

# Element Diffusion in the Solar Interior

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## Abstract

We study the diffusion of helium and other heavy elements in the solar interior by solving exactly the set of flow equations developed by Burgers for a multi-component fluid, including the residual heat-flow terms. No approximation is made concerning the relative concentrations and no restriction is placed on the number of elements considered. We give improved diffusion velocities for hydrogen, helium, oxygen and iron, in the analytic form derived previously by Bahcall and Loeb. These expressions for the diffusion velocities are simple to program in stellar evolution codes and are expected to be accurate to  $\sim 15\%$ . We find that the inclusion of the residual heat flow terms leads to an increase in the hydrogen diffusion velocity. We compare our numerical results with those obtained analytically by Bahcall and Loeb using a simplified treatment, as well as with those derived numerically by Michaud and Proffitt. We find that for conditions characteristic of the sun, the results of Bahcall and Loeb for the hydrogen diffusion velocity are smaller than our more accurate numerical results by  $\sim 30\%$ , except very near the center where the error becomes larger. The Michaud and Proffitt results differ from the numerical results derived here by  $\lesssim 15\%$ . Our complete treatment of element diffusion can be directly incorporated in a standard stellar evolution code by means of an exportable subroutine, but, for convenience, we also give simple analytical fits to our numerical results.

*Subject headings:* diffusion, stars: interiors, stars: abundances, Sun: interior

# 1. Introduction

Precise solar evolution calculations must be carried out to compare model results with observations of solar neutrino fluxes and of p-mode oscillation frequencies. In particular, element diffusion affects the element abundances, the mean molecular weight, and the radiative opacity in the core of the sun, and therefore affects the calculated neutrino fluxes and oscillation frequencies. The characteristic time for elements to diffuse a solar radius under solar conditions is of the order of  $6 \times 10^{13}$  yrs, much larger than the age of the sun. Element diffusion therefore introduces only a small correction to standard solar model calculations. Bahcall and Pinsonneault (1992a,b) showed that helium diffusion increases the predicted event rates by about 11% in the chlorine solar neutrino experiment, by 3% in the gallium experiment, and by 12% in the Kamiokande and SNO experiments, while increasing the inferred primordial helium abundance by 0.4% and decreasing the calculated depth of the convection zone by 2%. Christensen-Dalsgaard, Proffitt and Thompson (1993) calculated the sound speed as a function of radius in the solar model and concluded that helium diffusion causes a significant difference in the computed radial profile of the sound speed. Guenther, Pinsonneault, and Bahcall (1993) demonstrated that helium diffusion has a characteristic which depends upon the degree and frequency of the p-mode being discussed and which has a typical amplitude of order 1-3 MHz.

Since the effects of diffusion are small, there is in principle no need for very high accuracy in its treatment. However, discrepancies appear between various results in the literature, depending on the approximations made. Previous studies of element diffusion in the sun (Aller and Chapman 1960, Michaud *et al.* 1976, Noerdlinger 1977, 1978, Cox, Guzik and Kidman 1989, Paquette *et al.* 1986, Bahcall and Loeb 1990, Proffitt and Michaud 1991, Michaud and Proffitt 1992, Bahcall and Pinsonneault 1992a,b, Christensen-Dalsgaard, Proffitt and Thompson 1993, Guenther, Pinsonneault and Bahcall 1993, Vauclair and Vauclair 1982 and references therein) have usually included one or more of the following simplifying assumptions: neglecting thermal diffusion, or treating it using a simplified empirical formula; neglecting the presence of heavy elements when calculating helium diffusion; assuming a negligible helium abundance when calculating the diffusion of heavier elements; adopting a single constant value for all Coulomb logarithms. In this pa-

per, we provide a simple but complete treatment of the problem, making none of the above approximations, and we compare our results with those obtained under different simplifying assumptions. In particular, we compare our results with those obtained by Bahcall and Loeb (1990) (hereafter BL) and those obtained by Michaud and Proffitt (1992) (hereafter MP). BL made most of the above simplifying assumptions. In particular, they used empirical results for the thermal diffusion coefficients and a single value for the Coulomb logarithms, equal to 2.2. MP solved the Burgers equations and then represented the effects of the residual heat flow vectors by an ad-hoc correction to the results obtained when neglecting those heat fluxes. The principal difference between this work and most previous studies is that we solve the Burgers equations exactly and then represent the numerical results by simple analytic functions, rather than trying to obtain analytic solutions by various approximations.

Element diffusion in stars is driven by pressure gradients (or gravity), temperature gradients, composition gradients, and radiation pressure<sup>1</sup>. Gravity tends to concentrate the heavier elements towards the center of the star. In a pure hydrogen-helium plasma, helium diffuses towards the center of the star, while hydrogen diffuses outwards. As we will show in §4 (see also Bahcall and Loeb 1990), the local rate of change of the hydrogen mass fraction is equal and opposite to the rate of change of the helium mass fraction. This follows from the condition of momentum conservation. The light electrons also tend to rise, but are held back by an electric field which counteracts gravity. Temperature gradients lead to thermal diffusion, which tends to concentrate more highly charged and more massive species towards the hottest region of the star, its center. Concentration gradients oppose the above processes. Radiation pressure causes negligible diffusion in the solar core and will be neglected in this paper.

We study the relative diffusion of hydrogen, helium, and heavier elements, such as oxygen and iron. In contrast to many previous studies, no approximation is made concerning the relative concentrations of the various species, and no restriction is placed on

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<sup>1</sup> In this work, we ignore the effects of meridional circulation. It has been shown that meridional circulation velocities are several orders of magnitude smaller than the diffusion velocities in the solar interior (see, e.g., Michaud and Vauclair 1991).

the number of elements considered. Our method is therefore applicable to a wide variety of astrophysical problems, such as the diffusion of elements in white dwarf envelopes (see, e.g., Fontaine and Michaud 1979, Pelletier *et al.* 1986) and in globular cluster stars (see, e.g., Chaboyer *et al.* 1992). In this paper, we concentrate on calculating the diffusion velocities in the temperature and density ranges relevant to the sun, although our exportable subroutine can be used to calculate diffusion velocities in red giants and in white dwarfs.

Burgers (1969) has provided a complete and straightforward set of equations to describe the evolution of a multi-component fluid. In order to include the effects of thermal diffusion, he introduced the so-called “residual heat flow vectors”. Here, we will use the Burgers equations, including the residual heat fluxes, to describe the plasma in the solar interior. Even though these equations can in principle be solved analytically, the algebraic complexity increases rapidly with the number of species considered. For example, because of computational limitations, Noerdlinger (1977) included only three species (hydrogen, helium, and electrons) and adopted a single constant for all the Coulomb logarithms. In contrast, we solve the full set of Burgers equations numerically, and place no restriction on the number of species. The Coulomb logarithm is obtained by calculating the collision integrals using a pure Coulomb potential with a long-range cutoff at the Debye length. However, the result obtained for the Coulomb logarithm is valid only for plasmas that are sufficiently hot and rarefied, i.e., such that the plasma parameter  $\Lambda$  is much larger than unity. For conditions characteristic of the solar interior, the Coulomb logarithms are small, and can even become negative for collisions between heavy elements. For such plasmas, the collision integrals can be calculated numerically using a screened Debye-Huckel Coulomb potential. The results can then be fitted to simple analytic functions. We adopt an expression for the “effective” Coulomb logarithm obtained by Iben and MacDonald (1985) by fitting numerical results from Fontaine and Michaud (1979b).

