A COMPARISON BETWEEN EXPLICIT AND IMPLICIT MODELLING OF TRANSIENT CREEP STRAIN IN CONCRETE UNIAXIAL CONSTITUTIVE RELATIONSHIPS

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ABSTRACT

Transient creep strain has to be included within the constitutive relationships for concrete at high temperatures. However, the necessity of taking into account this term explicitly is not clearly defined. In the Eurocode 2 uniaxial concrete material model, transient creep is included implicitly. This paper aims to highlight the capabilities and limitations of concrete uniaxial models at elevated temperatures for thermo-mechanical behaviour modelling, depending on the implicit or explicit consideration of transient creep strain in the model.

The characteristics inherent to the two types of models are described and compared. It appears that one of the major limitations of implicit models concerns the unloading stiffness because implicit models treat transient creep as reversible. Based on numerical analysis performed on loaded concrete columns subjected to natural fire, it is shown that the stress-temperature paths experienced by structural concrete are varied and complicated and that concrete material models cannot handle properly these complex situations of unsteady temperatures and stresses without explicit consideration of transient creep.

The paper proposes a new formulation of the Eurocode 2 concrete material model that contains an explicit term for transient creep. The new model is implemented in the software SAFIR and validated against experimental data of the mechanical strain developed by concrete cylinders under different unsteady temperatures and loads. It is shown that the actual material behaviour is better matched with the new explicit model than with the current implicit Eurocode 2 model. Finally, a comparison is given between experimental and calculated results on an axially restrained concrete column subjected to heating and cooling.

INTRODUCTION

Structural fire engineers frequently use numerical analysis to assess the performance of building structures in accidental fire situations. For these numerical simulations, temperature dependent constitutive relationships are required for the load bearing materials used in the structure such as, for instance, concrete. Since the pioneered works of Anderberg & al.[1] and Schneider[2], concrete uniaxial constitutive models have been available for linear structural members such as beams and columns. However, the concrete behaviour at high temperatures is still under intense investigation as it includes particular and complex phenomena.

Transient Creep Strain

In concrete, a particular phenomenon appears when subjected to high temperatures: the transient creep strain. Physically, the transient creep strain is the additional strain that develops irrecoverably during first-time

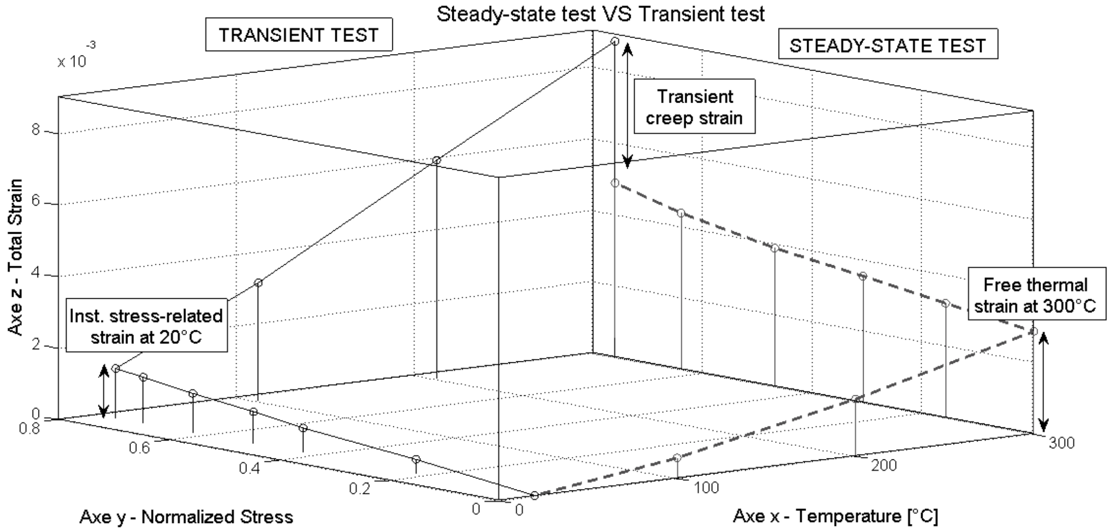

Figure 1: Transient creep strain

heating of concrete under load, compared to concrete loaded at elevated temperature[1]. This strain component depends on the temperature and on the stress applied during heating. Transient creep strain is highlighted by comparing the results of two experiments (see Figure 1). In the first one, called steady-state test, the concrete specimen is first heated uniformly to a pre-defined temperature and then loaded while the temperature is kept constant. In the second one, called transient test, the specimen is first loaded up to a given constant load and then heated while the load is kept constant. Note that for more readability, the strain components are added in Figure 1 although free thermal strain is opposed to the other strains in compression. Experimental data of steady-state and transient tests can be found in literature[2-3].

