Endogenous Growth and Regional Dynamics in an OLG Model with Land*

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Abstract

This paper examines the existence condition of a balanced growth path in an overlapping generations model in which production uses three inputs, physical capital, human capital and land, with increasing returns to scale. Human capital is the engine of economic growth. It is shown that, unlike standard economic geography models, increasing returns verifying balanced growth always lead to regional convergence. Physical capital mobility turns out to be an overwhelming convergence force.

Keywords: endogenous growth, human capital, land, overlapping generations, regional dynamics.
JEL Classification numbers: E13, O41, R11.

1 Introduction

The objective of this paper is to study endogenous growth and regional dynamics in an overlapping generations model in which production uses three inputs: physical capital, human capital and land. In particular, we want to examine the effect of increasing returns on income distribution in a two-region economy where physical capital is perfectly mobile across regions.

Benchmark neo-classical growth models predict that per capita incomes of regions that are identical in their structural characteristics converge with one another in the long run independently of their initial conditions (Barro and Sala-I-Martin (1995) and Galor (1996)). The law of diminishing returns implies that resources will move to places where the returns are high until the relative quantities of resources are equalized across locations. If this law explains well the allocation of labor and land in pre-industrial societies, it does not say much why economic growth in the industrial era has created so much inequality across countries in the world (Lucas 2002). Neither does it account for economic agglomerations within countries. The endogenous growth and economic geography literatures both point to the importance of increasing returns. The theories of endogenous growth have worked out to combine increasing returns and market competition (diminishing returns) as in Romer (1986) and Lucas (1988). Models in economic geography suggest increasing returns and transportation costs in a monopolistic competition framework (Fujita, Krugman, and Venables (1999) and Fujita and Thisse (2002)).

The remaining difficulty is to have increasing returns and room for convergence when capital is mobile across locations. A possible way we explore here is to add a non-reproducible factor in the production function. In a two-country OLG model with land and convex technology, Crettez, Michel, and Vidal (1998) show that capital mobility and labor mobility do not yield equivalent results in terms of welfare if countries have different population sizes, as is the case in the real world. Only labor mobility allows of equalization of standard of living. In standard economic geography models (Fujita, Krugman, and Venables 1999), in which there are increasing returns and no physical capital, it is the opposite. Labor mobility of the skilled workers is one of the main sources of inequality in living standards.

In this paper, we want to go further in the analysis by considering the characteristics of core-periphery models in a growth model and study the effect of increasing returns and capital mobility on regional dynamics. We built an overlapping generations model with physical capital, human capital and land. Human capital accumulates from the past stock of human capital and education spending made by each generation. We introduce the possibility of non-convex technologies. Therefore, a condition, that we determine below, is required to keep a balanced growth path. We compare different long run growth rates depending on the technologies. Then, we study the regional dynamics in a two-country framework when physical capital is perfectly mobile. Finally, we compare our results with those of the core-periphery models.

The paper is organized as follows. Section 2 defines the model. Section 3 presents the

effect of land when technologies are convex. Section 4 introduces non-convexities and determines the balanced growth condition. Section 5 analyzes regional dynamics. Finally, section 6 concludes.

2 The model

Our model is an extension of the overlapping generations model of Allais (1947) and Diamond (1965). In this closed economy there are three generations living for three periods. For simplicity, the growth rate of the population is zero and the size of the population is normalized to one. When young, the individuals benefit from education spending and build their human capital. When adult, the households work, consume and invest a part of their income in physical capital which is rented and used by the firms in the next period. They also devote resources (education spending) to the accumulation of human capital which will benefit to their children. When old, they consume the return of their savings and die.

Moreover, each household owns a piece of land and a share of the firms. They thus receive land rents and profits. As there is no land market, they transmit their property rights over land to their children when they are old. As a result, only the working generation owns land.

Each household is owner of the firms and receives interest on the capital rental. At the firm's level, firms buy inputs and produce the same single good in perfectly competitive markets. Each firm needs to locate its production activities on a piece of land. Therefore, land enters in the production and is priced at its marginal productivity that depends on the aggregate level of production.

The single good produced in this economy can either be consumed by the adult and the old generations or accumulated by the young households as capital for future production.

2.1 Technology

At each date the representative firm at the aggregate level produces a single good under a technology with constant or non-constant returns to scale (social returns). There are three factors of production: physical capital, human capital and land.

