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Anomalous perturbative transport in tokamaks due to drift-Alfvén-wave turbulence

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The method developed in Thoul, Similon, and Sudan [Phys. Plasmas **1**, 579 (1994)] is used to calculate the transport due to drift-Alfvén-wave turbulence, in which electromagnetic effects such as the fluttering of the magnetic field lines are important. Explicit expressions are obtained for all coefficients of the anomalous transport matrix relating particle and heat fluxes to density and temperature gradients in the plasma. Although the magnetic terms leave the transport by trapped electrons unaffected, they are important for the transport by circulating electrons.

I. INTRODUCTION

Drift-wave turbulence is believed to be a major source of anomalous transport in tokamaks (see Thoul *et al.*[1], hereafter Paper I, and references therein). However, turbulent magnetic fluctuations have also been suggested as a possible source of anomalous transport in magnetically confined plasmas. There have been several attempts to evaluate the transport resulting from these magnetic fluctuations,[2] but all used the quasilinear theory and therefore neglected the effects of the self-consistency constraints (quasineutrality and Ampère laws). Terry, Diamond, and Hahm[3] have included these self-consistency constraints in a study based on the clump theory.[4] They constructed a model for the evolution of the "incoherent fluctuations" that result from mode coupling processes, and are therefore not described by quasilinear theories. They reached the conclusion that the magnetic fluctuations do not contribute to the relaxation of the average electron distribution function, and therefore do not contribute to the radial transport of any velocity moment.

In this paper we examine the role of magnetic fluctuations in the evaluation of the incremental anomalous fluxes. We show that the electron heat transport produced by electromagnetic fluctuations does not vanish, in disagreement with the results of Terry, Diamond, and Hahm.[3,5] Qualitatively, our results can be understood as follows. Electrons diffuse because of resonant processes such as Compton scattering. Because of the resonance condition $v_{\parallel\,\text{res}}^2=(\omega/k_\parallel)^2\ll v_e^2$ [the propagator is $\pi\delta(\omega-k_\parallel v_\parallel)$], these electrons have small parallel energy. However, they create self-consistent electromagnetic fields, and other electrons are exchanged to maintain quasineutrality. These other electrons may have parallel velocities $v_\parallel$ of order the thermal velocity v_e, because they are not constrained to be resonant. This explains how there can be transport of electron parallel energy without transport of particles. The same reasoning applies to the transport of perpendicular energy.

II. ANOMALOUS TRANSPORT FROM DRIFT-ALFVÉN-WAVE TURBULENCE

A. Basic equations

As in the electrostatic regime (see Paper I), the electrons will be described by the drift-kinetic equation[6,7] (hereafter DKE).

In the electromagnetic regime, where the plasma parameter $\beta \gtrsim m_e/m_i$, additional terms corresponding to the magnetic field fluctuations appear in the DKE. The electromagnetic DKE is given by

$$\frac{\partial f}{\partial t}+\mathbf{v}_\mathbf{E}\cdot\nabla f+v_\parallel\,\mathbf{b}\cdot\nabla f-\frac{e}{m_e}\,\mathbf{E}\cdot\mathbf{b}\,\frac{\partial f}{\partial v_\parallel}=0, \tag{1}$$

where $f(\mathbf{x},v_\parallel,\mu,t)$ is the electron distribution function, $\mu=m_e v_\perp^2/2B$ is the electron magnetic moment, parallel $\parallel$ and perpendicular $\perp$ subscripts refer to the direction of the magnetic field, $\mathbf{v}_E=(c/B)\mathbf{E}\times\mathbf{b}$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity, $\mathbf{b}$ is the direction of the magnetic field, and e and m_e are the electron charge and mass. We neglect the magnetic shear and curvature (see Paper I). The electromagnetic fluctuations modify the DKE as follows. The direction $\mathbf{b}$ of the magnetic field is no longer fixed; it is a function of both position and time,

$$\mathbf{b}(\mathbf{x},t)\equiv\frac{\mathbf{B}(\mathbf{x},t)}{|\mathbf{B}(\mathbf{x},t)|}. \tag{2}$$

Correspondingly, the electric field has a magnetic component, and must be calculated as

$$\mathbf{E}=-\nabla\phi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}, \tag{3}$$

where ϕ is the electrostatic potential and $\mathbf{A}$ is the magnetic vector potential. The parallel gradient has now a variable direction, since it is defined with respect to $\mathbf{b}(\mathbf{x},t)$. The parallel electric field has a magnetic component $\mathbf{E}\cdot\mathbf{b}=-\nabla_\parallel\phi-(1/c)(\partial A_\parallel/\partial t)$. Finally, the $\mathbf{E}\times\mathbf{B}$ drift velocity $\mathbf{v}_E$ has a magnetic component $(-1/B)\times(\partial\mathbf{A}/\partial t)\times\mathbf{b}$.

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We can write the magnetic field as

$$\mathbf{B}(\mathbf{x},t)\equiv\mathbf{B}_0+\widetilde{\mathbf{B}}(\mathbf{x},t), \tag{4}$$

where the fluctuating part $\widetilde{\mathbf{B}}$ is given in terms of the magnetic vector potential by $\widetilde{\mathbf{B}}=\nabla\times\widetilde{\mathbf{A}}$. We use Coulomb's gauge, in which $\nabla\cdot\widetilde{\mathbf{A}}=0$. In terms of Fourier components, this implies $k_\parallel \widetilde{A}_\parallel+\mathbf{k}_\perp\cdot\widetilde{\mathbf{A}}_\perp=0$. For drift waves, the parallel and perpendicular wave numbers are such that $k_\parallel\sim(qR)^{-1}$ and $k_\perp\sim\rho_s^{-1}$ (see Paper I), so that $k_\parallel/k_\perp\ll 1$. Therefore, we can neglect the perpendicular component of $\widetilde{\mathbf{A}}$, since $|\widetilde{A}_\perp|=(k_\parallel/k_\perp)\widetilde{A}_\parallel\ll\widetilde{A}_\parallel$. The magnetic field can then be written as

$$\begin{aligned}&\widetilde{\mathbf{B}}_\perp=\nabla\times(\widetilde{A}_\parallel\mathbf{b}_0)=\nabla\widetilde{A}_\parallel\times\mathbf{b}_0,\\&\widetilde{\mathbf{B}}_\parallel=0,\end{aligned} \tag{5}$$

and the fluctuating electric field as

$$\begin{aligned}&\widetilde{\mathbf{E}}_\perp=-\nabla_\perp\widetilde{\phi},\\&\widetilde{\mathbf{E}}_\parallel=-\nabla_\parallel\widetilde{\phi}-\frac{1}{c}\frac{\partial\widetilde{A}_\parallel}{\partial t}\mathbf{b}_0.\end{aligned} \tag{6}$$

In this approximation, the $\mathbf{E}\times\mathbf{B}$ velocity $\mathbf{v}_E$ is unaffected by the magnetic component of the electric field. Therefore, the DKE for the trapped electrons is not modified by these electromagnetic effects, since the last two terms in Eq. (1) disappear after bounce averaging (see Paper I).