It should be relatively easy to incorporate our complete treatment of element diffusion into any standard solar evolution code<sup>2</sup>. However, we have obtained simple analytic fits to the exact results, which can provide a convenient alternative. These fits can be expressed as follows: Following BL’s notations (see footnotes 3 and 5) and using BL’s dimensionless

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<sup>2</sup> Our FORTRAN routine will be made available upon request.

variables (see §2), the mass fraction of element  $s$  satisfies the equation

$$\frac{\partial X_s}{\partial t} = -\frac{1}{\rho r^2} \frac{\partial}{\partial r} [r^2 X_s T^{5/2} \xi_s(r)], \quad (1)$$

where the partial derivatives are evaluated in the local rest frame of a mass shell in the star, i.e., in Lagrangian coordinates. The function  $\xi_s(r)$  is related to the diffusion velocity  $w_s$  of species  $s$  through

$$\xi_s(r) = w_s(r) \rho(r) / T^{5/2}(r). \quad (2)$$

We have obtained the following results for the diffusion velocities of hydrogen, oxygen and iron in the solar interior:

$$\xi_s(r) = A_p(s) \frac{\partial \ln p}{\partial r} + A_T(s) \frac{\partial \ln T}{\partial r} + A_H(s) \frac{\partial \ln C_H}{\partial r} \quad (3)$$

with

$$\begin{cases} A_p(H) = -2.09 + 3.15 X - 1.07 X^2, \\ A_T(H) = -2.18 + 3.12 X - 0.96 X^2, \\ A_H(H) = -1.51 + 1.85 X - 0.85 X^2, \end{cases} \quad (4)$$

for the hydrogen diffusion coefficients,

$$\begin{cases} A_p(O) = 0.15 + 1.34 X - 0.89 X^2, \\ A_T(O) = 0.53 + 1.99 X - 0.72 X^2, \\ A_H(O) = 0.08 + 0.58 X - 0.28 X^2, \end{cases} \quad (5)$$

for the oxygen diffusion coefficients, and

$$\begin{cases} A_p(Fe) = 0.25 + 1.31 X - 0.87 X^2, \\ A_T(Fe) = 0.65 + 1.99 X - 0.75 X^2, \\ A_H(Fe) = 0.09 + 0.53 X - 0.27 X^2, \end{cases} \quad (6)$$

for the iron diffusion coefficients. These fits were obtained by using a constant value for each Coulomb logarithm, equal to its value at the center of the sun, and are accurate to better than 15% for the hydrogen and oxygen diffusion velocities, and better than 20% for the iron diffusion velocity. Of course, the fits can only have a limited domain of applicability, whereas the numerical routine is completely general.

This paper is organized as follows. In §2, we introduce the notation and basic equations. In §3, we describe the method of solution. In §4, we give the results for the hydrogen and helium diffusion coefficients, for a fixed value of the temperature and density, characteristic of the solar core. We compare these results with those obtained by BL and MP. In §5, we give the results for the heavy element diffusion coefficients, obtained under the same conditions. In §6, we give the diffusion velocities in the sun, and again we compare our results with those obtained by BL and MP. In §7, we give analytical expressions for our numerical results. In §8, we compare our expression for the electric field with the value obtained by Braginskii. Finally, in §9, we give a summary of the most important results.

## 2. Basic Equations

Each species of particles  $s$  is described by a distribution function  $F_s(\mathbf{x}, \mathbf{v}, t)$  normalized to unit integral, a mean number density  $n_s$ , an ionic charge  $q_s \equiv Z_s e$ , and a mass  $m_s$ . All species are assumed to be at the same temperature  $T$  and in an overall hydrostatic equilibrium, since the temperature and pressure equilibration timescales are much shorter than the diffusion times. The mass and charge densities are  $\rho_s = n_s m_s$  and  $\rho_{es} = n_s q_s$ . The mean fluid velocity of each species is defined by

$$\mathbf{u}_s = \int \mathbf{v} F_s d\mathbf{v}. \quad (7)$$

The mean fluid velocity is given by

$$\mathbf{u} = \frac{1}{\rho} \sum_s \rho_s \mathbf{u}_s, \quad (8)$$

where  $\rho = \sum_s \rho_s$  is the total mass density. The diffusion velocity for species  $s$  is defined by

$$\mathbf{w}_s = \mathbf{u}_s - \mathbf{u}, \quad (9)$$

and is therefore measured relative to the mean velocity of the fluid as a whole. We define the “residual heat flow vector” for species  $s$  by (Burgers, 1969):

$$\mathbf{r}_s = \left[ \frac{m_s}{2k_B T} \int F_s(\mathbf{v} - \mathbf{u}) |\mathbf{v} - \mathbf{u}|^2 d\mathbf{v} - \frac{5}{2} \mathbf{w}_s \right], \quad (10)$$

where  $k_B$  is Boltzmann's constant. The cross-section for Coulomb scattering between particles of species  $s$  and of species  $t$  ( $s$  can be equal to  $t$ ) is given by

$$\sigma_{st} = 2\sqrt{\pi}e^4 Z_s^2 Z_t^2 (k_B T)^{-2} \ln \Lambda_{st}, \quad (11)$$

where  $\ln \Lambda_{st}$  is the Coulomb logarithm ( $\Lambda_{st}$  is the ‘‘plasma parameter’’), a correction factor taking into account the logarithmic contribution of binary encounters with impact parameters up to the Debye shielding length. For the Coulomb logarithm, we adopt the following expression, obtained by Iben and MacDonald (1985) using numerical results from Fontaine and Michaud (1979b),

$$\ln \Lambda_{st} = \frac{1.6249}{2} \ln \left[ 1 + 0.18769 \left( \frac{4k_B T \lambda}{Z_s Z_t e^2} \right) \right], \quad (12)$$

where  $\lambda = \max(\lambda_D, a_0)$ ,  $\lambda_D = (k_B T / 4\pi e^2 \sum_s n_s Z_s^2)^{1/2}$  is the Debye length, and  $a_0 = (3/4\pi \sum_{ions} n_i)^{1/3}$  is the interionic distance. The friction coefficient between species  $s$  and  $t$  is

$$K_{st} = (2/3) \mu_{st} (2k_B T / \mu_{st})^{1/2} n_s n_t \sigma_{st} \quad (13)$$

where  $\mu_{st} \equiv m_s m_t / (m_s + m_t)$  is the reduced mass for species  $s$  and  $t$ .