We assume that the production function of the representative firm is given by

$$Y_t = K_t^\alpha H_t^\gamma N^\mu, \tag{1}$$

where Y_t is the output at time t, K_t is physical capital and H_t the amount of human capital used by the firm. Physical capital is assumed to be fully depreciated after one period. N is the land endowment of this economy. This factor represents business estate where economic activities are located. It is assumed to be fixed over time and to enter the

aggregate production function. The parameters α, γ and μ are the productivity elasticities of physical capital, human capital and land respectively. Each of these parameters are assumed to be positive and smaller than one.

The problem of the firm is to maximize profits. Therefore, an interior solution (maximum of profits) exists if the production function is concave, i.e., if the returns with respect to the reproducible factors are non increasing. The condition is thus:

$$\alpha + \gamma \leqslant 1 \tag{2}$$

2.2 Human capital

The production function for the human capital accumulation is defined by

$$H_{t+1} = \Psi e_t^{\theta} H_t^{\eta}, \quad 0 < \theta, \eta < 1, \quad \Psi > 0, \tag{3}$$

where θ and η are the elasticities of human capital production with respect to education spending and to the past stock of human capital respectively, and Ψ is a scale technological parameter. The returns to scale of human capital accumulation are decreasing if $\theta + \eta < 1$, constant if $\theta + \eta = 1$, and increasing if $\theta + \eta > 1$.

The stock of human capital at time $t + 1$ is assumed to depend on contemporaneous education spending, e_t, financed by the young adult generation and on the human capital stock of the previous period, H_t. As in Lucas (1988), it is assumed that the production of human capital does not require physical capital because education is known to be relatively intensive in human capital.

An interior solution for an optimal choice of education spending is obtained if the private marginal returns to investment in human capital are decreasing, i.e.:

$$\theta \leqslant 1 \tag{4}$$

2.3 Preferences

The representative consumer maximizes a logarithmic utility function of the type:

$$u = \ln c_t + \beta \ln d_{t+1} + \lambda \ln e_t \tag{5}$$

subject to the following budget constraint,

$$
\begin{aligned}
c_t + s_t + e_t &= w_t H_t + \pi_t N \\
d_{t+1} &= R_{t+1} s_t
\end{aligned}
$$

Utility depends on consumption when young, c_t, consumption when old, d_{t+1}, and on the amount devoted to the offspring's education, e_t. We therefore assume that the parents enjoy giving their children education resources as in Glomm and Ravikumar (1992). The parameter λ indicates the parents' degree of altruism. The parameter β is the psychological discount factor. The adults supply inelastically one unit of labor and earn $w_t H_t$, where w_t is the regional wage per unit of human capital and H_t is the regional level of human capital. They also receive $\pi_t N$ as land rent. Their income is allocated to consumption and savings, s_t, for future consumption. When old agents spend all their saving and accrued interest on consumption.

2.4 Profits

The maximization problem of the representative firm is defined by

$$\{K_t, H_t\} = \arg\max \{K_t^\alpha H_t^\gamma N^\mu - w_t H_t - R_t K_t - \pi_t N\}$$

where R_t is the interest factor and w_t the wage per unit of effective labor and π_t is the rent per unit of land.

The representative firm maximizes its profits subject to the constraint of technology. Therefore, these profits depend on the utilized technology. When returns to scale (social returns) are non-constant, profits are non-null. In this case, we assume that (positive or negative) profits are redistributed to land owners. Therefore, π_t will represent the remuneration of the land factor and also the remaining part of the share in output.

2.5 Optimal behaviors

The representative consumer-producer chooses optimally c_t, e_t, d_{t+1} and H_t. As a representative firm, he chooses the human capital input, H_t, according to (1). The human capital accumulates according to (2). As a representative consumer, he chooses c_t, d_{t+1}, e_t, and therefore, s_t, according to (4).

Since profits have a maximum by the concavity of the production function, the production factors are paid at their marginal productivities. Hence, the first order conditions of the firm's program (1) are:

$$R_t = \alpha K_t^{\alpha-1} H_t^\gamma N^\mu, \tag{6}$$

$$w_t = \gamma K_t^\alpha H_t^{\gamma-1} N^\mu, \tag{7}$$

where R_t is the factor of interest and w_t is the regional wage per unit of effective labor.

The young adult land owners receive the land rent equal to the marginal productivity of this factor and the remaining part of the share in output:

$$\pi_t = (1 - \alpha - \gamma) K_t^\alpha H_t^\gamma N^{\mu-1}. \tag{8}$$

When returns to scale to reproducible factors are non-constant, the remaining part is positive or negative. When they are constant, the remaining part is null.