As in the electrostatic case (see Paper I), we treat the ions as fluid background. The relation between the ion density $\widetilde{n}_i$ and the electrostatic potential $\widetilde{\phi}$ is still given by

$$n_i/n_0=(e\phi/T_0)\chi_i(|\phi|^2), \tag{7}$$

[Eq. (4) of Paper I]. To close our system of equations, we use the quasineutrality law,

$$n_i(\mathbf{x},t)=n_e(\mathbf{x},t), \tag{8}$$

and Ampère's law,

$$\nabla^2\mathbf{A}_\parallel=-\frac{4\pi}{c}(\mathbf{j}_{\parallel e}+\mathbf{j}_{\parallel i})\simeq-\frac{4\pi}{c}\mathbf{j}_{\parallel e}, \tag{9}$$

where $\mathbf{j}_{\parallel e}(\mathbf{x},t)=-e\int f\mathbf{v}_\parallel\,d\mathbf{v}$ and $\mathbf{j}_{\parallel i}(\mathbf{x},t)=en_i\mathbf{v}_{\parallel i}$. The parallel ion current can be neglected compared to the parallel electron current.

B. Transport equations

As in the electrostatic case, we use the separation of length scales and time scales between turbulence and transport to obtain two separate systems of equations (see Sec. II B in Paper I). In the following calculations, we will use a caret for quantities that vary on the transport scales, and an overtilde for turbulent quantities. Subscripts 0 will be used for the equilibrium values of the parameters. The electron distribution function f is written as the sum of an equilibrium Maxwellian distribution, $f_0=n_0(2\pi T_0/m_e)^{-3/2}\exp(-m_e v^2/2T_0)$, where n_0 and T_0 are the local electron density and temperature, a fluctuating part $\widetilde{f}$, and the response to the modulation $\hat{f}$,

$$f=f_0+\widetilde{f}+\hat{f}. \tag{10}$$

The equations describing the response functions $\hat{f}$ and $\hat{n}_i$ are obtained by taking the low-frequency and long-wavelength limit of Eqs. (1) and (7), and adding the external sources $\hat{\xi}$ and $\hat{\xi}^i$ (see Paper I). This gives

$$\frac{\partial\hat{f}}{\partial t}+\left\langle\widetilde{\mathbf{v}}_E\cdot\nabla\widetilde{f}+v_\parallel\widetilde{\mathbf{b}}\cdot\nabla\widetilde{f}-\frac{e}{m_e}\widetilde{\mathbf{E}}\cdot\widetilde{\mathbf{b}}\frac{\partial f_0}{\partial v_\parallel}\right\rangle=\hat{\xi} \tag{11}$$

and

$$\hat{n}_i=n_0(e\hat{\phi}/T_0)\chi_i+\hat{\xi}^i. \tag{12}$$

On the transport scales, the quasineutrality and Ampère laws, Eqs. (8) and (9), become

$$\hat{n}_i=\hat{n}_e=\int\hat{f}\,d\mathbf{v} \tag{13}$$

and

$$\nabla^2\hat{A}_\parallel=-\frac{4\pi}{c}(\hat{j}_{\parallel e}+\hat{j}_{\parallel i})\simeq\frac{4\pi e}{c}\int\hat{f}v_\parallel\,d\mathbf{v}. \tag{14}$$

Equation (11) can be rewritten in terms of the incremental anomalous flux,

$$\hat{\mathbf{\Gamma}}\equiv\left\langle(\widetilde{\mathbf{v}}_E+v_\parallel\widetilde{\mathbf{b}})\widetilde{f}+\frac{e}{m_e}\widetilde{\phi}\widetilde{\mathbf{b}}\frac{\partial f_0}{\partial v_\parallel}\right\rangle, \tag{15}$$

as

$$\frac{\partial\hat{f}}{\partial t}+\nabla\cdot\hat{\mathbf{\Gamma}}=\hat{\xi}. \tag{16}$$

If we separate $\widetilde{f}$ into an adiabatic and a nonadiabatic contribution, as $\widetilde{f}=(e\widetilde{\phi}/T_0)f_0+\widetilde{h}$, the anomalous flux becomes

$$\begin{aligned}\hat{\mathbf{\Gamma}}=&\langle(\widetilde{\mathbf{v}}_E+v_\parallel\widetilde{\mathbf{b}})\widetilde{h}\rangle+\left\langle(\widetilde{\mathbf{v}}_E+v_\parallel\widetilde{\mathbf{b}})\frac{e\widetilde{\phi}}{T_0}f_0\right\rangle\\&+\left\langle\frac{e}{m_e}\widetilde{\phi}\widetilde{\mathbf{b}}\frac{\partial f_0}{\partial v_\parallel}\right\rangle.\end{aligned} \tag{17}$$

The fluxes of particles, $\hat{\mathbf{\Gamma}}_n$, and heat, $\hat{\mathbf{q}}$, are given by

$$\hat{\mathbf{\Gamma}}_n\equiv\int\hat{\mathbf{\Gamma}}\,d\mathbf{v} \tag{18}$$

and

$$\hat{\mathbf{q}}=T_0\int\hat{\mathbf{\Gamma}}\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v} \tag{19}$$

[also see Eqs. (13)–(17) in Paper I]. They are obtained by taking *even velocity moments* of Eq. (17). The last term on the right-hand side of Eq. (17) is odd in $v_\parallel$, since both $\widetilde{\phi}$ and $\widetilde{\mathbf{b}}$ are velocity independent and f_0 is a Maxwellian. Therefore, that term does not contribute to the transport of either particles or heat. Similarly, the second term in the second bracket is odd in $v_\parallel$ and does not contribute to the transport. We will omit these terms in the following calculations, although one should remember that they can contribute to the transport of quantities derived from odd velocity moments, such as the current. The first term in the second bracket is divergenceless and does not contribute to the relaxation of the density and temperature profiles. We will also omit this term in our calculations. Therefore, we will write $\hat{\Gamma}$ as

$$\hat{\Gamma}=\langle(\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})\tilde{h}\rangle. \tag{20}$$