The Burgers equations for mass, momentum, and energy conservation can then be written as

$$\frac{\partial n_s}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n_s w_s) = \left( \frac{\partial n_s}{\partial t} \right)_{nucl.}, \quad (14)$$

$$\frac{dp_s}{dr} + \rho_s g - \rho_{es} E = \sum_{t \neq s} K_{st} \left[ (w_t - w_s) + 0.6(x_{st} r_s - y_{st} r_t) \right], \quad (15)$$

and

$$\frac{5}{2} n_s k_B \frac{dT}{dr} = \sum_{t \neq s} K_{st} \left\{ \frac{3}{2} x_{st} (w_s - w_t) - y_{st} \left[ 1.6 x_{st} (r_s + r_t) + Y_{st} r_s - 4.3 x_{st} r_t \right] \right\} - 0.8 K_{ss} r_s. \quad (16)$$

In these equations,  $\mathbf{g} \equiv -(GM(r)/r^2)\hat{e}_r$  is the gravitational acceleration,  $g \equiv |\mathbf{g}|$ ,  $\mathbf{E}$  is the electric field,  $E \equiv |\mathbf{E}|$ ,  $x_{st} = \mu_{st}/m_s$ ,  $y_{st} = \mu_{st}/m_t$ , and  $Y_{st} = 3y_{st} + 1.3x_{st}m_t/m_s$ . The numerical coefficients in equations (15) and (16) are related to the collision integrals and were obtained using a pure Coulomb potential with a long-range cutoff at the Debye

length. More accurate results can be obtained by using numerical values derived from calculations using a screened Coulomb potential. We have assumed spherical symmetry and included a term for composition changes due to nuclear burning in equation (14). Using equation (15), it is straightforward to show that

$$\sum_s \left( \frac{dp_s}{dr} + \rho_s g - \rho_{es} E \right) = 0, \quad (17)$$

or

$$\frac{dp}{dr} + \rho g - \rho_e E = 0 \quad (18)$$

where  $p \equiv \sum_s p_s$  and  $\rho_e \equiv \sum_s \rho_{es}$  are the total pressure and total charge density. The departure from local charge neutrality is very small, with  $\rho_e E / \rho g \sim Gm_p^2 / e^2 \sim 10^{-37}$  (see discussion and eqs.(22)-(23) in BL). Equation (18) therefore reduces to the familiar equation of hydrostatic equilibrium,

$$dp/dr = -\rho g. \quad (19)$$

In addition, the following constraints must be satisfied: charge neutrality,

$$\sum_s q_{es} n_s = 0; \quad (20)$$

current neutrality,

$$\sum_s q_{es} n_s w_s = 0, \quad (21)$$

and local mass conservation,

$$\sum_s m_s n_s w_s = 0. \quad (22)$$

Note that equation (22) follows from the fact that the diffusion equations are solved in the rest frame of the plasma. The set of linear equations (15)-(16) and (21)-(22) forms a closed system for the diffusion velocities  $\mathbf{w}_s$ , the residual heat flow vectors  $\mathbf{r}_s$ , the gravitational acceleration  $\mathbf{g}$  and the electric field  $\mathbf{E}$  in terms of the pressure, temperature, and concentration gradients. Since we already know the value of  $g = GM(r)/r^2$ , this relation provides a useful check on the numerical results.



If we ignore thermal diffusion, the electric field is given by  $E = -(1/en_e)(\partial p_e/\partial r)$ . When thermal effects are included, the electric field can be written as (see eq. (57) in BL)

$$eE = -\frac{1}{n_e} \frac{\partial p_e}{\partial r} - \alpha_e k_B \frac{\partial T}{\partial r}. \quad (23)$$

Using his two-component fluid equations, Braginskii (1965) has obtained the values  $\alpha_e \approx 0.71$  for a pure hydrogen plasma and  $\alpha_e \approx 0.9$  for a pure helium plasma.

### 3. Method of solution

The system of equations (15)-(16) and (21)-(22) can be solved numerically. If there are  $S$  species in the system ( $S - 1$  ions plus electrons), there are  $S$  momentum equations (15) and  $S$  energy equations (16). The unknowns are the  $S$  drift velocities  $w_s$  and the  $S$  heat fluxes  $r_s$ . The gravitational acceleration and the electric field are also treated as unknowns, and we use the two additional equations for mass and charge conservation, equations (21) and (22), to help determine  $g$  and  $E$ . Note that the hydrostatic equilibrium condition (eq. 19) should be satisfied automatically, providing a useful check on the numerical results.

We now rewrite the basic equations in a dimensionless form that is better suited to a numerical treatment. The radius  $r$  is expressed in units of  $R_\odot$ , the mass density  $\rho$  in units of  $100 \text{ g cm}^{-3}$  and temperature  $T$  in units of  $10^7 \text{ K}$ , characteristic values at the center of the sun, and the time  $t$  is in units of  $\tau_0 = 6 \times 10^{13} \text{ yrs}$ , a characteristic diffusion time in the sun (see, e.g., Kippenhahn and Weigert p.60, or eq. (9) in BL). We write the Burgers equations (15)-(16) and the constraints (21)-(22) as

$$\frac{p}{K_0} \left[ \alpha_i \frac{d \ln p}{dr} + \nu_i \frac{d \ln T}{dr} + \sum_{\substack{j=1 \\ j \neq e, 2}}^S \gamma_{ij} \frac{d \ln C_j}{dr} \right] = \sum_{j=1}^{2S+2} \Delta_{ij} W_j, \quad (24)$$

where the following notations have been introduced. The concentration of species  $s$  is defined by

$$C_s \equiv n_s/n_e. \quad (25)$$

It is related to the mass fractions  $X_s \equiv m_s n_s/\rho$  by

$$C_s = \frac{X_s/A_s}{\sum_i Z_i X_i/A_i} \quad (26)$$

or inversely

$$X_s = \frac{A_s C_s}{\sum_i A_i C_i}, \quad (27)$$

where  $A_s$  is the atomic number of species  $s$ , and the sum is over all species, including the electrons. For the electrons,

$$A_e \equiv m_e/m_0, \quad (28)$$

where  $m_e$  and  $m_0$  are the electron and atomic masses, and

$$C_e \equiv 1. \quad (29)$$

The constant  $K_0$  is given by

$$K_0 = 1.144 \times 10^{-40} T^{-3/2} n_e^2, \quad (30)$$

where  $T$  and  $\rho$  are expressed in the characteristic units defined above. We use the ideal gas equation of state,  $p_s = n_s k_B T$ , and equations (25),(28) and (29) to write

$$\frac{p}{K_0} = 2.00 \frac{T^{5/2}}{\rho} \left( \sum_s C_s \right) \left( \sum_s A_s C_s \right), \quad (31)$$

where we have written the electron number density in terms of the mass density,  $\rho = m_0 n_e \sum_s A_s C_s$ . The variables  $W_i$  are

$$W_i = \begin{cases} w_i & \text{for } i = 1, \dots, S \\ r_{i-S} & \text{for } i = S+1, \dots, 2S \\ K_0^{-1} n_e e E & \text{for } i = 2S+1 \\ K_0^{-1} n_e m_0 g & \text{for } i = 2S+2. \end{cases} \quad (32)$$