The first order necessary conditions of the household program (4) are:

$$s_t = \frac{\beta}{1 + \beta + \lambda}(w_t H_t + \pi_t N), \tag{9}$$

$$e_t = \frac{\lambda}{1 + \beta + \lambda}(w_t H_t + \pi_t N). \tag{10}$$

Thus, savings and education are functions of the aggregate wage and land rent. At the optimum, the relationship between education and savings is linear:

$$e_t = \frac{\lambda}{\beta}s_t$$

2.6 Equilibrium

At equilibrium the total stock of physical capital is built from savings of the adult generation:

$$K_{t+1} = s_t. \tag{11}$$

The dynamics will be analyzed in terms of three stationary variables: the physical-human capital ratio k_t, the growth factor of human capital $x_t = H_{t+1}/H_t$, and the growth factor of the economy $g_t = Y_{t+1}/Y_t$.

Equilibrium requires a stationary physical-human capital ratio that should be defined as:

$$k_t = \frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}}. \tag{12}$$

It is calculated from equation (6).

The marginal productivity of the production factors can be rewritten as:

$$R_t = \alpha \frac{Y_t}{K_t} = \alpha k_t^{\alpha-1} N^\mu$$

$$w_t = \gamma \frac{Y_t}{H_t} = \gamma k_t^{\alpha-1} N^\mu \frac{K_t}{H_t}$$

$$\pi_t = (1 - \alpha - \gamma)\frac{Y_t}{N} = (1 - \alpha - \gamma)k_t^{\alpha-1} N^\mu \frac{K_t}{N}$$

An equilibrium can now be characterized as follows:

Given initial conditions $\{K_0, H_0\}$ satisfying (11), an equilibrium is a vector of positive quantities $(K_t, H_t, c_t, d_t, s_t, e_t, \pi_t)_{t \geqslant 0}$ and prices $(R_t, w_t)_{t \geqslant 0}$ such that equations (1) to (12) hold.

Equations (1) to (12) can be reduced to a system of three non-linear difference equations of the first order, describing the dynamics of the physical-human capital ratio k_t, the growth rate of human capital accumulation x_t and the growth rate of the economy g_t:

$$k_{t+1} = \frac{\beta}{\Psi \lambda^{\frac{\gamma\theta}{1-\alpha}}} \left(\frac{(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^{1 - \frac{\gamma\theta}{1-\alpha}} H_t^{\frac{-\eta\gamma}{1-\alpha} + \frac{\gamma}{1-\alpha}\left(1 - \frac{\gamma\theta}{1-\alpha}\right)} \tag{13}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^\theta H_t^{\frac{\theta\gamma + \eta(1-\alpha) - 1 + \alpha}{1-\alpha}} \tag{14}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \beta^\alpha(\Psi\lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+\theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)} H_t^{\gamma\eta + \frac{\gamma}{1-\alpha}(\alpha+\theta\gamma-1)} \tag{15}$$

Equation (13) gives the dynamics of the physical-human capital ratio, equation (14) the growth rate of the human capital stock and equation (15) the growth rate of the economy. The objective is to study the steady state value of the growth rate of the economy and find the conditions under which it is a positive constant. It also may not be equal to the growth rate of the human capital accumulation.

3 Constant returns to scale in production and human capital accumulation

In an OLG model with a non-reproducible factor such as land, constant returns to scale technologies in the production and in the human capital accumulation do not result in long run growth.

Production technology $(\alpha + \gamma + \mu = 1)$ The production function (1) can be rewritten in the form of a Cobb-Douglas production function:

$$Y_t = K_t^\alpha H_t^{1-\alpha-\mu} N^\mu,$$

where the factor elasticities sum up to one and, hence, $\gamma = 1 - \alpha - \mu$.

Human capital accumulation technology $(\theta + \eta = 1)$ The production function of human capital (3) can be rewritten in the form of a Cobb-Douglas function:

$$H_{t+1} = \Psi e_t^\theta H_t^\eta,$$

where $\eta = 1 - \theta$.