C. Turbulence

To calculate the right-hand side of Eq. (20), we need to solve the equations describing the turbulent variables. The fluctuations are described by the following set of nonlinear equations:

$$\left(\frac{\partial}{\partial t}+v_\parallel \nabla_\parallel\right)\tilde{f}+\tilde{\mathbf{v}}_E\cdot\nabla f_0+\frac{e}{m_e}\left(\nabla_\parallel \tilde{\phi}+\frac{1}{c}\frac{\partial \tilde{A}_\parallel}{\partial t}\right)\frac{\partial f_0}{\partial v_\parallel}$$
$$=-\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}-v_\parallel \tilde{\mathbf{b}}\cdot\nabla\tilde{f}-\frac{e}{m_e}\tilde{\mathbf{E}}\cdot\tilde{\mathbf{b}}\frac{\partial f_0}{\partial v_\parallel}, \tag{21}$$

$$\frac{\tilde{n}_i}{n_0}=\frac{e\tilde{\phi}}{T_0}\chi_i, \tag{22}$$

$$\tilde{n}_i=\tilde{n}_e=\int \tilde{f}\,d\mathbf{v}, \tag{23}$$

$$\nabla^2\tilde{A}_\parallel=-\frac{4\pi}{c}(\tilde{j}_{\parallel e}+\tilde{j}_{\parallel i})\simeq\frac{4\pi e}{c}\int \tilde{f}v_\parallel\, d\mathbf{v}, \tag{24}$$

obtained from Eqs. (1), (7), (8), and (9) by using the expansions (4), (5), and (10). Equations (21)–(24) are the electromagnetic equivalent of Eqs. (20)–(23) in Paper I. We have kept the nonlinear $\mathbf{E}\times\mathbf{B}$ drift, $\tilde{\mathbf{v}}_E\cdot\nabla\tilde{\mathbf{f}}$, the magnetic fluttering term, $v_\parallel \tilde{\mathbf{b}}\cdot\nabla\tilde{f}$, and the nonlinear parallel acceleration, $(-e/m_e)\tilde{\mathbf{E}}\cdot\tilde{\mathbf{b}}(\partial f_0/\partial v_\parallel)$, on the right-hand side of Eq. (21).

Equations (21), (23), and (24) can be rewritten in terms of the nonadiabatic distribution function $\tilde{h}$ and the diamagnetic velocity,

$$\mathbf{v}_*=-\frac{cT_0}{eB_0}\mathbf{b}_0\times\left[\frac{\nabla n_0}{n_0}+\frac{\nabla T_0}{T_0}\left(\frac{v^2-3v_e^2}{2v_e^2}\right)\right], \tag{25}$$

as

$$\left(\frac{\partial}{\partial t}+v_\parallel \nabla_\parallel\right)\tilde{h}+\frac{ef_0}{T_0}\left(\frac{\partial}{\partial t}+\mathbf{v}_*\cdot\nabla\right)\left(\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel\right)$$
$$=-\frac{c}{B}\nabla_\perp\left(\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel\right)\times\mathbf{b}_0\cdot\nabla_\parallel \tilde{h}, \tag{26}$$

$$\tilde{n}_i=\tilde{n}_e=n_0\frac{e\tilde{\phi}}{T_0}+\int \tilde{h}\,d\mathbf{v}, \tag{27}$$

and

$$\frac{e}{T_0}\nabla^2\tilde{A}_\parallel=\frac{k_D^2}{n_0}\int \tilde{h}\frac{v_\parallel}{c}\,d\mathbf{v}, \tag{28}$$

where $k_{\rm D}=\sqrt{4\pi n_0e^2/T_0}$ is the Debye wave number.

Considerable simplification occurs because the $v_\parallel$ and $v_\perp$ dependences in Eqs. (26)–(28) decouple. Indeed, we note that Eqs. (26)–(28) are linear in $\tilde{h}$ and depend explicitly on the electron perpendicular velocity $v_\perp$ only through f_0 or the product $f_0\mathbf{v}_*$. Therefore, the dependence on $v_\perp$ is of the form $\exp(-v_\perp^2/2v_e^2)$ or $v_\perp^2\exp(-v_\perp^2/2v_e^2)$. This suggests that we look for a solution of Eqs. (26)–(28) of the type

$$\tilde{h}(v_\parallel,v_\perp^2)=\tilde{p}(v_\parallel)f_{0\perp}+\tilde{q}(v_\parallel)f_{0\perp}\frac{v_\perp^2-2v_e^2}{2v_e^2}, \tag{29}$$

where $\tilde{p}$ and $\tilde{q}$ are new functions that depend only on $v_\parallel$, and

$$f_{0\perp}=\frac{n_0}{2\pi v_e^2}\exp\left(\frac{-v_\perp^2}{2v_e^2}\right). \tag{30}$$

We substitute expression (29) for $\tilde{h}$ in Eq. (26) and integrate over $v_\perp$. This gives an equation for $\tilde{p}$,

$$\left(\frac{\partial}{\partial t}+v_\parallel \nabla_\parallel\right)\tilde{p}+\left[\frac{\partial}{\partial t}+\left(\mathbf{v}_*^n+\mathbf{v}_*^T\frac{v_\parallel^2-v_e^2}{2v_e^2}\right)\cdot\nabla_\perp\right]\frac{e}{T_0}$$
$$\times\left(\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel\right)f_{0\parallel}$$
$$=-\frac{c}{B}\nabla_\perp\left(\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel\right)\times\mathbf{b}_0\cdot\nabla_\parallel \tilde{p}, \tag{31}$$

where

$$f_{0\parallel}=\frac{1}{\sqrt{2\pi}v_e}\exp\left(\frac{-v_\parallel^2}{2v_e^2}\right), \tag{32}$$

and we have defined $\mathbf{v}_*^n=-(cT_0/eB_0)\mathbf{b}_0\times(\nabla n_0/n_0)$, and $\mathbf{v}_*^T=-(cT_0/eB_0)\mathbf{b}_0\times(\nabla T_0/T_0)=\eta\mathbf{v}_*^n$ (where $\eta\equiv d\ln T_0/d\ln n_0$). We now multiply Eq. (31) by $f_{0\perp}$, subtract it from Eq. (26) and divide the result by $(v_\perp^2-2v_e^2)/2v_e^2$. This gives an equation for $\tilde{q}$,

$$\left(\frac{\partial}{\partial t}+v_\parallel \nabla_\parallel\right)\tilde{q}+(\mathbf{v}_*^T\cdot\nabla_\perp)\frac{e}{T_0}\left(\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel\right)f_{0\parallel}$$
$$=-\frac{c}{B}\nabla_\perp\left(\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel\right)\times\mathbf{b}_0\cdot\nabla_\parallel \tilde{q}. \tag{33}$$