If we define  $C \equiv \sum_i C_i$ , the coefficients on the left-hand-side of equation (24) are given by

$$\alpha_i = \begin{cases} \frac{C_i}{C} & \text{for } i = 1, 2, \dots, S \\ 0 & \text{for } i = S+1, \dots, 2S+2, \end{cases} \quad (33)$$

$$\nu_i = \begin{cases} 2.5 \frac{C_{i-S}}{C} & \text{for } i = S+1, \dots, 2S \\ 0 & \text{for } i = 1, \dots, S \text{ and } i = 2S+1, 2S+2, \end{cases} \quad (34)$$

and

$$\gamma_{ij} = \begin{cases} \frac{C_i}{C} \left[ \left( \delta_{ij} - \frac{C_j}{C} \right) - \left( \delta_{i2} - \frac{C_2}{C} \right) \frac{Z_j C_j}{Z_2 C_2} \right] & \text{for } i = 1, \dots, S \\ 0 & \text{for } i = S+1, \dots, 2S+2. \end{cases} \quad (35)$$

The coefficients on the right-hand-side of equation (24) are given by

$$\Delta_{ij} = \begin{cases} -\sum_{k \neq i} \kappa_{ik} & \text{for } j = i \\ \kappa_{ij} & \text{for } j = 1, \dots, S \text{ and } j \neq i \\ \sum_{k \neq i} 0.6\kappa_{ik}x_{ik} & \text{for } j = i + S \\ -0.6\kappa_{i,j-S}y_{i,j-S} & \text{for } j = S + 1, \dots, 2S \text{ and } j \neq i + S \\ Z_i C_i & \text{for } j = 2S + 1 \\ -A_i C_i & \text{for } j = 2S + 2 \end{cases} \quad (36)$$

for  $i=1, \dots, S$ , by

$$\Delta_{ij} = \begin{cases} \sum_{k \neq j} 1.5\kappa_{i-S,k}x_{i-S,k} & \text{for } j = i - S \\ -1.5\kappa_{i-S,j}x_{i-S,j} & \text{for } j = 1, \dots, S \text{ and } j \neq i - S \\ -\sum_{k \neq i} \kappa_{i-S,k}y_{i-S,k}(1.6x_{i-S,k} + Y_{i-S,k}) & \text{for } j = i \\ -0.8\kappa_{i-S,i-S} & \text{for } j = i \\ 2.7\kappa_{i-S,j-S}y_{i-S,j-S}x_{i-S,j-S} & \text{for } j = S + 1, \dots, 2S \text{ and } j \neq i \\ 0 & \text{for } j = 2S + 1, 2S + 2 \end{cases} \quad (37)$$

for  $i=S+1, \dots, 2S$ , by

$$\Delta_{ij} = \begin{cases} Z_j C_j & \text{for } j = 1, \dots, S \\ 0 & \text{for } j = S + 1, \dots, 2S + 2 \end{cases} \quad (38)$$

for  $i=2S+1$ , and finally by

$$\Delta_{ij} = \begin{cases} A_j C_j & \text{for } j = 1, \dots, S \\ 0 & \text{for } j = S + 1, \dots, 2S + 2 \end{cases} \quad (39)$$

for  $i=2S+2$ . In these expressions, the coefficient  $\kappa_{st}$  is defined by

$$\kappa_{st} = \left( \frac{A_s A_t}{A_s + A_t} \right)^{1/2} C_s C_t Z_s^2 Z_t^2 \ln \Lambda_{st}. \quad (40)$$

It is related to the friction coefficient through  $K_{st} = K_0 \kappa_{st}$ . We have used the constraint of charge neutrality to eliminate the concentration gradient of species 2 in equation (24),

$$\frac{d \ln C_2}{dr} = - \sum_{\substack{j=1 \\ j \neq e, 2}}^S \frac{Z_j C_j}{Z_2 C_2} \frac{d \ln C_j}{dr}. \quad (41)$$

Since  $p/K_0$  is proportional to  $T^{5/2}/\rho$  (see eq. 31), all the velocities will be proportional to  $T^{5/2}/\rho$ . Therefore, we introduce the function  $\xi_s$  (following BL), such that

$$w_s = (T^{5/2}/\rho)\xi_s. \quad (42)$$

The rate of change of the element mass fractions due to diffusion is now written in dimensionless form as

$$\frac{\partial X_s}{\partial t} = -\frac{1}{\rho r^2} \frac{\partial}{\partial r} [r^2 X_s T^{5/2} \xi_s(r)], \quad (43)$$

the generalization of equation (1) of BL to the case of arbitrary concentrations and a more accurate treatment of the plasma physics.

Equations (24) are linear. Therefore, we can combine linearly the solutions obtained by keeping only one of the gradients different from zero. We write the results in terms of generalized diffusion coefficients  $A_p(s)$ ,  $A_T(s)$  and  $A_t(s)$  for species  $s$ , as

$$\xi_s(r) = A_p(s) \frac{\partial \ln p}{\partial r} + A_T(s) \frac{\partial \ln T}{\partial r} + \sum_{t \neq e, 2} A_t(s) \frac{\partial \ln C_t}{\partial r}. \quad (44)$$

If  $\ln \Lambda$  is assumed identical for all the interactions, the coefficients  $A_p$ ,  $A_T$ , and  $A_t$  are functions of the mass fractions only. If  $\ln \Lambda$  is defined by equation (12), these coefficients also depend on the charges, the temperature, and the density.

## 4. Hydrogen and helium diffusion

First, we consider the diffusion of hydrogen and helium, neglecting the presence of heavier elements. We calculate the hydrogen diffusion velocity. The helium diffusion velocity is then simply obtained from the constraint that there is no mean fluid velocity,  $\sum_s X_s w_s = 0$ . Neglecting the electron mass compared to the proton mass, we have  $w_\alpha = -(X/Y)w_H$ . The rate of change of the helium number density is therefore equal and opposite to the rate of change of the hydrogen number density,  $(\partial Y/\partial t) = -(\partial X/\partial t)$ . Helium diffuses towards the center of the star, whereas hydrogen diffuses outwards.

In the absence of heavy elements, the function  $\xi_H$  is given by<sup>3</sup>

$$\xi_H = A_p(H) \frac{\partial \ln p}{\partial r} + A_T(H) \frac{\partial \ln T}{\partial r} + A_H(H) \frac{\partial \ln C_H}{\partial r}. \quad (45)$$

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<sup>3</sup> Note that BL define  $\xi_H$  with the opposite sign. They also write  $\xi_H$  in terms of the mass fraction gradient instead of the number concentration gradients. These are simply related by  $\partial \ln C_s/\partial r = \partial \ln X_s/\partial r - (\sum_i Z_i X_i/A_i)^{-1} \sum_j (Z_j X_j/A_j) \partial \ln X_j/\partial r$  or inversely,  $\partial \ln X_s/\partial r = \partial \ln C_s/\partial r - (\sum_i A_i C_i)^{-1} \sum_j A_j C_j \partial \ln C_j/\partial r$ . If hydrogen and helium are the only elements,  $\partial \ln X/\partial r = (1 + X) \partial \ln C_H/\partial r$ .

