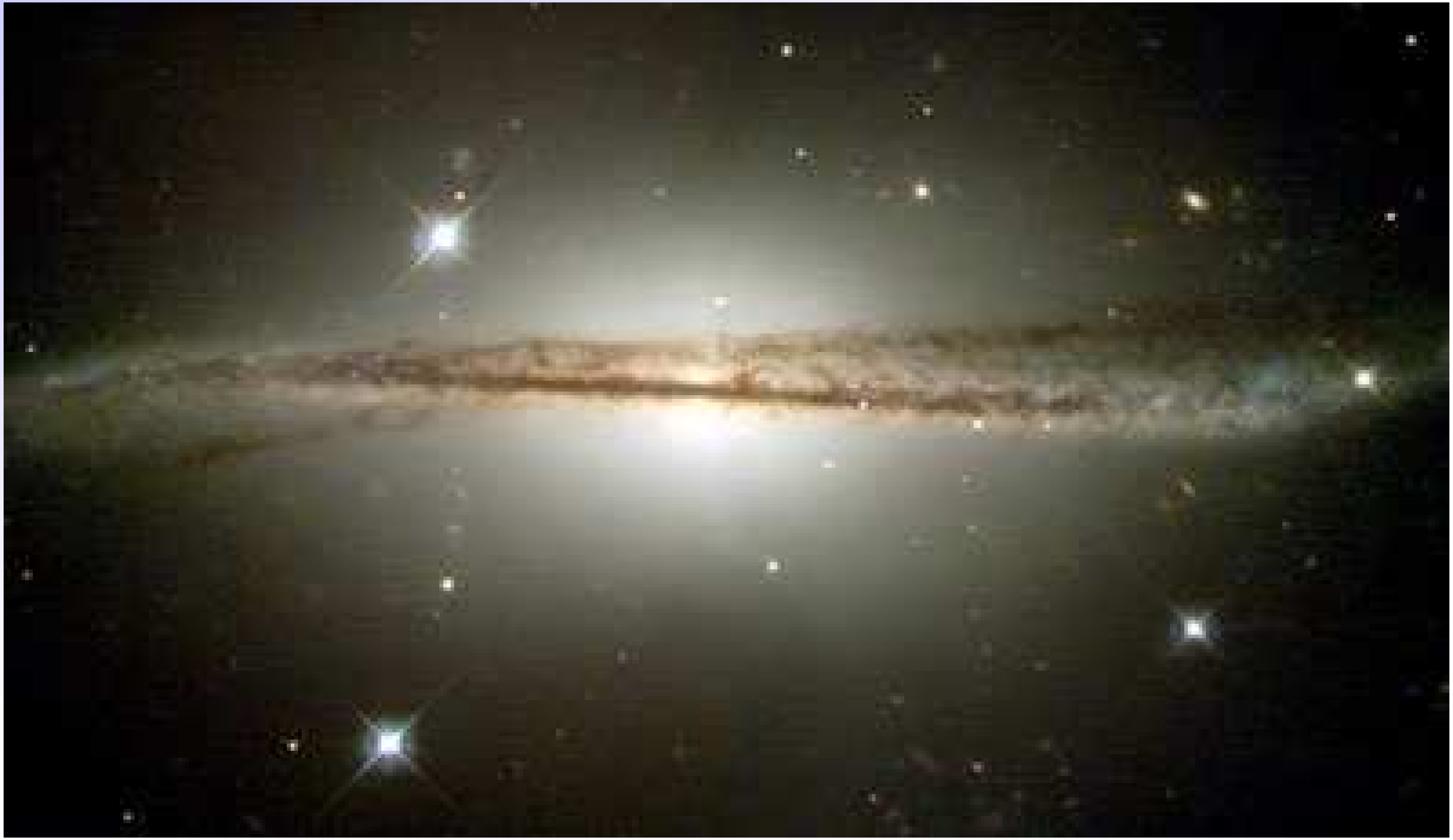
9ème cours de Mécanique Analytique (4/11/2010)