Equations (31) and (33), together with the definition (29), are equivalent to Eq. (26), but are considerably simpler to solve, since they involve only one dimension in velocity space. The quasineutrality and Ampère laws [Eqs. (23) and (24)], can be rewritten as

$$\frac{\tilde{n}_i}{n_0}=\frac{\tilde{n}_e}{n_0}=\frac{e\tilde{\phi}}{T_0}+\int \tilde{p}(v_\parallel)dv_\parallel \tag{34}$$

and

$$\frac{e}{T_0}\nabla^2\tilde{A}_\parallel=k_{\rm D}^2\int \tilde{p}(v_\parallel)\frac{v_\parallel}{c}dv_\parallel, \tag{35}$$

which are both independent of $\tilde{q}$. Therefore, Eqs. (31), (34), and (35) form a closed system of equations for $\tilde{p}$, $\tilde{\phi}$, and $\tilde{A}_\parallel$. The potentials $\tilde{\phi}$ and $\tilde{A}_\parallel$ do not depend on $\tilde{q}$, which is only passively convected according to Eq. (33).

We can similarly write the anomalous flux $\hat{\Gamma}$ in terms of $\tilde{p}$ and $\tilde{q}$. Substituting expression (29) for $\tilde{h}$ in Eq. (20), we obtain

$$\hat{\Gamma}=\langle(\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})\tilde{p}(v_\parallel)f_{0\perp}\rangle + \left\langle (\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})\tilde{q}(v_\parallel)f_{0\perp}\frac{v_\perp^2-2v_e^2}{2v_e^2}\right\rangle. \quad (36)$$

The flux of electrons $\hat{\Gamma}_n$ is obtained by integrating $\hat{\Gamma}$ over velocity space [cf. Eq. (18)]. We obtain

$$\hat{\Gamma}_n=n_0\left\langle \int (\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})\tilde{p}(v_\parallel)dv_\parallel \right\rangle. \quad (37)$$

The heat flux [cf. Eq. (19)] can be written as $\hat{\mathbf{q}}=\hat{\mathbf{q}}_\parallel+\hat{\mathbf{q}}_\perp$, with

$$\hat{\mathbf{q}}_\parallel=T_0\int \hat{\Gamma}\frac{v_\parallel^2-v_e^2}{2v_e^2}d\mathbf{v} =n_0T_0\left\langle \int (\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})\tilde{p}\frac{v_\parallel^2-v_e^2}{2v_e^2}dv_\parallel \right\rangle, \quad (38)$$

and

$$\hat{\mathbf{q}}_\perp=T_0\int \hat{\Gamma}\frac{v_\perp^2-2v_e^2}{2v_e^2}d\mathbf{v} =n_0T_0\left\langle \int (\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})\tilde{q}\,dv_\parallel \right\rangle. \quad (39)$$

D. Statistical solution

We now follow the method developed in Paper I, Sec. II C, to evaluate the flux $\hat{\Gamma}$, given by Eq. (36). In the DIA formalism, this flux can be written as

$$\hat{\Gamma}_{\rm DIA}=\left\langle (\tilde{\mathbf{v}}_E^{(1)}+v_\parallel \tilde{\mathbf{b}}^{(1)})f_{0\perp}\left(\tilde{p}+\tilde{q}\frac{v_\perp^2-2v_e^2}{2v_e^2}\right) + (\tilde{\mathbf{v}}_E+v_\parallel \tilde{\mathbf{b}})f_{0\perp}\left(\tilde{p}^{(1)}+\tilde{q}^{(1)}\frac{v_\perp^2-2v_e^2}{2v_e^2}\right)\right\rangle, \quad (40)$$

where the superscript (1) is used for the beat fluctuations. Following the procedure of Paper I, we find the electromagnetic equivalent of Eqs. (37)–(40) of Paper I for the forced beat fluctuations. These can be written (in Fourier space) as

$$g_{k'}^{-1}\tilde{p}_{k'}^{(1)}+b_{k'}^pf_{0\parallel}(e\tilde{\Phi}_{k'}^{(1)}/T_0) =-W_{k,k_0}(e/T_0)(\tilde{\Phi}_k^*\hat{p}_{k_0}-\hat{\Phi}_{k_0}\tilde{p}_k^*), \quad (41)$$

$$g_{k'}^{-1}\tilde{q}_{k'}^{(1)}+b_{k'}^qf_{0\parallel}(e\tilde{\Phi}_{k'}^{(1)}/T_0) =-W_{k,k_0}(e/T_0)(\tilde{\Phi}_k^*\hat{q}_{k_0}-\hat{\Phi}_{k_0}\tilde{q}_k^*), \quad (42)$$

$$\tilde{n}_{ik'}^{(1)}=n_0(e\tilde{\phi}_{k'}^{(1)}/T_0)\chi_i', \quad (43)$$

$$\tilde{n}_{ik'}^{(1)}=\tilde{n}_{ek'}^{(1)}=n_0\left(\frac{e\tilde{\phi}_{k'}^{(1)}}{T_0}\right)+n_0\int \tilde{p}_{k'}^{(1)}dv_\parallel, \quad (44)$$

and

$$-\frac{k^2}{k_{\rm D}^2}\frac{e}{T_0}\tilde{A}_{\parallel k'}^{(1)}=\int \tilde{p}_{k'}^{(1)}\frac{v_\parallel}{c}dv_\parallel. \quad (45)$$

In Eqs. (41) and (42) we have used a new, velocity-dependent, fluctuating potential $\tilde{\Phi}$, defined by

$$\tilde{\Phi}\equiv\tilde{\phi}-\frac{v_\parallel}{c}\tilde{A}_\parallel. \quad (46)$$