Interesting state of the art reviews of the transient creep strain models can be found in recent literature, e.g. Li & al.[4], Law & al.[5] and Youssef & al.[6]. Several authors have proposed uniaxial models of concrete integrating explicitly a term for transient creep strain and in most of these models, the transient creep strain is linearly proportional to the applied stress and increases with temperature but not linearly, e.g. Anderberg & al.[1], Schneider[2], Diederichs (reported in Li & al.[4]) and Terro[7].

It is well-admitted in literature that transient creep has to be considered in any fire analysis involving concrete in compression[4,8]. However, the necessity of taking it into account by an explicit term in the strain decomposition has been questioned[9] and in the current Eurocode 2 (EC2) model[10], the transient creep has been incorporated implicitly in the stress-mechanical strain relationship. Law & al.[5] have recently shown that considering this term implicitly can have important implications on the Young modulus calculation of concrete but the implications on the behaviour of a complete structure is still a pending question.

The first objective of the study reported here was to highlight the capabilities and limitations of a uniaxial constitutive model for concrete depending on its implicit or explicit consideration of transient creep strain. The second objective was, if this proved to be necessary, to derive an explicit model that would encompass the characteristics of most models presented up to now in the literature and, for reasons that will be explained below, that would be as close as possible to the present Eurocode 2 model.

Implicit or Explicit Models

In implicit models, the total strain ε_{tot} is considered as the sum of free thermal strain ε_{th}, mechanical strain ε_m, and possibly basic creep strain ε_{cr} as expressed by Eq. [1].

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_m \left(+\varepsilon_{cr}\right) \tag{1}$$

Basic creep, defined as the strain that develops when only time is changing with all other conditions such as stress and temperature being constant, is generally omitted for the structural calculation of building structures in fire [4].

In explicit models, the total strain is split into free thermal strain ε_{th}, instantaneous stress-related strain ε_{σ} and transient creep strain ε_{tr} (and possibly basic creep strain ε_{cr}):

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{\sigma} + \varepsilon_{tr} \left(+\varepsilon_{cr}\right) \tag{2}$$

The instantaneous stress-related strain can in turn be divided in elastic and plastic strains: $\varepsilon_{\sigma} = \varepsilon_{el} + \varepsilon_p$. The mechanical strain is the sum of the instantaneous stress-related strain and the transient creep strain.

In explicit models, the stress σ is directly related to the instantaneous stress-related strain ε_{σ}. This relationship can be obtained experimentally at any temperature from a steady-state test, by subtracting the free thermal strain to the total strain (see Figure 1). Then, an explicit relationship for the calculation of the transient creep strain has to be included in the model. However in implicit models, the stress σ is directly related to the mechanical strain ε_m, without calculation of the transient creep strain. In the EC2 model, for instance, the relationship at a given temperature T between the stress and the mechanical strain is given for the ascending branch by Eq. [3]:

$$\frac{\sigma}{f_c(T)} = \frac{3\,\varepsilon_m^{\text{implicit}}}{\varepsilon_{c1,EC2}(T)\left(2+\left(\varepsilon_m^{\text{implicit}} \big/ \varepsilon_{c1,EC2}(T)\right)^3\right)} \tag{3}$$

with f_c the compressive strength and $\varepsilon_{c1,EC2}$ the peak stress strain (PSS)[10]. In this relationship, the value of the peak stress strain accounts for the transient creep strain.

The mechanical strain given by implicit models for a given stress-temperature state is the same, whether concrete has been heated and then loaded at constant temperature or loaded and then heated under constant stress and this is known not to correspond to experimental evidence (see Figure 1). Another major limitation of implicit models is that transient creep strain is recovered during eventual unloading. This is because, at a given temperature, the elastic modulus used for unloading is taken as the initial tangent of the constitutive curve in terms of (ε_m; σ)[5], see Figure 2.

In the tests made to derive the constitutive models, either the temperature or the stress is constant, whereas the other variable is increased. It is important to notice that, in real structures, the transient creep strain depends not only on temperature and stress but also on the stress-temperature path followed by the material. As a result, in explicit models, the relationship between the stress and the mechanical strain is not univocal at a given temperature as seems to be implied by Figure 2. In explicit models, the transient creep strain is not recovered during unloading and/or cooling and the modulus for unloading at a given temperature is taken as the initial tangent to the instantaneous stress-strain curve.