We have chosen helium as element number 2, i.e., we write the diffusion velocity in terms of the hydrogen concentration gradient, using equation (41) to eliminate the helium concentration gradient.

Two major simplifications are usually made when calculating the hydrogen and helium diffusion velocities in the absence of heavy elements. It is usually assumed that the Coulomb logarithms  $\ln\Lambda_{ij}$  are identical for all interactions. This allows the factorization of  $\ln\Lambda$  outside the function  $\xi_H$  (see, eg., Noerdlinger 1977, Bahcall and Loeb 1990). In that case, the coefficients  $A_p$ ,  $A_T$  and  $A_H$  depend only on the hydrogen (or helium) concentration, not on density, temperature and ionic charges. The second simplification is to ignore the residual heat fluxes  $\mathbf{r}_s$ . Then, the diffusion velocities are easier to calculate analytically, since there is no need for the heat equations (16) and the number of variables and equations is reduced by a factor of two. However, these simplifications can lead to large relative errors in the diffusion velocities. In particular it has been argued by MP that thermal diffusion can increase the diffusion velocities by 30%.

In figure 1a-c, we show the variation of the coefficients  $A_p$ ,  $A_T$ , and  $A_H$  with the hydrogen mass fraction  $X$ . To obtain these results, we have assumed  $T = 10^7$  K and  $\rho = 100$  g cm<sup>-3</sup>, typical values in the core of the sun. The exact results are represented with solid lines, the results obtained neglecting the heat fluxes are represented with short-dashed lines, and the results obtained by keeping the heat fluxes but using  $g \ln\Lambda = 2.2$  for all interactions<sup>4</sup> are represented with long-dashed lines. If the heat fluxes are totally neglected, the two coefficients  $A_p$  and  $A_H$  are underestimated, and  $A_T = 0$ . In figure 1d, we show the relative errors on the coefficients due to these approximations. The short-dashed line represents the error on  $A_p$  and  $A_H$  when the heat fluxes are neglected. It can be as high as 45% for small values of the hydrogen mass fraction (not relevant to the sun). The long-dashed lines represent the errors due to  $\ln\Lambda = 2.2$ . The errors on  $A_p$ ,  $A_H$  and  $A_T$  are smaller than 20%, except when  $X \sim 1$ . In the interior of the sun,  $X$  varies approximately between 0.3 and 0.7. For these values of  $X$ , the error does not exceed 20%.

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<sup>4</sup> This value is usually considered representative of the Coulomb logarithms in the solar interior (see, e.g., Noerdlinger 1977 and BL).

## 4.1 Comparison with Bahcall and Loeb

In order to keep the analytical calculations simple without neglecting the thermal effects one can use “effective” thermal diffusion coefficients (obtained through fits to the exact numerical results). In figure 2a, we show the ratio between the “exact” coefficients and those obtained by BL who neglected the residual heat fluxes, assumed a Coulomb logarithm of 2.2 for all the interactions, and used an “effective” thermal diffusion coefficient. The expressions obtained by BL are:

$$\xi_H^{BL}(r) = A_p^{BL} \frac{\partial \ln p}{\partial r} + A_T^{BL} \frac{\partial \ln T}{\partial r} + A_H^{BL} \frac{\partial \ln C_H}{\partial r} \quad (46)$$

with

$$A_p^{BL} = -5(1 - X)/4, \quad (47)$$

$$A_T^{BL} = -6(1 - X)(X + 0.32)/(1.8 - 0.9X)(3 + 5X), \quad (48)$$

and

$$A_H^{BL} = -(X + 3)/(3 + 5X). \quad (49)$$

The result for the thermal diffusion coefficient was obtained by fitting values obtained previously by Aller and Chapman (1960), Montmerle and Michaud (1976), and Noerdlinger (1978). For values of  $X$  between 0.3 and 0.7, the error made by BL on  $A_p$  and  $A_H$  is smaller than 40%, whereas the error on  $A_T$  is as large as 70%. However, as we will show in §6, large errors on  $A_T$  do not necessarily lead to large errors on the diffusion velocities, since the temperature gradient in the sun is much smaller than the pressure and concentration gradients.

It is important to notice that the heat fluxes affect not only the thermal diffusion coefficients, but also the pressure and composition gradient coefficients.

## 4.2. Comparison with Michaud and Proffitt

MP solved the Burgers equations with and without including the heat fluxes, then represented the effects of the heat fluxes by an ad-hoc correction to the results obtained when neglecting those heat fluxes. In our dimensionless variables, their results can be written as

$$\xi_H^{MP}(r) = A_p^{MP} \frac{\partial \ln p}{\partial r} + A_T^{MP} \frac{\partial \ln T}{\partial r} + A_H^{MP} \frac{\partial \ln C_H}{\partial r} \quad (50)$$

with

$$A_p^{MP} = -\frac{5}{4} \frac{(1-X)}{(0.7+0.3X)(\ln \Lambda_{xy}/2.2)}, \quad (51)$$

$$A_T^{MP} = -\frac{9}{8} \frac{(1-X)}{(0.7+0.3X)(\ln \Lambda_{xy}/2.2)}, \quad (52)$$

and

$$A_H^{MP} = -\frac{(X+3)}{(3+5X)(0.7+0.3X)(\ln \Lambda_{xy}/2.2)}, \quad (53)$$

where

$$\ln \Lambda_{xy} = -19.95 - \frac{1}{2} \ln \rho - \frac{1}{2} \ln \frac{X+3}{2} + \frac{3}{2} \ln T. \quad (54)$$

In figure 2b, we show in solid lines the ratio of our coefficients and those obtained by MP. The difference between our results and those obtained by MP is smaller than 15% for  $A_T$ , and smaller than 5% for  $A_p$  and  $A_H$ . This small discrepancy will be discussed in §6.

## 5. Heavy element diffusion

Because of the complexity caused by the addition of heavy elements, this problem has always been approached with additional simplifications (Vauclair *et al.* 1974, Noerdlinger 1978, BL, MP). One common simplification is to assume a negligible helium concentration, therefore reducing the problem to a two-species situation. However, this assumption is not valid in the interior of the sun, where the characteristic mass fractions of hydrogen, helium, and oxygen are of the order of 0.34, 0.64 and 0.01 respectively (see, e.g., Bahcall 1990).