The coefficients g_k, b_k^p, and b_k^q are given by

$$g_k=[-i(\omega-k_\parallel v_\parallel)]^{-1}, \quad (47)$$

$$b_k^p=-i\left[\omega-\omega_*^n\left(1+\eta\frac{v_\parallel^2-v_e^2}{2v_e^2}\right)\right], \quad (48)$$

and

$$b_k^q=i\eta\omega_*^n. \quad (49)$$

The nonlinear coupling coefficient W_{k,k_0} is defined by

$$W_{k,k_0}=(cT_0/eB_0)\mathbf{b}_0\cdot(\mathbf{k}\times\mathbf{k}_0). \quad (50)$$

The linear system (41)–(45) can be solved for $\tilde{p}_{k'}^{(1)}$, $\tilde{q}_{k'}^{(1)}$, $\tilde{\phi}_{k'}^{(1)}$, and $\tilde{A}_{\parallel k'}^{(1)}$. After some algebra, we find

$$\tilde{p}_k^{(1)}=-g_kb_k^pf_{0\parallel}\frac{e\tilde{\Phi}_k^{(1)}}{T_0}+W_{k,k_0}\frac{e}{T_0}g_k(\tilde{\Phi}_k\hat{p}_{k_0}-\hat{\Phi}_{k_0}\tilde{p}_k), \quad (51)$$

$$\tilde{q}_k^{(1)}=-g_kb_k^qf_{0\parallel}\frac{e\tilde{\Phi}_k^{(1)}}{T_0}+W_{k,k_0}\frac{e}{T_0}g_k(\tilde{\Phi}_k\hat{q}_{k_0}-\hat{\Phi}_{k_0}\tilde{q}_k), \quad (52)$$

$$\frac{e\tilde{\Phi}_k^{(1)}}{T_0}=\frac{W_{k,k_0}}{\chi_i-\chi_e^L}\left[\left(1+\frac{v_\parallel}{c}\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\right)\int \frac{e}{T_0}g_k(\tilde{\Phi}_k\hat{p}_{k_0}-\hat{\Phi}_{k_0}\tilde{p}_k)dv_\parallel +\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\left(1+\frac{v_\parallel}{c}\frac{\chi_i-\chi_{e\phi}^L}{\chi_{eA}^L}\right) \times\int \frac{e}{T_0}g_k\frac{v_\parallel}{c}(\tilde{\Phi}_k\hat{p}_{k_0}-\hat{\Phi}_{k_0}\tilde{p}_k)dv_\parallel\right], \quad (53)$$

where the coefficients χ_e^L, $\chi_{e\phi}^L$, and χ_{eA}^L are defined by Eqs. (B10)–(B11) in Appendix B, and we have used the separation of scales $|k_0|\ll|k|$, so that $k'=k_0-k\simeq -k$.

Equations (51)–(53) give the beat quantities $\tilde{p}_k^{(1)}$, $\tilde{q}_k^{(1)}$, and $\tilde{\Phi}_k^{(1)}$ in terms of the response functions $\hat{p}_{k_0}$, $\hat{q}_{k_0}$, and $\hat{\Phi}_{k_0}$, and the fluctuations $\tilde{p}_k$, $\tilde{q}_k$, and $\tilde{\Phi}_k$. In Fourier space, the anomalous flux $\hat{\Gamma}_{\rm DIA}$ is

$$\hat{\Gamma}_{\rm DIA}=-\frac{c}{B_0}\sum_k\left\langle (i\mathbf{k}'\tilde{\Phi}_{k'}^{(1)}\times\mathbf{b}_0)\left(\tilde{p}_k+\frac{v_\perp^2-2v_e^2}{2v_e^2}\tilde{q}_k\right)f_{0\perp} +(i\mathbf{k}'\tilde{\Phi}_k\times\mathbf{b}_0)\left(\tilde{p}_{k'}^{(1)}+\frac{v_\perp^2-2v_e^2}{2v_e^2}\tilde{q}_{k'}^{(1)}\right)f_{0\perp}\right\rangle. \quad (54)$$

In Eq. (54), we can substitute expressions (51) and (52) for $\widetilde{p}_{k'}^{(1)}$ and $\widetilde{q}_{k'}^{(1)}$ in terms of $\widetilde{\Phi}_{k'}^{(1)}$ and the sources $\hat{p}_{k_0}$, $\hat{q}_{k_0}$, and $\hat{\Phi}_{k_0}$. This gives

$$\hat{\Gamma}_{\rm DIA}=-i\frac{cT_0}{eB_0}\sum_k \mathbf{k}\times\mathbf{b}_0\Biggl\{\Biggl\langle\frac{e\widetilde{\Phi}_k^{(1)}}{T_0}\Biggl[\Biggl(\widetilde{p}_k^*+\frac{v_\perp^2-2v_e^2}{2v_e^2}\widetilde{q}_k^*\Biggr)\times f_{0\perp}+\frac{e\widetilde{\Phi}_k^*}{T_0}g_kf_0\Biggl(b_k^p+\frac{v_\perp^2-2v_e^2}{2v_e^2}b_k^q\Biggr)\Biggr]\Biggr\rangle-\frac{1}{-i\omega_0}W_{kk_0}g_kf_{0\perp}\Biggl(\hat{\xi}_p+\frac{v_\perp^2-2v_e^2}{2v_e^2}\hat{\xi}_q\Biggr)\times\Biggl\langle\Biggl(\frac{e}{T_0}\Biggr)^2\widetilde{\Phi}_k\widetilde{\Phi}_k^*\Biggr\rangle\Biggr\},\qquad(55)$$

where again we have used the separation of scale $|k_0|\ll|k|$, and we have replaced the response functions by their lowest-order approximations,

$$\hat{p}_{k_0}=\hat{\xi}_p/(-i\omega_0),$$
$$\hat{q}_{k_0}=\hat{\xi}_q/(-i\omega_0),\qquad(56)$$
$$\hat{\Phi}_{k_0}=0.$$

