

# AN IMPROVED METHOD OF PHOTOMETRIC MODE IDENTIFICATION: APPLICATIONS TO SLOWLY PULSATING B, $\beta$ CEPHEI, $\delta$ SCUTI AND $\gamma$ DORADUS STARS

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**Abstract.** We present an improved version of the method of photometric mode identification based upon the inclusion of non-adiabatic eigenfunctions determined in the stellar atmosphere, according to the formalism recently proposed by Dupret et al. (2002). We apply our method to  $\beta$  Cephei, Slowly Pulsating B,  $\delta$  Scuti and  $\gamma$  Doradus stars. Besides identifying the degree  $\ell$  of the pulsating stars, our method is also a tool for improving the knowledge of stellar interiors and atmospheres, by imposing constraints on the metallicity for  $\beta$  Cephei and SPBs, the characteristics of the superficial convection zone for  $\delta$  Scuti and  $\gamma$  Doradus stars and the limb-darkening law.

## 1. Magnitude Variation of a Non-Radial Pulsator

Methods of mode identification based on multi-colour photometry have been derived by different authors: Dziembowski (1977), Stamford and Watson (1981), Watson (1988), Garrido et al. (1990), Garrido (2000), Heynderickx et al. (1994), Cugier et al. (1994) and Balona and Evers (1999). In our study, we assume that the geometrical distortion of the photosphere is given by the Lagrangian displacement at  $T = T_{\text{eff}}$  (one-layer approximation) and that the atmosphere remains in radiative equilibrium during the pulsation. Under these hypotheses, the monochromatic magnitude variation for the mode  $(\ell, m)$  and wavelength  $\lambda$  is given by :

$$\delta m_{\lambda} = -\frac{2.5}{\ln 10} \epsilon P_{\ell}^m(\cos i) b_{\ell\lambda} \left[ (1 - \ell)(\ell + 2) \cos(\sigma t) \right. \\ \left. + f_T \cos(\sigma t + \psi_T) (\alpha_{T\lambda} + \beta_{T\lambda}) - f_g \cos(\sigma t) (\alpha_{g\lambda} + \beta_{g\lambda}) \right], \quad (1)$$

where  $\epsilon$  is the amplitude of the radial displacement,  $i$  is the inclination angle of the star,  $b_{\ell\lambda}$ ,  $\alpha_{T\lambda}$ ,  $\alpha_{g\lambda}$ ,  $\beta_{T\lambda}$  and  $\beta_{g\lambda}$  are defined in Balona and Evers (1999),  $f_T$  is the amplitude of local effective temperature variation ( $\delta T_{\text{eff}}/T_{\text{eff}}$ ) and  $f_g$  is the amplitude of local effective gravity variation ( $\delta g_e/g_e$ ) for a normalized radial displacement at the photosphere and  $\psi_T$  is the phase difference between the effective temperature variation and the radial displacement.



In our method, the coefficients  $f_T$ ,  $\psi_T$  and  $f_g$  are computed by a non-adiabatic code including the interaction with the atmosphere, as derived by Dupret et al. (2002). Our determination of the effective gravity variation is similar to the one proposed by Cugier and Daszynska (2001), but not assuming the Cowling approximation. We do not assume that the effective temperature variation is equal to the Lagrangian temperature variation, as Dupret et al. (2002) showed that these two quantities can be very different.

We improved the mode identification code of Garrido et al. (1990) which uses the phase-lag information, as well as the code of Heynderickx et al. (1994) based on amplitude ratios, so that they can now take our non-adiabatic predictions into account, following Eq. (1).

The stellar models used in this study were computed with the new Code Liégeois d'Evolution Stellaire (CLEs) written by R. Scuflaire.

## 2. $\beta$ Cephei Stars

We applied our method to the  $\beta$  Cephei star EN (16) Lac, using the photometric amplitudes with Johnson filters derived by Jerzykiewicz (1993). We identified the degrees  $\ell$  of the 3 frequencies  $f_1 = 5.9112 \text{ c d}^{-1}$ ,  $f_2 = 5.8551 \text{ c d}^{-1}$  and  $f_3 = 5.5033 \text{ c d}^{-1}$  as  $\ell = 0, 2$  and  $1$ , respectively. Our photometric mode identification is fully compatible with the spectroscopic mode identification (see Aerts et al. and Briquet et al., these proceedings). Moreover, we could constrain the metallicity of EN Lac, because the non-adiabatic results are dependent on it. The higher the metallicity, the more efficient the  $\kappa$  mechanism, which implies a more important decrease of the luminosity variation in the driving region and thus a smaller effective temperature variation (for a normalized displacement). The best agreement between the theoretical and observed amplitude ratios was obtained for a model with  $Z = 0.015$ . Values below  $Z = 0.015$  do not lead to excitation of the modes.