The functions  $\xi_s$  for the hydrogen and oxygen are now written as

$$\xi_H = A_p(H) \frac{d \ln p}{dr} + A_T(H) \frac{d \ln T}{T} + A_H(H) \frac{d \ln C_H}{dr} + A_O(H) \frac{d \ln C_O}{dr}, \quad (55)$$

and<sup>5</sup>

$$\xi_O = A_p(O) \frac{\partial \ln p}{\partial r} + A_T(O) \frac{\partial \ln T}{\partial r} + A_H(O) \frac{\partial \ln C_H}{\partial r} + A_O(O) \frac{\partial \ln C_O}{\partial r}. \quad (56)$$

It is interesting to show the variation of the diffusion coefficients as a function of the hydrogen mass fraction for fixed values of the temperature, density, and heavy element mass fraction. Indeed, as we will show in the next section, in the sun these parameters all vary with radius, and it is more difficult to extract the hydrogen mass fraction dependence itself. In figure 3, we show the four coefficients  $A_p(H)$ ,  $A_T(H)$ ,  $A_H(H)$  and  $A_O(H)$  as a function of the oxygen mass fraction  $X$ . These results were obtained using  $T = 10^7$  K,  $\rho = 100 \text{ g cm}^{-3}$  and  $Z = 0.01$ , where  $Z$  is the oxygen mass fraction. These values are typical in the solar interior. Note that now  $X$  has a maximum value determined by  $X_{max} = 1 - Z$ , because of the charge neutrality constraint. We notice that the coefficient  $A_O(H)$  is two orders of magnitude smaller than the other coefficients. This was expected, since we have chosen  $K_{HO}/K_{H\alpha} \sim Z/Y \sim 10^{-2}$ . The error made by neglecting the presence of oxygen when calculating the hydrogen diffusion velocity is smaller than 2%.

In figure 4 we show the oxygen diffusion coefficients  $A_p(O)$ ,  $A_T(O)$ ,  $A_H(O)$  and  $A_O(O)$ . Again, the coefficient  $A_O(O)$  is much smaller than the other three coefficients, and can be neglected to the level of precision desired in these calculations.

## 6. Diffusion velocities in the sun

We now calculate the diffusion velocities of hydrogen, helium, and heavier elements in the present solar interior ( $r < 0.7R_\odot$ ). We use values for the pressure, temperature, density and mass fractions of the contemporary sun, obtained from the standard solar model (table 4.4, in Bahcall 1990). Since we have the radial profile of the pressure, temperature, and mass fractions, we can calculate their gradients. The coefficients  $A_p$ ,  $A_T$ ,  $A_H$  and  $A_O$  are computed using the tabulated values of  $T$ ,  $\rho$ ,  $X$ ,  $Y$ , and  $Z$ . The iron abundance is assumed to be uniform and given by  $\log_{10}(n_{Fe}/n_H) = 6.82 - 12$ , and its ionization is 21.

In figure 5, we show the radial variation of the hydrogen diffusion coefficients. In figure 6, we show the relative importance of the different terms in the hydrogen diffusion

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<sup>5</sup> Note that BL define the function  $\xi_A$  such that  $\partial Z/\partial t = -(1/\rho r^2) \partial[r^2 X Z T^{5/2} \xi_A / (2 - X)]/\partial r$ . The function  $\xi_A$  is related to the function  $\xi_O$  through  $\xi_A = \xi_O(2 - X)/X$ .



velocity. The oxygen concentration gradient gives a negligible contribution to  $w_H$ . The temperature term is not negligible, but is smaller than the pressure term (between 25% and 50% of the pressure term). Therefore, a large error on the temperature diffusion coefficient does not necessarily lead to a large error for the diffusion velocities.

The timescale for a change in the element abundances can be characterized by  $t \equiv r/w_H$ . To obtain the time  $t$  in units of  $t_0 = 5 \times 10^9$  yrs, the age of the sun, we simply multiply the dimensionless time by  $t_0/\tau_0$ , where  $\tau_0$  is the characteristic diffusion time defined in § 3. The smaller the time  $t$ , the faster the element concentrations change. In figure 7, we show the variation of  $t$  with the radius. The fastest changes in the hydrogen concentration occurs at approximately  $0.05R_\odot$ , where  $t_{min} \sim 70 t_0$ .

### 6.1. Comparison with Bahcall and Loeb

In figure 8a, we compare our exact results with those obtained using the analytic BL formulae, equations (47)-(49) (eqs. 1-5 in BL). The BL formula underestimate the diffusion coefficients. The error on the pressure and concentration diffusion coefficients is smaller than 30%. The error on the temperature diffusion coefficient is of the order of 50%, except near the center where it becomes very large.

In figure 9a-c, we show in solid lines the results for the diffusion velocities of hydrogen, oxygen, and iron. The helium diffusion velocity  $w_\alpha$  is related to the hydrogen and oxygen diffusion velocities through the zero mean velocity constraint, equation (22),

$$w_\alpha = -(X_H w_H + X_O w_O + X_{Fe} w_{Fe})/X_{He}. \quad (57)$$

We also show in short-dashed lines the BL results for the hydrogen and oxygen diffusion velocities, given by equations (47)-(49) for the hydrogen velocity, and equations (2)-(5) in BL for the oxygen velocity. In the BL approximation, the helium velocity is given by  $w_\alpha = -(X_H/X_{He})w_H$ . The error in  $w_H$  due to the BL approximations is smaller than 30%, except near the center, where the error is as high as 70%. However, the error for the oxygen diffusion velocity is very large. This was expected, since BL neglected completely the presence of helium when calculating  $w_O$ . The BL results for  $w_O$  are therefore only valid when  $X \approx 1$ .

## 6.2. Comparison with Michaud and Proffitt

In figure 8b, we compare our results with those obtained using the MP formula, equations (51)-(53). The difference between our results and the results obtained by MP for the diffusion coefficients is smaller than 5% for  $A_p$  and  $A_H$ . Our thermal diffusion coefficient  $A_T$  is larger by about 20%. This discrepancy may result from the fact that we have used  $z_{st} = 0.6$  for the heat flux coefficient in equations (15) and (16). If the collision integrals are calculated using a screened Coulomb potential, the value obtained for  $z_{st}$  is about 2/3 smaller (Proffitt 1993).

In figure 9a-c, we show the results for the diffusion velocities of hydrogen, oxygen and iron. The MP result for the heavy elements diffusion velocities are given by

$$\begin{aligned} \xi_i = -\frac{2}{\sqrt{5}Z_i^2} & \left[ \frac{\frac{d}{dr} \left\{ \ln \left[ \frac{X_i}{5X+3} \left( \frac{1+X}{5X+3} \right)^{Z_i} \right] \right\} + \left[ 1 + Z_i - A_i \left( \frac{5X+3}{4} \right) \right] \frac{d \ln p}{dr}}{X(A_{ix}^{1/2}C_{ix} - A_{iy}^{1/2}C_{iy}) + A_{iy}^{1/2}C_{iy}} \right] \\ & + X\xi_H \left[ \frac{(A_{ix}^{1/2}C_{ix} - A_{iy}^{1/2}C_{iy})}{X(A_{ix}^{1/2}C_{ix} - A_{iy}^{1/2}C_{iy}) + A_{iy}^{1/2}C_{iy}} - 0.23 \right] \\ & + \frac{0.54(4.75X + 2.25)}{(\ln \Lambda_{xy} + 5)} \frac{d \ln T}{dr}, \end{aligned} \quad (58)$$

where  $A_{ij} \equiv A_i A_j / (A_i + A_j)$  is the reduced mass in atomic number units,  $C_{ij} = \ln[\exp(1.2 \ln \Lambda_{ij}) + 1] / 1.2$ , and  $\ln \Lambda_{ij}$  is given by equation (54). The difference between our results and those obtained by MP is smaller than 15% for the hydrogen diffusion velocity, and smaller than 20% for the oxygen diffusion velocity.