Using these expressions for the response functions, we can also rewrite Eq. (53) as

$$\frac{e\widetilde{\Phi}_k^{(1)}}{T_0}=\frac{W_{k,k_0}}{\chi_i-\chi_e^L}\frac{1}{-i\omega_0}\Biggl[\Biggl(1+\frac{v_\parallel}{c}\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggr)\int g_k\frac{e\widetilde{\Phi}_k}{T_0}\hat{\xi}_p\,dv_\parallel+\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggl(1+\frac{v_\parallel}{c}\frac{\chi_i-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggr)\int g_k\frac{v_\parallel}{c}\frac{e\widetilde{\Phi}_k}{T_0}\hat{\xi}_p\,dv_\parallel\Biggr].\qquad(57)$$

If we substitute this expression for $\widetilde{\Phi}_k^{(1)}$ in Eq. (55), we obtain the anomalous flux $\hat{\Gamma}_{\rm DIA}$ in terms of the fluctuation spectrum and the sources,

$$\hat{\Gamma}_{\rm DIA}=-i\frac{cT_0}{eB_0}\sum_k\mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega_0}\Biggl\{\Biggl\langle\frac{1}{\chi_i-\chi_e^L}\Biggl[\Biggl(1+\frac{v_\parallel}{c}\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggr)\int g_k\frac{e\widetilde{\Phi}_k}{T_0}\hat{\xi}_p\,dv_\parallel+\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggl(1+\frac{v_\parallel}{c}\frac{\chi_i-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggr)\times\int g_k\frac{v_\parallel}{c}\frac{e\widetilde{\Phi}_k}{T_0}\hat{\xi}_p\,dv_\parallel\Biggr]\Biggl[\Biggl(\widetilde{p}_k^*+\frac{v_\perp^2-2v_e^2}{2v_e^2}\widetilde{q}_k^*\Biggr)f_{0\perp}+\frac{e\widetilde{\Phi}_k^*}{T_0}g_kf_0\Biggl(b_k^p+\frac{v_\perp^2-2v_e^2}{2v_e^2}b_k^q\Biggr)\Biggr]\Biggr\rangle-g_kf_{0\perp}\Biggl(\hat{\xi}_p+\frac{v_\perp^2-2v_e^2}{2v_e^2}\hat{\xi}_q\Biggr)\Biggl\langle\Biggl|\frac{e\widetilde{\Phi}_k}{T_0}\Biggr|^2\Biggr\rangle\Biggr\}.\qquad(58)$$

Note that the electrostatic result (without trapped electrons) can be recovered by replacing $\widetilde{\Phi}_k$ by $\widetilde{\phi}_k$ (i.e., letting $\widetilde{A}_{\parallel\,k}\to0$) and $\chi_{e\phi}^L$ by χ_e^L in Eq. (58).

We can now proceed to the calculation of the various transport coefficients. The source terms are given by

$$\hat{\xi}_p=(-i\omega_0)f_{0\parallel}\,\hat{n}_0,$$
$$\hat{\xi}_q=0,\qquad(59)$$
$$\xi_i=\hat{n}_0,$$

for perturbations of the density profile, and

$$\hat{\xi}_p=(-i\omega_0)n_0f_{0\parallel}\,\frac{v_\parallel^2-v_e^2}{2v_e^2}\frac{\hat{T}_0}{T_0},$$
$$\hat{\xi}_q=(-i\omega_0)n_0f_{0\parallel}\,\frac{\hat{T}_0}{T_0},\qquad(60)$$
$$\hat{\xi}_i=0,$$

for perturbations of the temperature profile. The transport matrix coefficients are obtained by substituting one of these expressions for $\hat{\xi}_p$ and $\hat{\xi}_q$ in Eq. (58), and integrating over velocity. The calculations are straightforward but lengthy. The flux of electrons is given by

$$\hat{\Gamma}_n=n_0\Biggl(i\frac{cT_0}{eB_0}\Biggr)\sum_k\mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega_0}\int dv_\parallel\Biggl\{g_k\hat{\xi}_p\Biggl\langle\Biggl|\frac{e\widetilde{\Phi}_k}{T_0}\Biggr|^2\Biggr\rangle-\frac{1}{\chi_i-\chi_e^L}\Biggl\langle\Biggl[\Biggl(1+\frac{v_\parallel}{c}\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggr)\Biggl(\int dv_\parallel\;g_k\hat{\xi}_p\frac{e\widetilde{\Phi}_k}{T_0}\Biggr)+\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggl(1+\frac{v_\parallel}{c}\frac{\chi_i-\chi_{e\phi}^L}{\chi_{eA}^L}\Biggr)\Biggl(\int dv_\parallel\;g_k\frac{v_\parallel}{c}\hat{\xi}_p\frac{e\widetilde{\Phi}_k}{T_0}\Biggr)\Biggr]\Biggl(\widetilde{p}_k^*+g_kf_{0\parallel}\,b_k^p\frac{e\widetilde{\Phi}_k^*}{T_0}\Biggr)\Biggr\rangle\Biggr\}.\qquad(61)$$

The flux of parallel heat is given by

$$\hat{\mathbf{q}}_{\parallel}=n_0T_0\left(i\frac{cT_0}{eB_0}\right)\sum_k \mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega_0}\int dv_{\parallel}\,\frac{v_{\parallel}^2-v_e^2}{2v_e^2}\left\{g_k\hat{\xi}_p\left\langle\left|\frac{e\widetilde{\Phi}_k}{T_0}\right|^2\right\rangle-\frac{1}{\chi_i-\chi_e^L}\left\langle\left[\left(1+\frac{v_{\parallel}}{c}\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\right)\left(\int dv_{\parallel}\;g_k\hat{\xi}_p\frac{e\widetilde{\Phi}_k}{T_0}\right)\right.\right.\right.$$

$$\left.\left.\left.+\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\left(1+\frac{v_{\parallel}}{c}\frac{\chi_i-\chi_{e\phi}^L}{\chi_{eA}^L}\right)\left(\int dv_{\parallel}\;g_k\frac{v_{\parallel}}{c}\hat{\xi}_p\frac{e\widetilde{\Phi}_k}{T_0}\right)\right]\left(\widetilde{p}_k^*+g_kf_{0\parallel}\,b_k^p\frac{e\widetilde{\Phi}_k^*}{T_0}\right)\right\rangle\right\}. \tag{62}$$