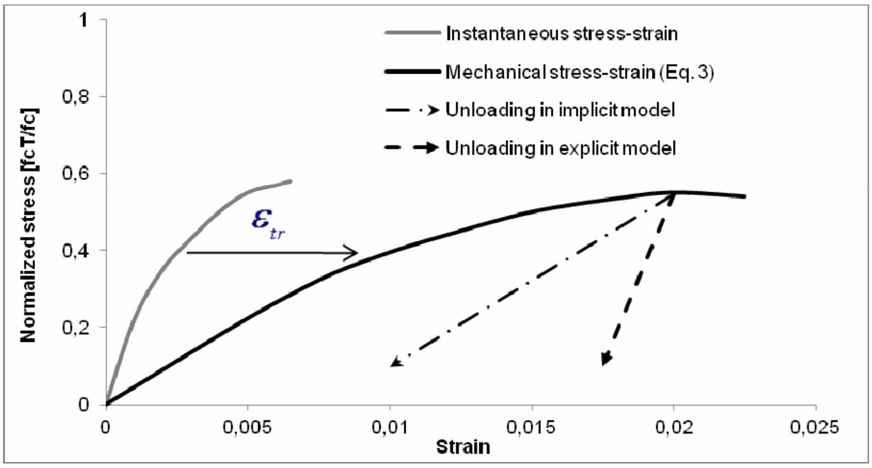

Figure 2: Strain components in implicit and explicit models at 500°C

POSSIBLE STRESS-TEMPERATURE PATHS IN A STRUCTURAL ELEMENT

The kind of demand that is being imposed on a material model may be quite different when it comes to modelling a structural element then when it is used to model experimental tests made on cylinder with a quite simple stress-strain-temperature history. Because of transient thermal gradients inherent to concrete sections, different points in the structure are expected to experience different and complex stress-strain-temperature histories. The following simple example illustrates this aspect and will serve as a starting point to establish the demand imposed on a constitutive model. All simulations have been performed with the software SAFIR[11] and with the current thermal and mechanical models of Eurocode 2, i.e. with an implicit model. The results would of course be quantitatively different with another model but the exercise has been performed to show the trends, not to obtain precise values.

The model is a circular siliceous concrete column of 4 m height, with a section of 300 mm diameter reinforced with four 16 mm diameter bars covered by 40 mm of concrete. The concrete has a compressive strength of 30 MPa and a tensile strength of 3 MPa whereas the steel of the bars has a yield strength of 500 MPa. The ultimate load of this column at room temperature is 2309 kN.

The temperature distribution in the sections was determined by a 2D non linear transient analysis. The column is first axially loaded with a load of 462 kN and then subjected to the natural fire curve shown in Figure 3. No collapse occurs during the numerical simulation.

The stress-temperature paths observed at different points across the section at mid level of the column are plotted in Figure 4 (compression is positive). Points A to F are distributed on a radius in the section, with point A at the centre and point F at the surface.

It can be observed in Figure 4 that the stress and temperature evolutions across the section during the fire are complex and significantly different depending on the position in the section. It is possible to extract five different situations from Figure 4. For each of these situations, it is discussed whether explicit or implicit constitutive models are able to take into account accurately the transient creep strain.

Situation I: increasing stress and temperature. Transient creep strain develops because the temperature increases under stress. However, the transient creep strain is overestimated by implicit models because these models calculate at any time the total transient creep strain on the base of the current value of the stress. On the contrary, it is possible with explicit models to perform an incremental calculation of transient creep strain.