Traînée de la
Galaxie du Grand Chien
Galaxie du
Grand Chien


Disque de la
Voie Lactée
Galaxie du
Grand Chien



- 3.7 La mécanique relativiste du point matériel

$$m_0 \frac{d\vec{v}}{dt} = \vec{F}$$

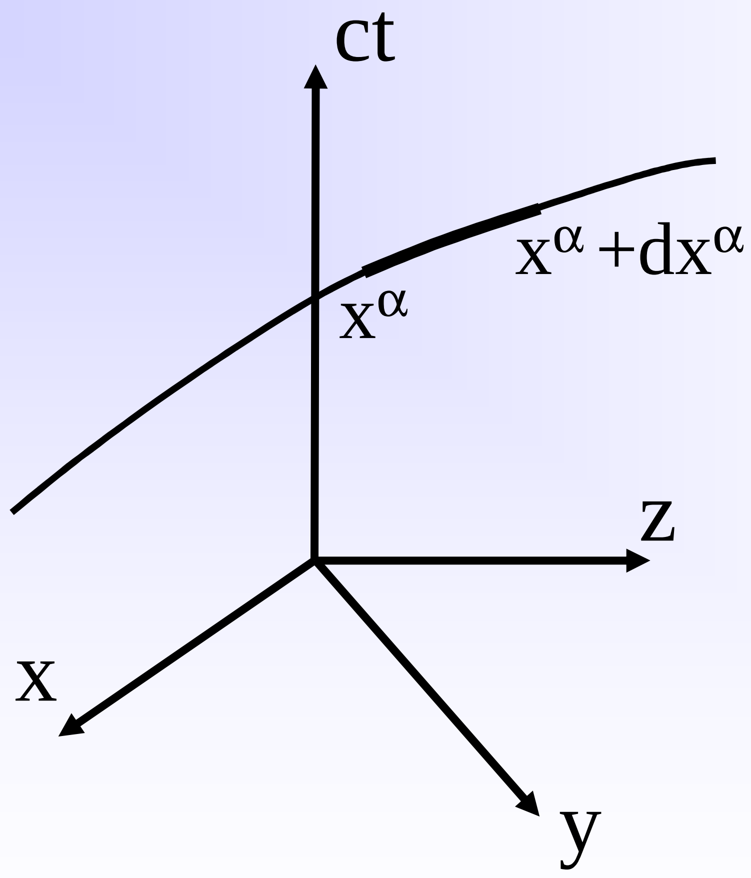
$$\cancel{\frac{d}{dt}} \qquad \frac{d}{d\tau}$$

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \qquad (3.41)$$

$$x^\alpha, \quad x^\alpha + dx^\alpha$$

$$x^\alpha = ct,\ x,\ y,\ z$$

• 3.7 La mécanique relativiste du point matériel



• 3.7 La mécanique relativiste du point matériel

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = (v^2 - c^2) dt^2 < 0$$

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2$$

Si $d\tau^2 = -ds^2/c^2$, alors

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}\, dt \qquad (3.42)$$

• 3.7 La mécanique relativiste du point matériel

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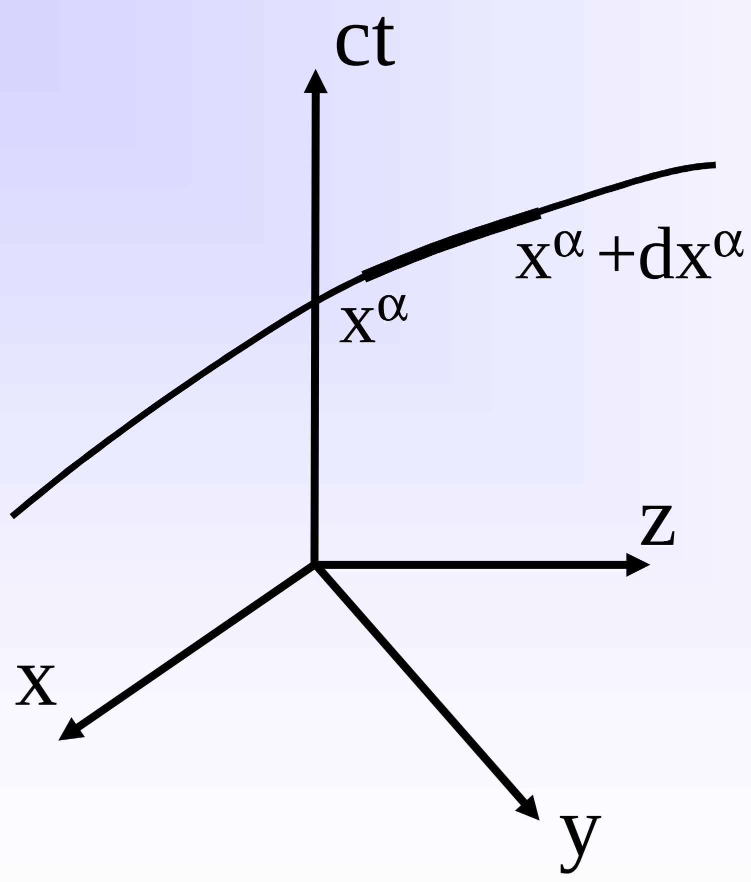
• 3.7 La mécanique relativiste du point matériel