Finally, the flux of perpendicular heat is given by

$$\hat{\mathbf{q}}_{\perp}=n_0T_0\left(i\frac{cT_0}{eB_0}\right)\sum_k \mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega_0}\int dv_{\parallel}\left\{g_k\hat{\xi}_q\left\langle\left|\frac{e\widetilde{\Phi}_k}{T_0}\right|^2\right\rangle-\frac{1}{\chi_i-\chi_e^L}\left\langle\left[\left(1+\frac{v_{\parallel}}{c}\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\right)\left(\int dv_{\parallel}\;g_k\hat{\xi}_p\frac{e\widetilde{\Phi}_k}{T_0}\right)\right.\right.\right.$$

$$\left.\left.\left.+\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}\left(1+\frac{v_{\parallel}}{c}\frac{\chi_i-\chi_{e\phi}^L}{\chi_{eA}^L}\right)\left(\int dv_{\parallel}\;g_k\frac{v_{\parallel}}{c}\hat{\xi}_p\frac{e\widetilde{\Phi}_k}{T_0}\right)\right]\left(\widetilde{q}_k^*+g_kf_{0\parallel}\,b_k^q\frac{e\widetilde{\Phi}_k^*}{T_0}\right)\right\rangle\right\}. \tag{63}$$

All the velocity integrals in these expressions can be written in terms of the integrals G_n^m, defined in Appendix A. For example, consider the test-particle contribution to the flux of electrons $\hat{\Gamma}_n$ due to a perturbation in the density profile, Eq. (59). We have

$$\hat{\Gamma}_n^{\mathrm{TP}}=n_0\left(i\frac{cT_0}{eB_0}\right)\sum_k \mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega_0}\int dv_{\parallel}\;g_k\hat{\xi}_p\left\langle\left|\frac{e\widetilde{\Phi}_k}{T_0}\right|^2\right\rangle$$

$$=n_0\left(i\frac{cT_0}{eB_0}\right)\sum_k \mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega}\hat{n}_0\left[G_0^0\left\langle\left|\frac{e\widetilde{\phi}_k}{T_0}\right|^2\right\rangle-2G_0^1\,\mathrm{Re}\left\langle\frac{e\widetilde{\phi}_k}{T_0}\frac{e\widetilde{A}_{\parallel\,k}^*}{T_0}\right\rangle+G_0^2\left\langle\left|\frac{e\widetilde{A}_{\parallel\,k}}{T_0}\right|^2\right\rangle\right]. \tag{64}$$

It is of interest to determine the regime in which the contribution from the electromagnetic terms to the transport is important. To evaluate the relative order of magnitude of the electromagnetic terms compared to the electrostatic terms, let us calculate the ratio $\widetilde{A}_{\parallel}/\widetilde{\phi}$ in the linear regime for the particular case of the TEXT tokamak. From Eqs. (B10) and (B11) in Appendix B, we find

$$\frac{\widetilde{A}_{\parallel}}{\widetilde{\phi}}=-\frac{\chi_e^L-\chi_{e\phi}^L}{\chi_{eA}^L}$$

$$=\frac{\chi_{eA}^L}{[-(k^2/k_{\mathrm{D}}^2)+(\omega_*^n/\omega-1)G_0^2+\eta(\omega_*^n/\omega)G_1^2]}$$

$$=\frac{(\omega_*^n/\omega-1)G_0^1+\eta(\omega_*^n/\omega)G_1^1}{[-(k^2/k_{\mathrm{D}}^2)+(\omega_*^n/\omega-1)G_0^2+\eta(\omega_*^n/\omega)G_1^2]}. \tag{65}$$

We can evaluate this expression numerically using the TEXT parameters (cf. Table I in Paper I). We obtain

$$\frac{\widetilde{A}_{\parallel}}{\widetilde{\phi}}\approx\beta\frac{m_i}{m_e}\frac{c^2}{v_e^2}\frac{\omega}{k_{\parallel}c}\approx\beta\times 10^4. \tag{66}$$

The ratio of the electromagnetic to the electrostatic contributions to the transport coefficient (64) is of the order

$$\frac{G_0^2\langle|e\widetilde{A}_{\parallel}/T_0|^2\rangle}{G_0^0\langle|e\widetilde{\phi}/T_0|^2\rangle}\approx\left(\frac{\omega}{k_{\parallel}v_e}\right)^3\left(\beta\frac{m_i}{m_e}\right)^2\approx 4\times 10^4\beta^2. \tag{67}$$

This ratio becomes of order unity when $\beta\approx 5\times 10^{-3}$. In the Texas Experimental Tokamak (TEXT),[8] a typical value is $\beta\approx 2\times 10^{-3}$. *The electromagnetic contribution to the transport by circulating electrons is therefore important, in spite of the small value of β.* However, we have shown in Paper I that the transport is dominated by the trapped electrons, which are not affected by the magnetic fluctuations.

III. SUMMARY AND DISCUSSION

We have used the new approach developed in Paper I to calculate the anomalous transport in tokamak plasmas due to electromagnetic drift-Alfvén-wave turbulence. This method is based on the direct-interaction approximation, a renormalized theory of turbulence that provides the response functions due to infinitesimal perturbations.

As in the electrostatic case, the theoretical expressions obtained for the transport coefficients are based on a specific set of equations describing the dynamics of the plasma and on the DIA, but do not require any further approximations.

We have shown that the electromagnetic contribution to the transport by circulating electrons is important. However, the transport is often dominated by the trapped electrons, which are not affected by the magnetic fluctuations.

The results obtained here were expressed in a form such that they can be easily compared with experiments. Since we need experimental data for the frequency and wave number dependence of the fluctuation spectrum, new experiments, yielding more detailed results on the turbulence characteristics, would be welcome. In particular, it would be very useful to obtain the information on both the fluctuations and the perturbative transport during the same experiment. This would allow a direct comparison between theoretical predictions and measured data.