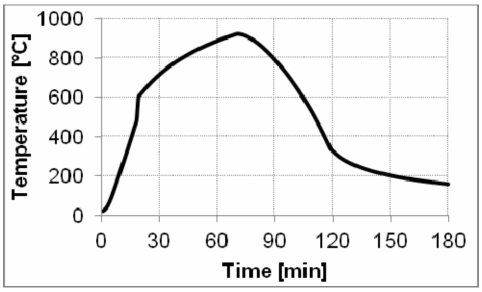

Figure 3: Natural fire applied to the column

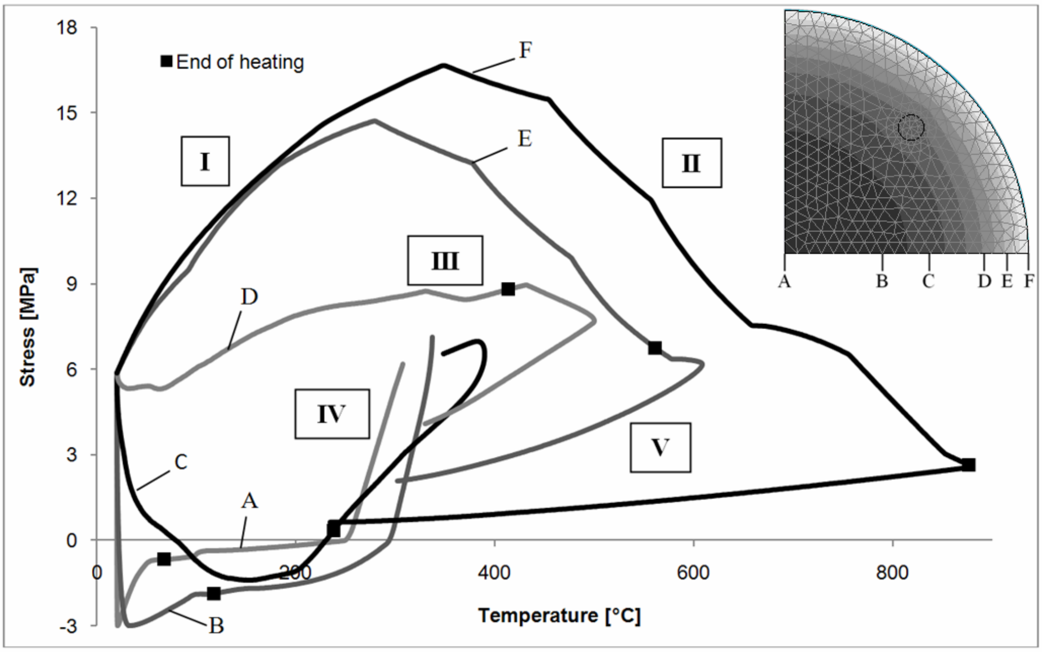

Figure 4: Stress-temperature path in different parts of the section

Situation II: decreasing stress and increasing temperature. In situations where the stress decreases, explicit and implicit models lead to very different results because the unloading stiffness considered by both models is different (see Figure 2). In implicit models, the transient creep strain is treated as reversible, which is in contradiction with its physical nature. Note that physically, it is generally assumed that the transient creep strain is the same for loading and unloading as long as the stress is in compression[4]. In other words, in explicit models, the transient creep strain is still incremented in situation II.

Situation III: (approximately) constant stress and increasing temperature. This situation corresponds to transient tests. Implicit and explicit models give the same mechanical strain for a given stress-temperature state reached after an evolution that matches situation III.

Situation IV: increasing stress and (approximately) constant temperature. This situation corresponds to steady-state tests. No transient creep develops. In explicit models, the mechanical strain reduces to the instantaneous stress-related strain. However in implicit models, transient creep strain is still implicitly included, leading to a highly underestimated stiffness.

Situation V: decreasing stress and decreasing temperature. Implicit and explicit models lead to different material behaviours because the unloading stiffness considered in the two models is different. In explicit models, the transient creep strain remains constant as it cannot be recovered and it does not develop under decreasing temperature. In implicit models, the transient creep strain decreases.

This example shows that implicit models reproduce correctly the behaviour of concrete only in a very particular situation, when the temperature increases and the stress is constant (situation III), and this situation is not so common, even in a simple element subjected to the heating phase of a fire. This is even more the case during the cooling phase of the fire.

It is thus preferable to utilize an explicit model for the sake of precision of the stress and stiffness calculated at the local level, i.e. in every point of integration in the structure. Whereas the difference between the utilization of both types of model will be noticeable in the global behaviour of the structural elements is another question.

It has been shown[2] that the amount of transient creep may significantly depend on the type of concrete. It is possible to determine precisely the properties of a well defined type of concrete to be used in well defined conditions, usually for a very important project, e.g. the concrete vessel for a nuclear reactor that will be subjected to a well defined fire scenario. For more general applications, generic properties of concrete have to be established. Generic properties are used, for instance, when the mechanical behaviour of two structural systems has to be compared, with no reference to a particular concrete mix, or at the preliminary stage of a design, when no information is yet available on the particular mix that will be used. Generic properties are also required for determining the fire resistance of an element in a small project, where the cost to conduct experimental tests would be by far outweigh the budget allocated for the design studies of the building.

The constitutive model of Eurocode 2 has imposed itself as one of the most widely used generic models in the last decade, in Europe and beyond. It has been proposed by a draft committee comprising several European experts, has proved to yield quite satisfactory results when applied to structural calculations (although most application where under ISO fire, which means under constantly increasing temperature) and it is well accepted by authorities and regulators. It was estimated that, if there is a chance to see a breakthrough in the utilization of explicit models, this could not be achieved by selecting one of the various particular models presented up to now, each with its own characteristics and some requiring particular tests, but rather by proposing an explicit model that would yield the same results as the present Eurocode implicit model when used in the situation of transient test. This model could then be seen as a new formulation of the Eurocode model and by called Explicit Transient Creep Eurocode model (ETC Eurocode model). It should of course encompass the most widely accepted characteristics of transient creep.

EXPLICIT TRANSIENT CREEP FORMULATION OF THE EUROCODE MODEL

Assumptions

The new Explicit Transient Creep (ETC) formulation was calibrated to yield the same mechanical strain as the EC2 model for a material first-time heated under constant stress (i.e. transient test). From Eq. [1] and [2], this leads to Eq. [4].

$$\varepsilon_m^{\text{implicit}} = \varepsilon_\sigma^{\text{explicit}} + \varepsilon_{tr}^{\text{explicit}} \qquad [4]$$

The elastic modulus of the material was taken as the initial tangent to the ENV curve[12] with the minimum value of the PSS, $\varepsilon_{c1,\min}$. Indeed, the ENV relationship with $\varepsilon_{c1,\min}$ is based[13] on steady-state tests made by Schneider[14] that do not include transient creep strain, see Figure 5. Relationships for the evolution of the elastic modulus with temperature presented by Felicetti & al.[15] are in line with the values given by ENV.