## 3. Slowly Pulsating B Stars

We applied our method to 13 SPBs observed with Geneva photometry by De Cat and Aerts (2002). We derived the effective temperatures and gravities of these stars, using the recent calibrations of Künzli et al. (1997). We computed theoretical models closest to these values. We performed non-adiabatic computations and we did the photometric mode identification of the dominant modes. In Figure 1, we illustrate the results obtained for the star HD 74560. We see that the dominant frequency of this star ( $f = 0.64472 \text{ c d}^{-1}$ ) is identified as an  $\ell = 1$  mode. This result is in full agreement with the spectroscopic mode identification achieved with the moment method (De Cat et al., 2003).

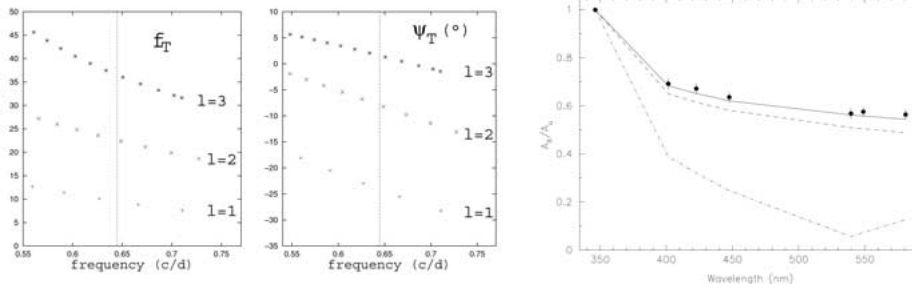


Figure 1. Model calculations for the SPB star HD 74560. On the left (resp. middle), non-adiabatic values of  $f_T$  (resp.  $\psi_T$ ) for different modes. On the right, observed amplitude ratios (bullets) and theoretical predictions (solid, dashed and dot-dashed lines for  $\ell = 1, 2$  and 3).

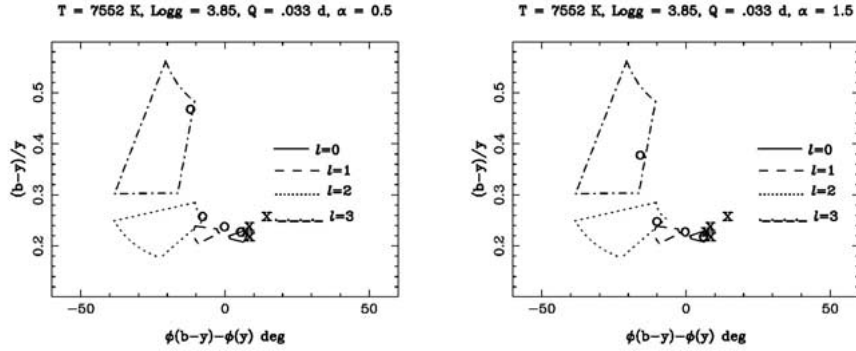


Figure 2. Strömgren filters phase-amplitude diagrams for two  $\delta$  Scuti models with  $\alpha = 0.5$  (left) and  $\alpha = 1.5$  (right). The different regions are for different degrees  $\ell$ , with  $R$  and  $\psi_T$  as free parameters (Garrido et al., 1990). The circles are our non-adiabatic predictions and the crosses are for observations of radial pulsators.

#### 4. $\delta$ Scuti Stars

We performed non-adiabatic computations for  $\delta$  Scuti models with different values of the mixing length parameter  $\alpha$ . Our non-adiabatic results (mainly the phase-lag  $\psi_T$ ) are very sensitive to the size of the thin superficial convection zone (linked to  $\alpha$ ), confirming the results of Balona and Evers (1999). In Figure 2, we illustrate typical results obtained for two  $\delta$  scuti models with  $\alpha = 0.5$  (left) and  $\alpha = 1.5$  (right). We refer to Moya et al. (these proceedings) for more details and a comparison between the non-adiabatic results obtained with and without inclusion of the atmosphere in the pulsation equations.

#### 5. $\gamma$ Doradus Stars

We also performed non-adiabatic computations for  $\gamma$  Doradus models with different  $\alpha$  and these results are extremely dependent on the size of the superficial

convection zone. At present, no theory is able to explain at the same time the excitation mechanism and the photometric observations. We refer to Moya et al. (these proceedings) for more theoretical results and to Aerts et al. (these proceedings) for the observations of two multi-periodic  $\gamma$  Doradus stars, with Geneva photometry.

## 6. Conclusions

Multi-colour photometry can be used efficiently for the identification of the degree  $\ell$  of pulsation modes. These observables are very sensitive to the degree of non-adiabaticity of the superficial layers. The theoretical non-adiabatic predictions depend much on the metallicity for  $\beta$  Cephei and SPBs and on the mixing-length parameter  $\alpha$  for  $\delta$  Scuti and  $\gamma$  Doradus stars. Therefore, a precise confrontation between theory and observations can constrain these parameters.

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