Physical-human capital ratio   Equilibrium requires a stationary physical-human capital ratio:

$$k_{t+1} = \frac{K_t}{H_t^{\frac{1-\alpha-\mu}{1-\alpha}}}.$$

Factor prices   With Cobb-Douglas technologies, marginal productivities of production factors are as follows:

$$
\begin{aligned}
R_t &= \alpha k_t^{\alpha-1} N^\mu \\
w_t &= (1 - \alpha - \mu) k_t^{\alpha-1} N^\mu \frac{K_t}{H_t} \\
\pi_t &= \mu k_t^{\alpha-1} N^\mu \frac{K_t}{N}
\end{aligned}
$$

Dynamic system   The system of three non-linear difference equations (13)-(15) becomes:

$$k_{t+1} = \frac{\beta}{\Psi \lambda^{\frac{(1-\alpha-\mu)\theta}{1-\alpha}}} \left(\frac{\mu k_t^\alpha N^\mu}{1 + \beta + \lambda} \right)^{1 - \frac{(1-\alpha-\mu)\theta}{1-\alpha}} H_t^{\frac{(1-\alpha-\mu)}{1-\alpha}\left(1 - \frac{(1-\alpha-\mu)\theta}{1-\alpha}\right)} \tag{16}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda \mu k_t^\alpha N^\mu}{1 + \beta + \lambda} \right)^{\frac{(1-\alpha-\mu)\theta}{1-\alpha}} H_t^{\frac{-\mu\theta}{1-\alpha}} \tag{17}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \beta^\alpha (\Psi \lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1 + \beta + \lambda} \right)^{\alpha + \theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)} H_t^{\frac{\gamma\theta\mu}{1-\alpha}} \tag{18}$$

Balance growth path condition Equation (18) gives the growth rate of this economy and allows us to derive the balanced growth condition.

Proposition 1 *In an OLG model with land and Cobb-Douglas technology in production and human capital accumulation, the growth rate of per capita income is zero in the long run.*

The proof is straightforward. There is a balanced growth path if and only if the growth rate (18) is equal to a constant. This happens when $H_t^{\frac{\gamma\theta\mu}{1-\alpha}} = 1$. This condition is met if and only if $\mu = 0.\blacksquare$

Since the production technology is Cobb-Douglas and $\mu > 0$, the returns to scale of the reproducible factors are decreasing. Moreover, the returns to scale of the human capital accumulation are constant and do not compensate for decreasing returns of the production technology. This precludes a balanced growth path. The reproducible factors are decreasing. As a result, there is no long run growth in this economy:

$$\bar{g}_1 = 0.$$

Interestingly, the presence of non-reproducible factors offers the possibility to use non-convexities and externalities in a growth model. Note that, if the elasticity of increasing returns or externalities is lower than $\frac{\gamma\theta\mu}{1-\alpha}$, then the long run growth rate remains null. In developing countries where land still accounts for a large share of national product, externalities coming from knowledge in a broad sense, for example, may not be large enough for their economies to experience sustained growth.

There are many ways to produce endogenous growth. Empirical work should make the selection among candidate theories. However, measurement difficulties lead to inconclusive results. In the following two sections, we propose to use non-convexities to yield various balanced growth paths and compare long run growth rates.

4   Long run growth

In this section, we want to obtain endogenous growth in an OLG model with land. We introduce increasing returns to scale in the output technology. This is the only difference with the model of the previous section. The engine of growth is human capital accumulation financed by the parents' education spending. The existence of a balanced growth path requires some conditions on the production function and the technology of human capital accumulation.

Production technology ($\alpha + \gamma \leqslant 1$) The production technology of the firm is defined by:

$$Y_t = K_t^\alpha H_t^\gamma N^\mu, \tag{19}$$

where the elasticities of the reproducible factors $\alpha + \gamma \leqslant 1$ and $\mu > 0$. However, the sum of the factor elasticities, $\alpha + \gamma + \mu$, is higher than one.

Human capital accumulation technology ($\theta + \eta \geqslant 1$)

$$H_{t+1} = \Psi e_t^\theta H_t^\eta. \tag{20}$$

Physical-human capital ratio Equilibrium requires a stationary physical-human capital ratio:

$$k_t = \frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}}.$$

Factor prices Marginal productivities of production factors are as follows:

$$\begin{aligned}
R_t &= \alpha k_t^{\alpha-1} N^\mu \\
w_t &= \gamma k_t^{\alpha-1} N^\mu \frac{K_t}{H_t} \\
\pi_t &= (1 - \alpha - \gamma) k_t^{\alpha-1} N^\mu \frac{K_t}{N}
\end{aligned}$$

The variables R_t, w_t, π_t are the equilibrium factor prices per unit of inputs K_t, H_t and N.