Transient creep models have been developed by several authors in literature and, generally, transient creep is proportional to the applied stress (Anderberg & al.[1], Schneider[2], Terro[7]). Adopting the same assumption, the formulation was developed according to Eq. [5].

$$\varepsilon_{tr} = \phi(T) \times \frac{\sigma}{f_{ck}} \qquad [5]$$

where $\phi(T)$ is a nonlinear function of temperature and f_{ck} is the compressive strength at 20°C.

Description of the ETC Model

The initial stiffness (i.e. the tangent to the curve at 0 stress) of the material subjected to steady-state test is assumed equal to the ENV elastic modulus, written here as $E_{\text{ENV}}(T)$. In case of transient test, the new model must be calibrated on the EC2 model, so in particular the tangent to the curve at 0 stress must be the same as that of the EC2 curve, denoted as $E_{\text{EC2}}^{\text{implicit}}(T)$. Transient creep strain is defined as the difference between the "transient test" curve and the "steady-state test" curve. As transient creep has been assumed linearly stress-dependent, it is graphically obtained on Figure 5 between the straight line of slope $E_{\text{EC2}}^{\text{implicit}}(T)$ and that of slope $E_{\text{ENV}}(T)$. Mathematically, it is given by Eq. [6].

$$\varepsilon_{tr}(T,\sigma) = \frac{\sigma}{E_{\text{EC2}}^{\text{implicit}}} - \frac{\sigma}{E_{\text{ENV}}} = \frac{2}{3} \frac{\left(\varepsilon_{c1,EC2} - \varepsilon_{c1,\min}\right)}{\left(f_c / f_{ck}\right)} \frac{\sigma}{f_{ck}} = \phi(T) \frac{\sigma}{f_{ck}} \qquad [6]$$

The function $\phi(T)$ is a growing function of temperature that is not reversible during cooling, as each of its components $\varepsilon_{c1,EC2}$; $\varepsilon_{c1,\min}$; f_c / f_{ck} are irrecoverable. This is in line with the definition of transient creep that is not recovered during the cooling phase. The function $\phi(T)$ components are given in the EC2 and ENV.

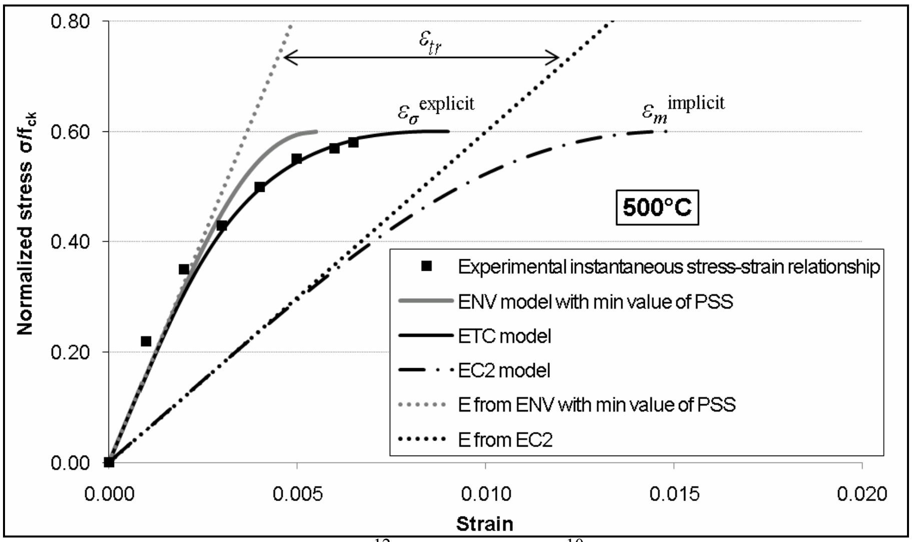

Figure 5: Comparison at 500°C of ENV[12], ETC and EC2[10] models and experimental data from Schneider[14]

The instantaneous stress-strain relationship of the model, obtained as the difference between the EC2 relationship and the transient creep given by Eq. [6] (see "ETC model" in Figure 5), is not exactly equal to the ENV relationship because the transient creep has been considered as linearly stress dependent. However, the initial stiffness of the new relationship is exactly equal to the ENV elastic modulus.