• 3.7 La mécanique relativiste du point matériel

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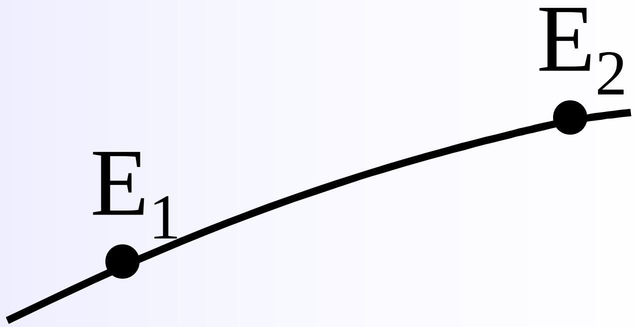
$$\Delta\tau = \int_{t_1}^{t_2} \sqrt{1 - \frac{v^2}{c^2}}\, dt \qquad (3.43)$$



$$\tau = \int \sqrt{1 - \frac{v^2}{c^2}}\, dt \qquad (3.44)$$

$$\left.\begin{array}{l} ct = ct \\ x = x(t) \\ y = y(t) \\ z = z(t) \end{array}\right\} x^\alpha = x^\alpha(t)$$

$$\tau = \tau(t) \quad , \qquad t = t(\tau)$$

$$x^\alpha = x^\alpha(\tau) \qquad (3.45)$$

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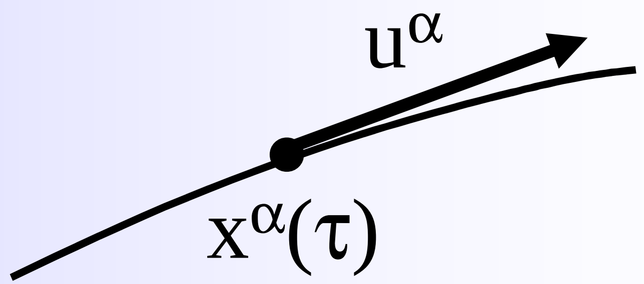
$$u^\alpha = \frac{dx^\alpha}{c\,d\tau}$$ (3.46)



$$\eta_{\alpha\beta}u^\alpha u^\beta = \eta_{\alpha\beta}\frac{dx^\alpha}{c\,d\tau}\frac{dx^\beta}{c\,d\tau}$$

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta = -c^2 d\tau^2$$

$$\eta_{\alpha\beta}u^\alpha u^\beta = -1$$ (3.47)

- 3.7 La mécanique relativiste du point matériel

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}\, dt$$

$$u^\alpha = \frac{dx^\alpha}{c\, d\tau} \qquad v^i = \frac{dx^i}{dt}$$

$$u^o = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad , \quad u^i = \frac{v^i/c}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$u^\alpha = \frac{(1,\ v^i/c)}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.48)$$

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$$m_0 c \frac{du^\alpha}{d\tau}$$

$$\boxed{m_0 c \frac{du^\alpha}{d\tau} = K^\alpha} \qquad (3.49)$$

$$\boxed{\eta_{\alpha\beta} K^\alpha u^\beta = 0} \qquad (3.50)$$

$$\begin{aligned} \eta_{\alpha\beta} K^\alpha u^\beta &= \eta_{\alpha\beta} m_o c \frac{du^\alpha}{d\tau} u^\beta \\ &= \frac{m_0\ c}{2} \frac{d}{d\tau} (n_{\alpha\beta} u^\alpha u^\beta) = 0 \end{aligned}$$

• 3.7 La mécanique relativiste du point matériel

$$m_0 c \frac{du^\alpha}{d\tau} = K^\alpha \text{ , avec } u^\alpha = \frac{dx^\alpha}{c\,d\tau}$$