Dynamic system The system of three non-linear difference equations (13)-(15) becomes:

$$k_{t+1} = \frac{\beta}{\Psi \lambda^{\frac{\gamma\theta}{1-\alpha}}} \left(\frac{(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^{1-\frac{\gamma\theta}{1-\alpha}} H_t^{\frac{-\eta\gamma}{1-\alpha} + \frac{\gamma}{1-\alpha}\left(1 - \frac{\gamma\theta}{1-\alpha}\right)} \tag{21}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^\theta H_t^{\frac{\theta\gamma+\eta(1-\alpha)-1+\alpha}{1-\alpha}} \tag{22}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \beta^\alpha (\Psi\lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+\theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)} H_t^{\gamma\eta + \frac{\gamma}{1-\alpha}(\alpha+\theta\gamma-1)} \tag{23}$$

The system (21)-(23) admits a balanced growth path under certain conditions:

Proposition 2 *An OLG model with land, decreasing returns to scale in the reproducible factors and increasing returns to scale in human capital accumulation admits a balanced growth path for conditional values.*

Proof:

A balanced growth path exists if and only if g_{t+1} in equation (23) or x_{t+1} in equation (22) are equal to constants. For x_{t+1}, this requires that:

$$\frac{\theta\gamma + \eta(1 - \alpha) - 1 + \alpha}{1 - \alpha} = 0$$

i.e.,

$$\theta = \frac{(1 - \eta)(1 - \alpha)}{\gamma}$$

However, the value of θ must satisfy the condition (4) of concavity for the human capital accumulation, namely:

$0 \leqslant \theta \leqslant 1 \iff 1 \geqslant \eta \geqslant 1 - \frac{\gamma}{1-\alpha}$.

Therefore, if the two following conditions are satisfied:

$$1 \geqslant \eta \geqslant 1 - \frac{\gamma}{1 - \alpha},$$

$$\theta = \frac{(1 - \eta)(1 - \alpha)}{\gamma}, \tag{24}$$

there exists a balanced growth path. $\blacksquare$

In particular, note that along the balanced growth path,

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \left(\frac{H_{t+1}}{H_t}\right)^{\gamma}.$$

Since the capital stock is equal to savings, i.e. a fraction of the output, then along the balanced growth path,

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t}.$$

This implies,

$$g_{t+1} = (x_{t+1})^{\frac{\gamma}{1-\alpha}}. \tag{25}$$

If $\gamma \neq 1 - \alpha$, i.e. if returns to scale to reproducible factors in the production function are decreasing, the growth rate of human capital accumulation and the growth rate of the economy are not equal.[1]

In the sector of human capital accumulation, the growth rate of the human capital stock is

$$\left(\frac{H_{t+1}}{H_t}\right)^{1-\eta} = \left(\frac{e_{t+1}}{e_t}\right)^{\theta}.$$

Since education spending is a fraction of output, then, along the balanced growth path,

$$g_{t+1} = (x_{t+1})^{\frac{1-\eta}{\theta}}. \tag{26}$$

Again the growth rate of the economy and the growth rate of the human capital stock are different. However, this equality is not necessarily the same as (25). Therefore, the condition for a balanced growth path is that the growth rates in both sectors are equal. Then, the condition implies,

$$\frac{\gamma}{1-\alpha} = \frac{1-\eta}{\theta}, \tag{27}$$

which, obviously, is equivalent to the condition (24).

We now propose two examples of economies with long run growth satisfying the condition (24). The difference between the two lies in the technology of human capital accumulation. In the first example, returns to human capital accumulation are constant while they are increasing in the second example. The balanced growth path in the second example necessitates decreasing returns to the reproducible production factors.

4.1 Increasing returns to scale in production and constant returns to scale in human capital accumulation

This is the limit case satisfying the conditions (24). The sum of the reproducible factors in the production technology, $\alpha + \gamma$, and the coefficients in the function of human capital accumulation, $\theta + \eta$, are both equal to 1.

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[1]It could be possible to have increasing returns to scale (social returns) to reproducible factors in the production function ($\gamma + \alpha > 1$) in a setting with knowledge spillovers that firms could not internalize. Therefore, private returns to reproducible factors would be constant while social returns would be increasing. In our setting, firms internalize all returns.

Physical-human capital ratio   Equilibrium requires a stationary physical-human capital ratio:

$$k_{t+1} = \frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}} = \frac{K_t}{H_t}.$$

Dynamic system   The system of two non-linear difference equations (13)-(15) becomes:

$$k_{t+1} = \frac{\beta}{\Psi\lambda^\theta} \left(\frac{(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^{1-\theta} \tag{28}$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^{\theta} \tag{29}$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \beta^\alpha(\Psi\lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+\theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)} \tag{30}$$

Equations (29) and (30) show that the system (28)-(30) admits a balanced growth path.