The mathematical expression of this instantaneous stress-strain relationship is approximated by a direct relationship $\sigma = f\left(\varepsilon_\sigma^{\text{explicit}}\right)$ with the same generic form as the current EC2 model, see Eq. [7]:

$$\frac{\sigma}{f_c(T)} = \frac{n\ \varepsilon_\sigma^{\text{explicit}}}{\varepsilon_{c1,ETC}(T)\left[(n-1)+\left(\frac{\varepsilon_\sigma^{\text{explicit}}}{\varepsilon_{c1,ETC}(T)}\right)^n\right]} \quad [7]$$

where n is a parameter to be determined and $\varepsilon_{c1,ETC}(T)$ is the PSS for the ETC relationship, given by Eq. [8]:

$$\varepsilon_{c1,ETC} = \varepsilon_{c1,EC2} - \phi\frac{f_c}{f_{ck}} = \frac{2\,\varepsilon_{c1,\min} + \varepsilon_{c1,EC2}}{3} \quad [8]$$

The ETC tangent modulus and the ETC initial stiffness (elastic modulus) are obtained directly by derivation of Eq. [7]. The parameter n is chosen to obtain the best possible correlation between Eq. [7] and the curve obtained as the difference between the EC2 relationship and the transient creep given by Eq. [6]. It was chosen to use a single value of n for all temperatures. A good indication to calibrate the parameter n is to calibrate the ETC initial stiffness E_{ETC} on the ENV elastic modulus with the minimal value of the PSS. This is done using Eq. [9]:

$$E_{ETC} = \frac{n\ f_c}{(n-1)\,\varepsilon_{c1,ETC}} = \frac{3\,f_c}{2\,\varepsilon_{c1,\min}} \Leftrightarrow \frac{n}{(n-1)} = \frac{3\,\varepsilon_{c1,ETC}}{2\,\varepsilon_{c1,\min}} = 1 + \frac{\varepsilon_{c1,EC2}}{2\,\varepsilon_{c1,\min}} \quad [9]$$

Good correlation in the range of temperatures from 100°C and 1100°C is obtained using $n = 2$. The initial stiffness of the ETC model is close to the elastic modulus of the ENV with $\varepsilon_{c1,\min}$.

Characteristics of the ETC Model

Figure 6 compares the transient creep of the present model with experimental data and models given in the literature (reported in Youssef & al.[6]) for the particular case of a specimen first subjected to a uniaxial compressive stress equal to 0.33 f_c and then heated at a constant rate. It can be seen that the present ETC model is reasonably close to the other models and to experimental data.

Finally, the ETC model presents the following characteristics:

- The ETC model has the same generic form as the current EC2 implicit model;
- The ETC initial stiffness is close to the elastic modulus of ENV with minimal value of the PSS, which leads to an accurate representation of the elastic modulus of the material;
- The transient creep strain calculated with the ETC model is comparable to other models found in literature (Figure 6);

- The instantaneous stress-strain relationships considered in the ETC model are consistent with experimental data obtained by steady-state tests (Figure 5);
- The mechanical stress-strain relationships obtained with the ETC model for a material first-time heated under constant stress (transient tests) are calibrated to yield the same results as the EC2.

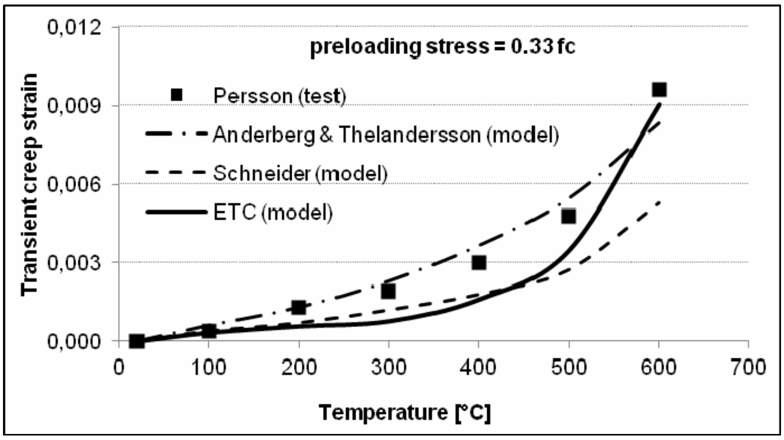

Figure 6: Comparison between different models of transient creep and experimental data

EXPERIMENTAL VALIDATION AT THE MATERIAL LEVEL FOR UNSTEADY TEMPERATURES AND LOADS

The ETC model is validated by a comparison between experimental results and the computed values of the mechanical strain developed by concrete specimens subjected to unsteady temperatures and loads. The considered experiments are taken from Schneider & al.[16]. The specimens are axially unrestrained cylinders with 80 mm diameter and 300 mm height. In all cases, the temperature is constantly increasing at heating rate of 2 °C/min. The compressive strength at 20°C is 38 MPa. The numerical calculations are performed with the nonlinear finite element software SAFIR[11] where the ETC model has been implemented.

The concrete cylinders are subjected to different stress-time relationships. The aim is to highlight the influence of the explicit consideration of transient creep strain on the mechanical strain calculation. The computed results given by the ETC model and the EC2 implicit model are compared to the measured results, see Figure 7. The observations are put in relation with the theoretical considerations discussed above.