## 7. Analytical fits

All the results shown above were obtained using the expression (12) for the “effective” Coulomb logarithms. If we use (as in the previous results) the correct charge, temperature and density dependent expression for the Coulomb logarithms, we cannot give a simple analytical fit, as in BL, for the diffusion coefficients in terms of the hydrogen mass fraction. Even though one could in principle incorporate a subroutine which solves the problem of element diffusion in a standard solar model evolution code, it is useful to give a simple analytical fit of the results obtained here in the solar interior. It is convenient to provide

to stellar evolution programs other types of input physics, such as opacities or equations of state, in the form of tabulated values, or in terms of approximate analytical fits. In order to provide a similarly convenient service for diffusion, we adopted a constant for each Coulomb logarithm, the value it has at the center of the sun. In figure 10, we show the values of the various Coulomb logarithms in the sun computed using equation (12). In figure 11a-c, we compare the diffusion coefficients obtained with these Coulomb logarithms with those obtained using a constant value of  $\Lambda$  (equal to its central value). In figure 11d, we show that the error on  $w_H$  is smaller than 4% in the solar core ( $r/R_\odot \leq 0.4$ ), and remains smaller than 15% up to the convection zone. The error on the heavy elements diffusion velocities are  $\lesssim 6\%$  in the solar core, and remain  $\lesssim 20\%$  up to the convection zone. We can now fit these results to second order polynomials. Since the presence of oxygen and iron have very little influence on the hydrogen velocity, we can assume that the diffusion coefficients depend only on  $X$ . We find:

$$\begin{cases} A_p(H) = -2.09 + 3.15 X - 1.07 X^2, \\ A_T(H) = -2.18 + 3.12 X - 0.96 X^2, \\ A_H(H) = -1.51 + 1.85 X - 0.85 X^2, \end{cases} \quad (59)$$

for the hydrogen diffusion coefficients,

$$\begin{cases} A_p(O) = 0.15 + 1.34 X - 0.89 X^2, \\ A_T(O) = 0.53 + 1.99 X - 0.72 X^2, \\ A_H(O) = 0.08 + 0.58 X - 0.28 X^2, \end{cases} \quad (60)$$

for the oxygen diffusion coefficients, and

$$\begin{cases} A_p(Fe) = 0.25 + 1.31 X - 0.87 X^2, \\ A_T(Fe) = 0.65 + 1.99 X - 0.75 X^2, \\ A_H(Fe) = 0.09 + 0.53 X - 0.27 X^2, \end{cases} \quad (61)$$

for the iron diffusion coefficients. The errors due to the polynomial fits are of the order of 0.2%. There is no need to use higher order polynomials, since the error made with these second order polynomials are already much smaller than the errors introduced by the simplified Coulomb logarithm.

For an easy comparison with BL and MP results, we can factorize the BL results and write the numerical fits as

$$A_p(H) = -\frac{5}{4}(1 - X)a_p, \quad (62)$$

$$A_T(H) = -\frac{6(1 - X)(X + 0.32)}{(1.8 - 0.9X)(3 + 5X)}a_t, \quad (63)$$

and

$$A_H(H) = -\frac{(X + 3)}{(3 + 5X)}a_H. \quad (64)$$

First order polynomial fits give the following analytical results:

$$\begin{cases} a_p = 1.66 - 0.82 X, \\ a_T = 4.46 - 3.65 X, \\ a_H = 1.63 - 0.74 X. \end{cases} \quad (65)$$

The error introduced by the polynomial fit is much smaller than the error due to the simplified Coulomb logarithm. Equations (62)-(65) can be used to improve existing diffusion subroutines that are based on the BL formalism.

It is important to remember that these fits are made for the standard parameters of the solar interior.

## 8. Hydrostatic equilibrium and electric field

As explained in § 2, we solve the system of equations (24) for the diffusion velocities, the residual heat flow vectors, the gravitational acceleration, and the electric field. On the other hand, the equation of hydrostatic equilibrium given by equation (19) must be satisfied. The numerical results for  $g$  can be written as

$$g = -\frac{p}{\rho} \left[ A_p(g) \frac{d \ln p}{dr} + A_T(g) \frac{d \ln T}{dr} + A_H(g) \frac{d \ln C_H}{dr} \right]. \quad (66)$$

In hydrostatic equilibrium,  $A_T = A_H = 0$  and  $A_p = 1$ . The equation of hydrostatic equilibrium is satisfied to  $10^{-7}$  by the numerical results.

Similarly, we write the results for the electric field as

$$E = \frac{-p_e}{en_e} \left[ A_p(E) \frac{d \ln p}{dr} + A_T(E) \frac{d \ln T}{dr} + A_H(E) \frac{d \ln C_H}{dr} \right]. \quad (67)$$

The coefficients in equation (67) should be compared with equation (23). We find an excellent agreement between our numerical results and equation (23) for the coefficients in front of the pressure and concentration gradients (with an error smaller than  $0.2 \times 10^{-2}$ ). Our values for the coefficient  $A_T(E)$  are slightly larger than the results obtained by Braginskii (1965) for  $\alpha_e$ . As shown in figure 12, we observe however the same tendency for this coefficient to increase slightly with an increase in the helium concentration. We get  $\alpha_e = 0.8$  for  $X = 1$ , and  $\alpha_e = 0.9$  for  $X = 0$ .

## 9. Summary and conclusions

We have developed a Fortran program to solve numerically the Burgers equations for an arbitrary number of species, without any approximation. For the discussion of solar conditions given here, we have neglected the radiative forces, but these forces could easily be incorporated in the numerical routine. The accuracy of the results for the diffusion velocities is limited only by the validity of the expression for the Coulomb logarithm. The diffusion velocities of hydrogen, oxygen and iron were calculated in the solar interior and compared with the results of Bahcall and Loeb (1990) and by Michaud and Proffitt (1992). The results of BL for the hydrogen diffusion velocity are smaller by  $\sim 30\%$ , except near the center, where the error is much larger. The results obtained by MP for the hydrogen and oxygen diffusion velocities differ by  $\lesssim 15\%$ .

We provide analytical fits of our numerical results for the diffusion coefficients as a function of the hydrogen mass fraction in the solar interior (eqs. 59- blf). These fits were obtained by assuming fixed values for the Coulomb logarithms, equal to their values at the center of the sun.