Si $K^\alpha = 0 \Rightarrow$ $u^\alpha = \text{constante}$

$$\frac{v^i/c}{\sqrt{1-\frac{v^2}{c^2}}} = \text{constante} \text{ , } \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \text{constante}$$

$$v^i = \text{constante}$$

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$$m_0 c \frac{du^i}{d\tau} = K^i \qquad (3.51)$$

$$\frac{d}{dt}(mv^i) = F^i \qquad (3.52)$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.53)$$

$$F^i = \sqrt{1 - \frac{v^2}{c^2}}\, K^i \qquad (3.54)$$

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$$\frac{d}{dt}(mv^i) = F^i$$

$$\boxed{\tfrac{d}{dt}(m\vec{v}) = \vec{F}} \qquad (3.55)$$

$$\boxed{p^\alpha = m_0 c u^\alpha} \qquad (3.56)$$

$$p^i = m_0 c u^i = m v^i \qquad (3.57)$$

$$\vec{p} = m\vec{v} \qquad (3.58)$$

$$\frac{d}{dt}\vec{p} = \vec{F} \qquad (3.59)$$

• 3.7 La mécanique relativiste du point matériel

$$m_0 c \frac{du^0}{d\tau} = K^0 \qquad (3.60)$$

$$\frac{d}{dt}(m\,c) = \sqrt{1-\frac{v^2}{c^2}}\,K^0 \qquad (3.61)$$

$$\eta_{\alpha\beta} K^\alpha u^\beta = \delta_{ij} K^i u^j - K^0 u^0 = 0$$

$$K^0 = \frac{\delta_{ij} K^i u^j}{u^0} = \frac{1}{c}\frac{\vec{F}\cdot\vec{v}}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

$$\boxed{\frac{d}{dt}(m\,c^2) = \vec{F}\cdot\vec{v}} \qquad (3.62)$$

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$$\boxed{\frac{d}{dt}(m\ c^2) = \vec{F} \cdot \vec{v}} \qquad (3.62)$$

$$\boxed{T = mc^2 - m_0c^2} \qquad (3.63)$$

$$\boxed{\frac{dT}{dt} = \vec{F} \cdot \vec{v}} \qquad (3.64)$$

$$\boxed{mc^2 = m_0c^2 + T} \qquad (3.65)$$

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$$T = (m - m_0)\, c^2 = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) m_0 c^2$$

$$T = \frac{1}{2} m_0 v^2 \left[1 + \frac{3}{4} \frac{v^2}{c^2} + \dots \right] \qquad (3.66)$$

$$p^\alpha = m_0 c u^\alpha = \left(\frac{1}{c} m c^2, m v^i \right) \qquad (3.67)$$

- 3.7 La mécanique relativiste du point matériel

$$p^\alpha = m_0 c u^\alpha = \left(\frac{1}{c} m c^2, m v^i\right) \qquad (3.67)$$

mc^2 $\qquad$ $\vec{p} = m\vec{v}$

$$\boxed{m^2 c^4 = m_0^2 c^4 + p^2 c^2} \qquad (3.68)$$

$$\begin{aligned} \eta_{\alpha\beta} p^\alpha p^\beta &= m_0^2 c^2 \, \eta_{\alpha\beta} u^\alpha u^\beta = -m_0^2 c^2 \\ &= -(p^0)^2 + \delta_{ij} p^i p^j \\ &= -m^2 c^2 + p^2 \end{aligned}$$