The first test successively represents Situation I (increasing stress), Situation II (decreasing stress) and finally Situation III (constant stress). At the beginning and until the peak stress, the difference between the two models is very small, see first row in Figure 7. Then, the stress rate becomes negative. During this second phase of the test (decreasing stress), the mechanical strain computed by the EC2 implicit model quickly decreases, because the transient creep strain is being recovered. On the contrary, the mechanical strain computed by the ETC model keeps on growing, though more and more slowly, because transient creep strain still develops in the material. The transient creep strain counterbalances the elastic unloading due to the stress decrease. During this phase, the behaviour predicted by the ETC model better matches the measured behaviour. This tends to confirm the fact that implicit models are not able to capture properly the actual unloading stiffness at elevated temperatures. At the end of the test, the stress is kept constant (Situation III) and both models predict exactly the same variation of the mechanical strain.

In the second test, the specimen is successively subjected to different constant stress levels while the temperature is increasing (Situation III). The transition between two stress levels is made by a “step”, i.e. a quasi-instantaneous variation from one stress level to another, see second row of Figure 7. At each stress step, the corresponding mechanical strain variations predicted by the two models are slightly different. Implicit models such as the EC2 model amplify the effect of a stress step on the mechanical strain variation. Indeed, the transient creep strain considered in implicit models is suddenly increased or decreased together with the elastic strain. On the contrary in explicit models, transient creep strain does not vary in such situations where the stress varies at constant temperature. It can be seen that the behaviour predicted by the ETC model better matches the experimental behaviour of the specimens, thanks to a better modelling of the material stiffness at constant high temperature.

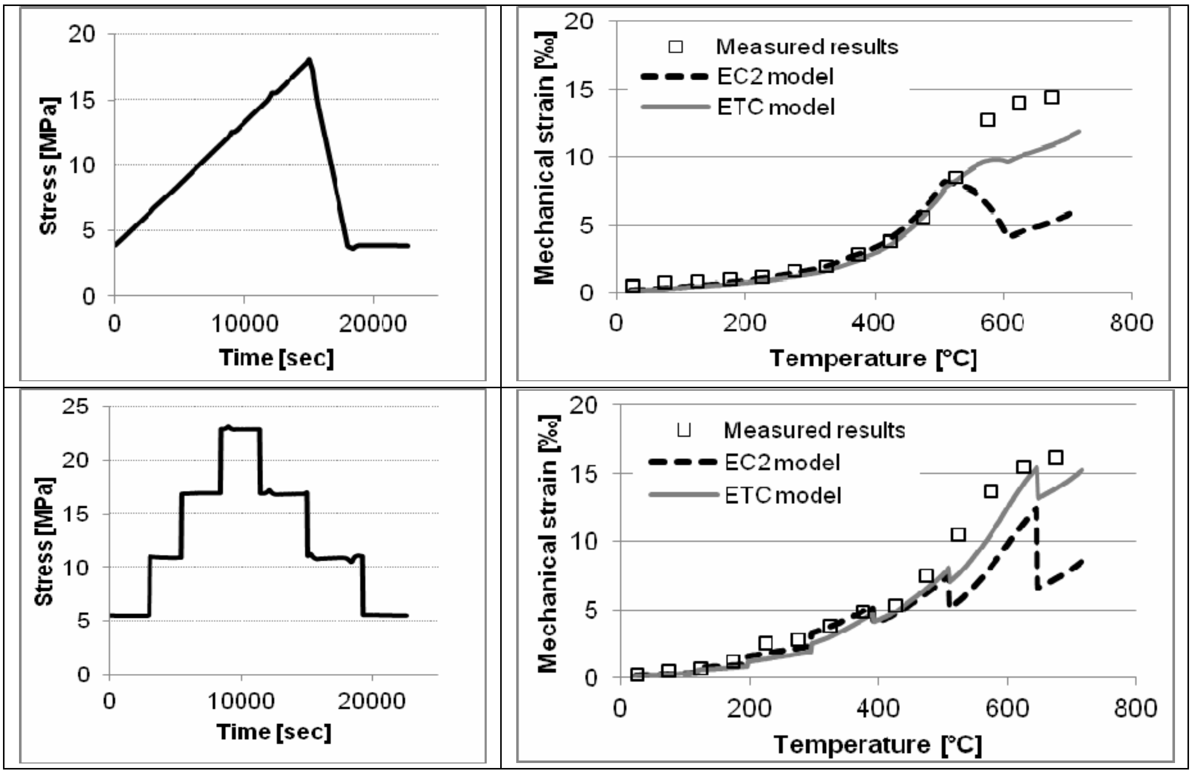

Figure 7: Mechanical strain-Temperature relationships: measured and computed results