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## References

- Aller, L. H., and Chapman, S. 1960, *Ap. J.*, **132**, 461.
- Bahcall, J. N. 1990, *Neutrino Astrophysics* (Cambridge University Press, Cambridge).
- Bahcall, J. N., and Loeb, A. 1990, *Ap. J.*, **360**, 267.
- Bahcall, J. N. and Pinsonneault, M. H. 1992a, *Ap. J.*, **395**, L119.
- . 1992b, *Rev. Mod. Phys.*, **64**, 885.
- Book, D. L. 1990, *NRL Plasma Formulary* (NRL Publication 177-4405).
- Braginskii, S. I. 1965, *Reviews of Plasma Physics*, Vol.1, p.205 (Consultants Bureau, New York).
- Burgers, J. M. 1969, *Flow Equations for Composite Gases* (Academic Press, New York).
- Chaboyer, B., Deliyannis, C. P., Demarque, P., Pinsonneault, M. H., and Sarajedini, A. 1992, *Ap. J.*, **388**, 372.
- Christensen-Dalsgaard, J., Proffitt, C. R., and Thompson, M. J. 1993, preprint.
- Cox, A. N., Guzik, J. A., and Kidman, R. B. 1989, *Ap. J.*, **342**, 1187.
- Fontaine, G., and Michaud, G. 1979, *Ap. J.*, **231**, 826.
- . 1979b, IAU Colloquium **59**, 192.
- Guenther, D. B., Pinsonneault, M. H., and Bahcall, J. N. 1993, preprint.
- Iben, I., and MacDonald, J. 1985, *Ap. J.*, **296**, 540.
- Ichimaru, S. 1982, *Rev. Mod. Phys.*, **54**, 1017.
- Kippenhahn, R., and Weigert, A. 1991, *Stellar Structure and Evolution* (Springer-Verlag, Berlin).
- Michaud, G., Charland, Y., Vauclair, S., and Vauclair, G. 1976, *Ap. J.*, **210**, 447.
- Michaud, G. and Proffitt, C. R. 1992, preprint.
- Michaud, G. and Vauclair, S. 1991, *Element Separation by Atomic Diffusion*, in Solar Interior and Atmosphere, ed. Cox, A.N., Livingston, W.C. and Matthews, M.S. (University of Arizona press, Tucson), p.312.
- Montmerle, T., and Michaud, G. 1976, *Ap. J. Suppl.*, **31**, 489.
- Noerdlinger, P. D. 1977, *Astr. Ap.*, **57**, 407.
- . 1978, *Ap. J. Suppl.*, **36**, 259.

Paquette, C., Pelletier, C., Fontaine, G., and Michaud, G. 1986, *Ap. J. Suppl.*, **61**, 177.

Pelletier, C., Fontaine, G., Wesemael, F., Michaud, G., and Wegner, G. 1986, *Ap. J.*, **307**, 242.

Proffitt, C. R. 1993, *personal communication*; see also Paquette et al. 1986.

Proffitt, C. R. and Michaud, G. 1991, *Ap. J.*, **380**, 238.

Vauclair, S., and Vauclair, G. 1982, *Ann. Rev. Astr. Ap.*, **20**, 37.

Vauclair, G., Vauclair, S., and Pamjatnikh, A. 1974, *Astr. Ap.*, **31**, 63.

## FIGURE CAPTIONS

FIG. 1.— Variation of the hydrogen diffusion coefficients with the hydrogen mass fraction in a pure hydrogen-helium plasma, with  $T = 10^7$  K and  $\rho = 100 \text{ gcm}^{-3}$ . The solid lines represent the results obtained using Burgers equations, with no approximation. The short-dashed lines represent the results obtained when neglecting the heat fluxes. The long-dashed lines are the results obtained by using a single value for all the Coulomb logarithms, equal to 2.2. The dash-dot lines are the results obtained when neglecting the heat fluxes **and** using a single value for all the Coulomb logarithms, equal to 2.2. (a) Pressure gradient coefficient  $A_p$ . (b) Temperature gradient coefficient  $A_T$ . (c) Hydrogen concentration gradient coefficient  $A_H$ . (d) Relative errors due to the approximations. The short-dashed line is the error on  $A_p$  and  $A_H$  when the heat fluxes are neglected. The long-dashed lines are the errors made by using  $\ln \Lambda = 2.2$ , and the dash-dot line is the error on  $A_p$  and  $A_H$  when both approximations are made.

FIG. 2.— (a) Ratio of the exact hydrogen diffusion coefficients and those obtained by various approximations, in terms of the hydrogen mass fraction  $X$ , with  $T = 10^7$  K and  $\rho = 100 \text{ gcm}^{-3}$ . (a) Comparison with Bahcall and Loeb (1990). (b) Comparison with Michaud and Proffitt (1992). The solid lines and the dashed lines are the results obtained using equations (54) and equation (12) respectively for the Coulomb logarithms in the Michaud and Proffitt formulae (51)-(53).

FIG. 3.— Variation of the hydrogen diffusion coefficients with the hydrogen mass fraction in a hydrogen-helium-oxygen plasma, with  $T = 10^7$  K,  $\rho = 100 \text{ gcm}^{-3}$ , and  $Z = 0.01$ .

FIG. 4.— Variation of the oxygen diffusion coefficients with the hydrogen mass fraction in a hydrogen-helium-oxygen plasma, with  $T = 10^7$  K,  $\rho = 100 \text{ gcm}^{-3}$ , and  $Z = 0.01$ .

FIG. 5— Hydrogen diffusion coefficients in the present sun, as a function of radius.



FIG. 6— Contributions to the hydrogen diffusion velocity in the sun due to each gradient.

FIG. 7— Local diffusion time of hydrogen, oxygen, and iron as a function of radius, in units of the age of the sun.

FIG. 8— (a) Ratio of the exact hydrogen diffusion coefficients and those obtained by Bahcall and Loeb (1990) as a function of the radius. (b) Ratio of the exact hydrogen diffusion coefficients and those obtained by Michaud and Proffitt (1992) as a function of the radius.

FIG 9— Diffusion velocities in the contemporary sun. The solid lines are the exact results; The short-dashed lines are the results of Bahcall and Loeb (1990) (see eqs. 1-5 in BL); The long-dashed lines are the results of Michaud and Proffitt (1992). (a) Hydrogen diffusion velocity. (b) Oxygen diffusion velocity. (c) Iron diffusion velocity.

FIG. 10— Coulomb logarithms in the sun.

FIG. 11— Diffusion coefficients in the sun, as a function of radius. The solid lines represent the results obtained using Burgers equations, with no approximation. The dashed lines represent the results obtained using  $\ln \Lambda = 3.2$  for all the interactions. (a) Pressure gradient coefficient  $A_p$ . (b) Temperature gradient coefficient  $A_T$ . (c) Hydrogen concentration gradient coefficient  $A_H$ . (d) Relative error on the hydrogen and oxygen diffusion velocities made by using  $\ln \Lambda = 3.2$ .

FIG. 12— Thermal coefficient for the electric field (see eq. 23)