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APPENDIX A: INTEGRALS

The electromagnetic integrals are defined by

$$G_n^m \equiv \int_{-\infty}^{+\infty} \frac{f_{0\|}}{1-k_\| v_\| /\omega} \left(\frac{v_\|^2 - v_e^2}{2v_e^2}\right)^n \left(\frac{v_\|}{c}\right)^m dv_\| . \tag{A1}$$

These integrals obey the following recurrence relation:

$$G_n^m = \frac{\omega}{k_\| c}\left[G_n^{m-1} - \int_{-\infty}^{+\infty} f_{0\|} \left(\frac{v_\|^2 - v_e^2}{2v_e^2}\right)^n \left(\frac{v_\|}{c}\right)^{m-1} dv_\| \right]. \tag{A2}$$

Furthermore, it is easy to show that $G_0^0 = H_0$, $G_1^0 = H_1$, and $G_2^0 = H_2 - H_0$, where

$$H_n = \frac{1}{n_0}\int \frac{f_0}{1-k_\| v_\| /\omega}\left(\frac{v^2 - 3v_e^2}{2v_e^2}\right)^n d\mathbf{v}. \tag{A3}$$

APPENDIX B: LINEARIZED ELECTRON EQUATIONS

The linearized electromagnetic drift-kinetic equations are obtained from Eqs. (31)–(33) by letting the right-hand sides go to zero. If we Fourier transform these equations and eliminate the adiabatic part of the density fluctuations by using $\tilde{f} = f_0(e\tilde{\phi}/T_0) + \tilde{h}$, we obtain

$$g_k^{-1}\tilde{p}_k + b_k^p f_{0\|}\,(e\tilde{\Phi}_k/T_0) = 0, \tag{B1}$$

and

$$g_k^{-1}\tilde{q}_k + b_k^q f_{0\|}\,(e\tilde{\Phi}_k/T_0) = 0, \tag{B2}$$

where

$$g_k = [-i(\omega - k_\| v_\|)]^{-1}, \tag{B3}$$

$$b_k^p = -i\left(\omega - \omega_*^n - \frac{v_\|^2 - v_e^2}{2v_e^2}\omega_*^T\right) = -i\left[\omega - \omega_*^n\left(1 + \eta\frac{v_\|^2 - v_e^2}{2v_e^2}\right)\right], \tag{B4}$$

$$b_k^q = i\omega_*^T = i\eta\omega_*^n, \tag{B5}$$

and

$$\tilde{\Phi}_k = \tilde{\phi}_k - \frac{v_\|}{C}\tilde{A}_{\|\,k}. \tag{B6}$$

The electron susceptibility χ_e^L is defined by

$$\tilde{n}_e^L = \int \tilde{f}^L d\mathbf{v} = n_0 \frac{e\tilde{\phi}}{T_0}\chi_e^L. \tag{B7}$$

Therefore, we have here

$$\frac{e\tilde{\phi}}{T_0}\chi_e^L = \frac{1}{n_0}\int\left(\tilde{h}^L + \frac{e\tilde{\phi}}{T_0}f_0\right)d\mathbf{v}, \tag{B8}$$

which becomes

$$\frac{e\tilde{\phi}}{T_0}\chi_e^L = \frac{e\tilde{\phi}}{T_0} + \int \tilde{p}^L(v_\|)dv_\| , \tag{B9}$$

after integrating over $v_\perp$. Replacing $\tilde{p}$ by its value obtained from Eq. (B1), we obtain

$$\frac{e\tilde{\phi}}{T_0}\chi_e^L = \frac{e\tilde{\phi}}{T_0}\left[1 + \left(\frac{\omega_*^n}{\omega} - 1\right)G_0^0 + \eta\frac{\omega_*^n}{\omega}G_1^0\right] - \frac{e\tilde{A}_\|}{T_0}\left[\left(\frac{\omega_*^n}{\omega} - 1\right)G_0^1 + \eta\frac{\omega_*^n}{\omega}G_1^1\right] \equiv \frac{e\tilde{\phi}}{T_0}\chi_{e\phi}^L - \frac{e\tilde{A}_\|}{T_0}\chi_{eA}^L, \tag{B10}$$

where the integrals G_n^m are defined by Eq. (A1) in Appendix A, and the last identity defines the quantities $\chi_{e\phi}^L$ and χ_{eA}^L. Solving the linear system formed by Eqs. (B1), (34), and (35), we obtain

$$\chi_e^L = \left[1 + \left(\frac{\omega_*^n}{\omega} - 1\right)G_0^0 + \eta\frac{\omega_*^n}{\omega}G_1^0\right] - \left[\left(\frac{\omega_*^n}{\omega} - 1\right)G_0^1 + \eta\frac{\omega_*^n}{\omega}G_1^1\right]^2\left[-\frac{k^2}{k_D^2} + \left(\frac{\omega_*^n}{\omega} - 1\right)G_0^2 + \eta\frac{\omega_*^n}{\omega}G_1^2\right]^{-1}. \tag{B11}$$

The coefficient χ_T^L is given by

$$\tilde{T}_e^L = \frac{2T_0}{3n_0}\int \tilde{f}^L\frac{v^2 - 3v_e^2}{2v_e^2}d\mathbf{v} = \frac{2}{3}T_0\frac{e\tilde{\phi}}{T_0}\chi_T^L, \tag{B12}$$

which becomes

$$\frac{e\tilde{\phi}}{T_0}\chi_T^L = \int \tilde{p}^L\frac{v_\|^2 - v_e^2}{2v_e^2}dv_\| + \int \tilde{q}^L dv_\| = \frac{e\tilde{\phi}}{T_0}\left[\left(\frac{\omega_*^n}{\omega} - 1\right)G_1^0 + \eta\frac{\omega_*^n}{\omega}(G_2^0 + G_0^0)\right] - \frac{e\tilde{A}_\|}{T_0}\left[\left(\frac{\omega_*^n}{\omega} - 1\right)G_1^1 + \eta\frac{\omega_*^n}{\omega}(G_2^1 + G_0^1)\right]. \tag{B13}$$

Replacing $\widetilde{A}_{\parallel}$ by its value in terms of $\widetilde{\phi}$, obtained by solving the linear system, we can rewrite this as

$$\chi_T^L=\left[\left(\frac{\omega_*^n}{\omega}-1\right)G_1^0+\eta\,\frac{\omega_*^n}{\omega}\,(G_2^0+G_0^0)\right]-\left[\left(\frac{\omega_*^n}{\omega}-1\right)G_1^1+\eta\,\frac{\omega_*^n}{\omega}\,(G_2^1+G_0^1)\right]\left[\left(\frac{\omega_*^n}{\omega}-1\right)G_0^1+\eta\,\frac{\omega_*^n}{\omega}\,G_1^1\right]\times\left[-\frac{k^2}{k_{\mathrm{D}}^2}+\left(\frac{\omega_*^n}{\omega}-1\right)G_0^2+\eta\,\frac{\omega_*^n}{\omega}\,G_1^2\right]^{-1}. \qquad \text{(B14)}$$

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