AXIALLY RESTRAINED REINFORCED CONCRETE COLUMN SUBJECTED TO HEATING AND COOLING

An experimental fire test made at South China University of Technology on an axially restrained concrete column, described by Wu et al.[17], was simulated using the software SAFIR. The experimental data were compared with the computed results obtained respectively with the EC2 concrete model and the ETC concrete model. The column is 2340 mm height but only the central portion of 1650 mm is exposed to fire. The column has a t-shape C30 concrete cross section (Figure 8) reinforced with 12 longitudinal steel bars of HRB400 with a diameter of 10 mm. The column was axially restrained (axial restraint k_l of 34.5 MN/m) using a restraining beam. The column was initially concentrically loaded with a load of 375 kN (load level 0.34) and then subjected to ISO834 standard fire on all sides. The fire was stopped when approximately 50% of the working load was transferred from the column to the restraining beam, followed by a cooling phase.

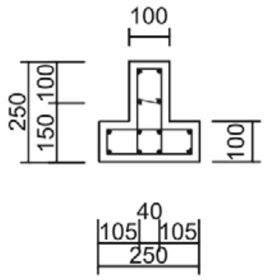

Figure 8: T-shape cross section

Thermal parameters for concrete and steel recommended by EC2 were used in the heat transfer analysis by SAFIR. For the structural analysis, initial eccentricity of 3 mm was introduced. The axial restraint stiffness remains unchanged during the simulation. The deformation behaviour can be observed in Figure 9 and the evolution of the axial load in Figure 10.

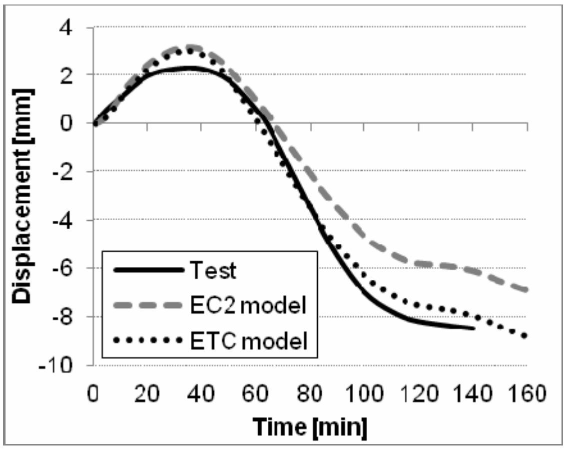

Figure 9: Displacement-time relationship

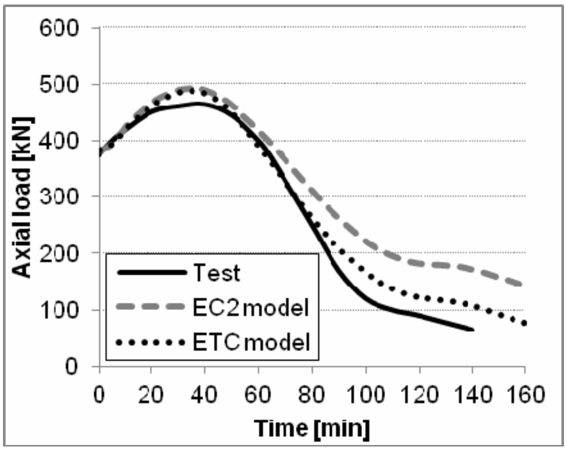

Figure 10: Axial load-time relationship

The ETC model and the EC2 model lead to comparable results during the expanding phase of the column. Then during the contracting phase, the behaviour predicted by the ETC model tends to differ from the behaviour predicted by the EC2 model; the effect of the explicit consideration of transient creep on the structural behaviour becomes notable. The difference between the behaviours predicted by the ETC and the EC2 models is particularly significant during the cooling phase. The ETC model matches better than the EC2 model the actual behaviour of the structure.

CONCLUSION

- Even in a simple concrete element subjected to the heating phase of a fire, the stress and temperature evolutions across the section are complex and significantly different depending on the position in the section.
- Concrete models that include implicitly the transient creep strain, such as the current Eurocode 2 model, have inherent limitations that prevent them from accurately representing the mechanical strains developed in concrete members subjected to fire. Especially, implicit models are not able to capture properly the actual unloading stiffness at elevated temperatures.
- The ETC model proposed in this paper is a new formulation of the generic EC2 concrete model that contains an explicit term for consideration of the transient creep. The ETC model brings a

supplementary accuracy without removing the generic characteristic of the EC2 model. The improvement may be significant as indicated by comparisons against experimental data performed at the material level and for a simple structural element. The utilization of the ETC model should be particularly recommended when modelling the cooling phase of a fire because it is able to capture the irreversibility of transient creep.

- In future works, more experimental and numerical comparisons have to be performed in order to quantify the consequences of the explicit consideration of transient creep on the global behaviour of concrete subjected to fire.

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