• 3.7 La mécanique relativiste du point matériel

$$m_0 = 0 \quad (3.69)$$

$$mc^2 = h\nu \quad (3.70)$$

$$\boxed{p = mc = \frac{h\nu}{c}} \quad (3.71)$$

$$\boxed{m^2c^4 = m_0^2c^4 + p^2c^2} \quad (3.68)$$

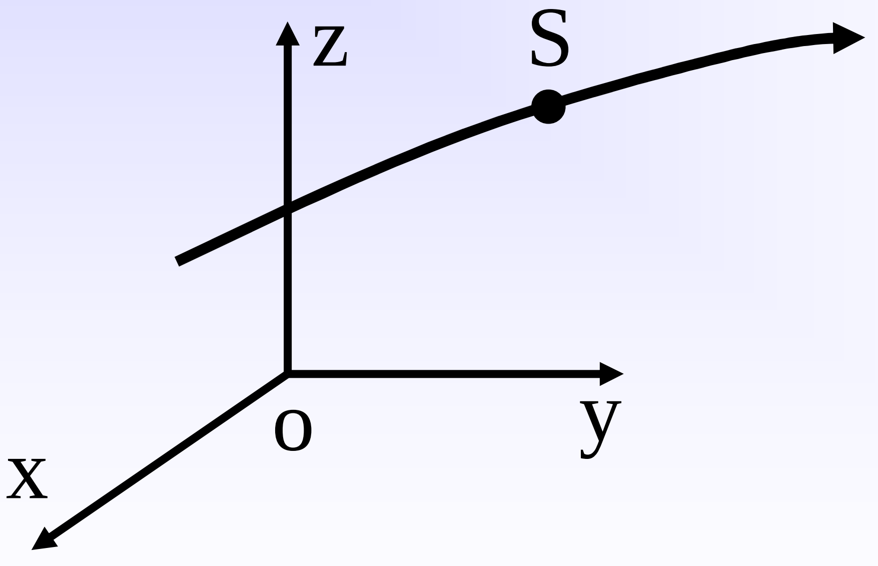
$$p^\alpha = (\frac{h\nu}{c}, \frac{h\nu}{c} l^i) \quad (3.72)$$

$$\vec{l}\,(\delta_{ij}\, l^i l^j = 1)$$

• 3.7 La mécanique relativiste du point matériel

3.7 Exercices

1. L'effet Doppler général



$$E_1 \ : x(t_1), y(t_1), z(t_1), t_1; \qquad E_2, 0, 0, 0, t_2$$

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$$t_2 = t_1 + \frac{r(t_1)}{c}$$

$\vec{r}(t)$ $(x(t), y(t), z(t))$

t_1 $t_1 + dt_1$ t_2 $t_2 + dt_2$

$$dt_2 = dt_1 + \frac{1}{c}\left(\frac{\vec{r}\cdot\vec{v}}{r}\right)_{t_1} dt_1$$

• 3.7 La mécanique relativiste du point matériel

$$dt_2 = dt_1 + \frac{1}{c}\left(\frac{\vec{r}\cdot\vec{v}}{r}\right)_{t_1} dt_1$$

$$d\tau_1 = \left(\sqrt{1-\frac{v^2}{c^2}}\right)_{t_1} dt_1$$

$$d\tau_2 = dt_2$$

$$\frac{d\tau_2}{d\tau_1} = \left[\frac{1+\frac{v_r}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right]_{t_1}$$

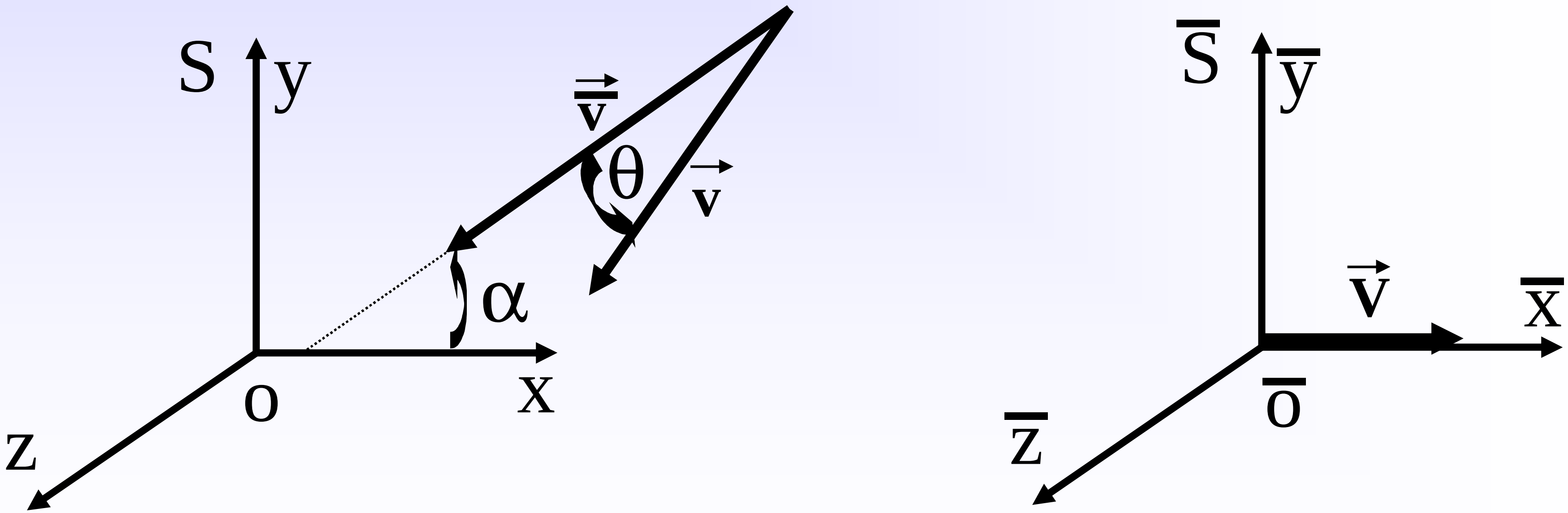
$$v_r = \frac{\vec{V}\cdot\vec{r}}{r} = \text{vitesse radiale de la source}$$