Since the elasticities of the reproducible factors sum up to one and the returns to human capital accumulation are constant ($\theta = 1-\eta$), per capita income grows linearly. Along the balanced growth path, the stock of physical capital and the growth rate of the economy are positive constants:

$$\bar{k} = \left(\frac{\beta}{\Psi\lambda^{1-\eta}} \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^\eta \right)^{\frac{1}{1-\alpha\eta}}$$

$$\bar{x}_2 = \Psi \left(\frac{\lambda(1-\alpha)\bar{k}^\alpha N^\mu}{1+\beta+\lambda} \right)^{1-\eta} > 0$$

$$\bar{g}_2 = \beta^\alpha(\Psi\lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+(1-\eta)\gamma} \bar{k}^{\alpha(\alpha+(1-\eta)\gamma-1)} > 0$$

The growth rate of this economy, $\bar{g}_2$, is increasing with the elasticity of education spending, ($\theta = 1-\eta$). Moreover, the share of land in output, μ, is no longer a problem for sustained growth. By benefitted from overall steady productivity, land affects positively the growth rate of the economy.

4.2 Increasing returns to scale in production and human capital accumulation

The sum of the reproducible factors in the production technology, $\alpha + \gamma$, are smaller than 1, while the coefficients in the function of human capital accumulation, $\theta + \eta$, are higher than 1. The decreasing returns to scale of the reproducible factors are offset by increasing returns in the accumulation of human capital.

Physical-human capital ratio   Equilibrium requires a stationary physical-human capital ratio:

$$k_{t+1} = \frac{K_t}{H_t^{\frac{\gamma}{1-\alpha}}}.$$

Dynamic system   If the balanced growth path conditions (24) are satisfied, the system of two non-linear difference equations (21)-(23) becomes:

$$k_{t+1} = \frac{\beta}{\Psi \lambda^{\frac{\gamma\theta}{1-\alpha}}} \left(\frac{(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^\eta$$

$$x_{t+1} = \frac{H_{t+1}}{H_t} = \Psi \left(\frac{\lambda(1-\alpha)k_t^\alpha N^\mu}{1+\beta+\lambda} \right)^\theta$$

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = \beta^\alpha (\Psi \lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+\theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)}$$

Along the balanced growth path, the stock of physical capital and the growth rate of the economy are positive constants:

$$\bar{k} = \left(\frac{\beta}{\Psi \lambda^{1-\eta}} \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^\eta \right)^{\frac{1}{1-\alpha\eta}}$$

$$\bar{x}_3 = \Psi \left(\frac{\lambda(1-\alpha)\bar{k}^\alpha N^\mu}{1+\beta+\lambda} \right)^\theta$$

$$\bar{g}_3 = \beta^\alpha (\Psi \lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+\theta\gamma} \bar{k}^{\alpha(\alpha+\theta\gamma-1)},$$

where $\theta > 1 - \eta$.

Then, if $\Psi < 1$, we can conclude that:

$$\bar{g}_3 > \bar{g}_2 > 0.$$

Per capita income grows faster when reproducible production factors are decreasing and returns to human capital accumulation are increasing.

5 Capital mobility and regional dynamics

In this section, we study an economy composed of two regions A and B, having the same preferences. Physical capital is perfectly mobile across regions while human capital is immobile. We consider this as a realistic assumption. Physical capital is incommensurably more mobile than labor in developed as well as developing economies. The model of this section aims at reproducing some characteristics of economic geography models to compare their results. In the standard economic geography models (core-periphery models[2]), there are two regions and skilled labor moves freely to either of them offering the highest return. Production exhibits increasing returns in the manufactured sector employing skilled labor. The traditional sector employs unskilled and produces at constant returns to scale. Trade across regions incurs transportation costs. Therefore, increasing returns and labor mobility are the divergence (or agglomeration) forces while monopolistic competition, immobility of unskilled labor and transportation costs are the convergence forces. To make relevant comparisons with our setup, in which competition is perfect, we will study the core-periphery model when competition is the least monopolistic.[3] In this case, divergence (or agglomeration) always arises when the decrease in transportation costs attains a certain threshold value. In our model, increasing returns are the divergence force while capital mobility is the convergence force. This is the main difference with the core-periphery model: production in our framework uses physical capital which turns out to be an overwhelming convergence force.