$$\frac{d\tau_2}{d\tau_1} = \frac{\tau}{\tau_0} = \frac{\lambda}{\lambda_0}$$

$$\frac{\lambda}{\lambda_0} = \left[\frac{1+\frac{v_r}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right]_{instant\ d'émission}$$

• 3.7 La mécanique relativiste du point matériel

2. Formule de l'aberration relativiste

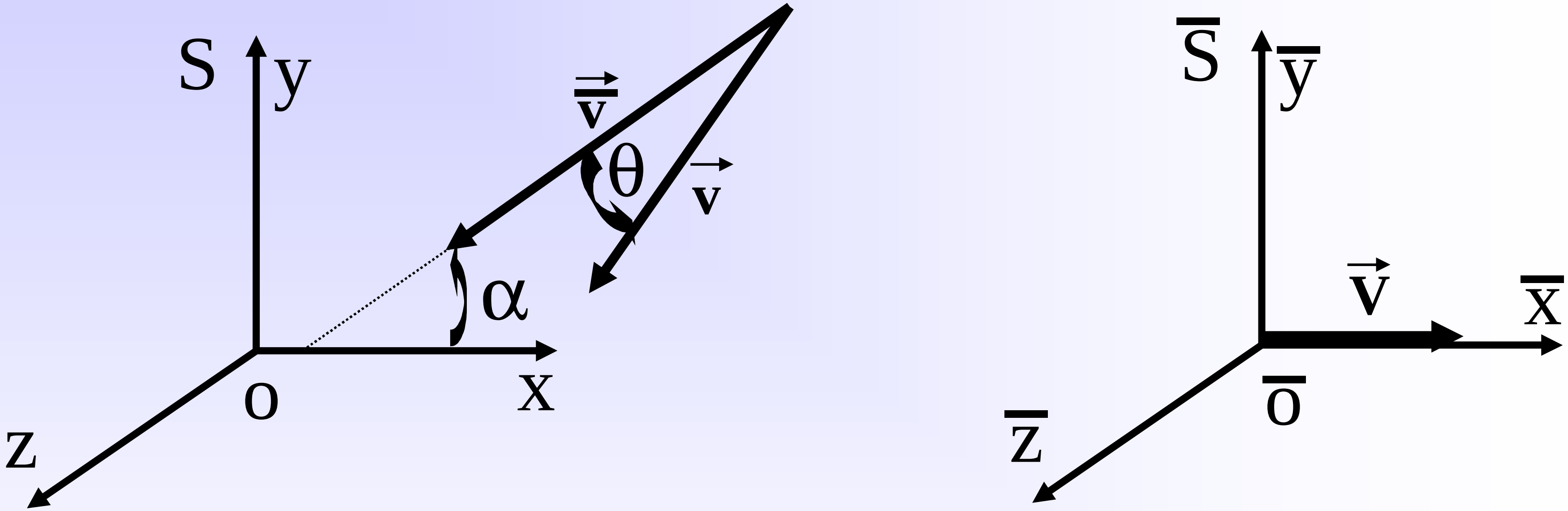


Rappel !

$$\vec{v} = \frac{\sqrt{1-\frac{V^2}{c^2}}\,\vec{v} + \left[1 + \left(1 - \sqrt{1-\frac{V^2}{c^2}}\right)\frac{\vec{V}.\vec{v}}{V^2}\right]\vec{V}}{1 + \frac{\vec{V}.\vec{v}}{c^2}} \qquad (3.11)$$

• 3.7 La mécanique relativiste du point matériel

2. Formule de l'aberration relativiste



$$\sin\theta = \frac{\frac{V}{c} - (1 - \sqrt{1 - \frac{V^2}{c^2}})\cos\alpha}{1 - \frac{V}{c}\cos\alpha}\sin\alpha$$

$$\sin\theta = \frac{V}{c}\sin\alpha(1 + \frac{V}{2c}\cos\alpha) + \ldots$$