The main features of the two-region OLG model are as follows:

Production technology   In each region, the production technology of the firm is defined by:

$$Y_{i,t} = K_{i,t}^{\alpha} H_{i,t}^{\gamma} N_i^{\mu}, \qquad i = (A, B)$$

Human capital accumulation technology   Each region accumulates human capital as follows:

$$H_{i,t+1} = \Psi e_{i,t}^{\theta} H_{i,t}^{\eta}, \;\; 0 < \theta \; < 1, \;\; \Psi > 0,$$

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[2]See Krugman (1991) for more details.

[3]This happens when the varieties of goods produced by monopolistic firms are not very differentiated.

Equilibrium At equilibrium the interest factor is identical in both regions since physical capital is perfectly mobile:

$$R_{A,t} = R_{B,t} = R_t. \tag{31}$$

The total stock of physical capital is built from savings of the adult generation:

$$K_{t+1} = K_{A,t+1} + K_{B,t+1} = s_{A,t} + s_{B,t}.$$

Physical-human capital ratio Equilibrium requires a stationary physical-human capital ratio:

$$k_{i,t+1} = \frac{K_{i,t}}{H_{i,t}^{\frac{\gamma}{1-\alpha}}}.$$

Dynamics In this two-region economy, the growth rate of each region is driven by the regional accumulation of human capital. Physical capital moves freely across regions subject to the equilibrium condition on the capital market. The dynamics of this economy will be analyzed with four equation describing the dynamics of the physical-human capital ratio $k_{i,t}$, the ratio of workers' human capital from both regions $v_t = H_{B,t}/H_{A,t}$, the ratio of regional outputs $z_t = Y_{B,t}/Y_{A,t}$ and the growth factor in one region, say A, $g_A = H_{A,t}/H_{A,t-1}$:

$$\frac{k_{B,t}}{k_{A,t}} = \left(\frac{N_B}{N_A}\right)^{\frac{\mu}{1-\alpha}} \tag{32}$$

$$v_{t+1} = \frac{H_{B,t+1}}{H_{A,t+1}} = \left(\frac{N_B}{N_A}\right)^{\frac{\mu\theta}{1-\alpha}} v_t^{\frac{\theta\gamma+\eta(1-\alpha)}{1-\alpha}} \tag{33}$$

$$z_t = \frac{Y_{B,t}}{Y_{A,t}} \tag{34}$$

$$g_{A,t+1} = \frac{Y_{A,t+1}}{Y_{A,t}} = \beta^\alpha (\Psi\lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda}\right)^{\alpha+\theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)} H_t^{\gamma\eta+\frac{\gamma}{1-\alpha}(\alpha+\theta\gamma-1)} \tag{35}$$

Balanced growth path condition The conditions for long run growth in this economy are the same as previously (see conditions (24)).

5.1 Regional disparities in land endowments

The land endowment, N_i, represents the geographic characteristics of region i. These geographic peculiarities are fixed or exhaustible resources that economic activities can use as inputs. For instance, geographic location, soil, coast, climate, ports, water and mineral resources, etc., are natural endowments unevenly distributed on earth that are useful for economic development. Regional disparities in terms of such land endowments may explain some of the income inequalities in the world. In our framework, if the two regions have different endowments in land, there cannot be a balanced growth path for both regions. The dynamics of both regions may result in a situation where the relative weight of one region may tend to 0.

Proposition 3 *In a two-region OLG model with land, increasing returns and mobility of physical capital, there is a balanced growth path and regional convergence if and only if both regions grow at the same rate. If regional per capita incomes grow at different rates, the dynamics may show that the relative weight of one region tends to 0.*

Proof:

Using equation (31) and the fact that $R_t = \alpha \frac{Y_{i,t}}{K_{i,t}}$, we can rewrite equation (34) as the following equality:

$\frac{Y_{B,t}}{Y_{A,t}} = \frac{K_{B,t}}{K_{A,t}}.$

Then, writing $Y_{i,t}$ as a function of $K_{i,t}$ and R_t yields:

$Y_{i,t} = \frac{R_t K_{i,t}}{\alpha}.$

Substituting for $Y_{i,t}$ in the production function,

$K_{i,t} = \left(\frac{\alpha H_{i,t}^{\gamma} H_{i,t}^{\mu}}{R_{i,t}} \right)^{\frac{1}{1-\alpha}}$

As a result,

$$z_t = \frac{K_{B,t}}{K_{A,t}} = \left(\frac{H_{B,t}^{\gamma} N_B^{\mu}}{H_{A,t}^{\gamma} N_A^{\mu}} \right)^{\frac{1}{1-\alpha}}.$$

Replacing the regional stocks of human capital by the ratio v_t, we obtain

$$z_t = v_t^{\frac{\gamma}{1-\alpha}} \left(\frac{N_B}{N_A} \right)^{\frac{\mu}{1-\alpha}}.$$

The dynamic equation of the ratio v_t is:

$v_{t+1} = \left(\frac{e_{B,t}}{e_{A,t}} \right)^{\theta} v_t^{\eta},$

where $\frac{e_{B,t}}{e_{A,t}}$ is a constant fraction of the output ratio $\frac{Y_{B,t}}{Y_{A,t}}$. Then,

$$v_{t+1} = z_t^\theta v_t^\eta.$$

Substituting for z_t yields

$$v_{t+1} = \left(\frac{N_B}{N_A}\right)^{\frac{\mu}{1-\alpha}} v_t^{\eta + \frac{\theta\gamma}{1-\alpha}}$$

Taking the balanced growth condition (24) into account,

$$v_{t+1} = \left(\frac{N_B}{N_A}\right)^{\frac{\mu}{1-\alpha}} v_t. \tag{36}$$

If $N_B = N_A$, then v_t and z_t are constants, there is a balanced growth path and both regions grow at the same rate.

If $N_B < N_A$, then $\lim v_t = 0$ and $\lim z_t = 0$, regional per capita incomes grow at different rates. One region tends to 0 in terms of relative income.∎

The dynamics of this two-region economy is eventually determined by the land differential regardless of initial conditions in physical or human capital. If the land endowments are not equal between regions, the two regions grow in the long run at different rates. As a result, there is no steady states for the ratio of human capital regional stocks, v_t, and for the ratio of regional outputs, z_t. There is a balanced growth path if and only if both regions grow at the same rate. This happens only when both regions are endowed with the same amount of land.

As land endowments are fixed over time, initial conditions on the distribution of this resource across regions determine once for all the long run results. The regions are deterministically locked in. Although natural resources were determinant factors for economic development in the industrialization era, economic growth nowadays relies more on knowledge-based inputs than natural resources. Moreover, the case of the relative weight of one region going to 0 seems implausible. Therefore, these results hardly offer a candidate theory to account for regional income disparities.

5.2 Inequality of initial human capital stocks across regions

In this section and the following, we will assume that the land endowments are identical between regions. However, the initial conditions on the human capital stocks are different across regions. In an OLG model without land, unequal initial human capital stocks between regions would lead to regional divergence. The region with an initial lower human capital stock would never catch up the leading region. The introduction of a fixed factor, such as land, allows for both long run growth and regional convergence regardless of initial conditions in human capital. The equilibrium is defined by:

$$\bar{k}_B = \bar{k}_A$$

$$\bar{z} = 1$$

$$\bar{g}_A = \beta^\alpha (\Psi \lambda^\theta)^\gamma \left(\frac{(1-\alpha)N^\mu}{1+\beta+\lambda} \right)^{\alpha+\theta\gamma} k_t^{\alpha(\alpha+\theta\gamma-1)}$$

Proposition 4 *In a two-region OLG model with land, increasing returns and mobility of physical capital, regional per capita incomes converge to the same balanced growth path regardless of the initial conditions in regional human capital stocks.*

If a region starts with a lower human capital stock, it will always catch up the leading region. The mechanism at work is the following: in the region with less human capital, the wage per unit of effective labor, i.e. its marginal productivity, is higher, attracting physical capital in this region. The influx of physical capital and the growing output fuel the process of accumulation of human capital and, hence, dampen down the marginal productivity of the region's human capital up to the level of the other region. In the end, both regions grow at the same rate in the long run.

In a balanced growth path framework, where physical capital is perfectly mobile across regions, increasing returns to scale and initial cross-regional inequality in human capital are not sufficient conditions to have a core-periphery result as in the standard economic geography models. Mobility of physical capital is a strong convergence force powerful enough to dominate the effect of increasing returns.

6   Conclusion

This paper addressed the question of the effect of increasing returns in an overlapping generations model on regional convergence. We first set up a model in which increasing returns to scale would be compatible with balanced growth. A way to do so is to add a fixed factor, e.g. land, in the production function using two reproducible factors. After determining the balanced growth path condition, we extended the model to a two-region framework to analyze regional dynamics. We assumed human capital immobility and physical capital mobility. This is in line with the empirical evidence showing that physical capital is much more mobile than labor. Our results show that, unlike core-periphery models, increasing returns verifying balanced growth always lead to regional convergence. Physical capital mobility turns out to be an overwhelming convergence force.

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