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A procedure for modelling asymptotic g-mode pulsators: The case of γ Doradus stars

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Mode identification is one of the first and main problems we encounter in trying to develop the complete potential of asteroseismology. In the particular case of g-mode pulsators, this is still an unsolved problem, from both the observational and theoretical points of view. Nevertheless, in recent years, some observational and theoretical efforts have been made to find a solution. In this work we use the latest theoretical and computational tools to understand asymptotic g-mode pulsators: 1) the Frequency Ratio Method, and 2) Time Dependent Convection. With these tools, a self-consistent procedure for mode identification and modelling of these g-mode pulsators can be constructed. This procedure is illustrated using observational information available for the γ Doradus star 9 Aurigae.

1 Introduction

Asteroseismology is currently being developed as an efficient instrument for the study of stellar interiors, and mode identification is one of the first and main difficulties we find in developing its complete potential. In the well-known solar case, the problem is solved with the help of: 1) full observations, allowing the knowledge of the complete spectrum for a given spherical degree ℓ, and 2) the theoretical advantage of pulsations in the high frequency asymptotic regime. However, there are a number of stars which do not pulsate in the asymptotic regime, such as the δ Scutis, where mode identification is a hard task. On the other hand, there are also stars pulsating in the g-mode asymptotic regime, like γ Doradus stars, where we can benefit from the analytical solution of the equations. However, problems arise from the observational side. The pulsational periods of these stars, of the order of days, and the day-night sequence, do not make the observation of such periods an easy task. In addition, the pulsation modes of these stars usually have very small amplitudes, introducing additional difficulties. Helioseismological tools cannot therefore simply be extrapolated to g-mode asymptotic pulsators, since we hardly have a series of consecutive modes; and mode identification cannot be obtained for these stars with techniques presently available.

Most of the major improvements in our knowledge of the pulsational behavior of γ Doradus stars are related to theoretical aspects: 1) those concerning the description of the excitation mechanism of their oscillation modes, the so called gravity modes (or g-modes), and 2) those dealing with mode identification, which significantly improve their asteroseismic potential. These improvements make the mode identification and modelling of asymptotic g-mode pulsators possible, in certain cases, based on: 1) photometric (white and multicolour) observations, 2) the Frequency Ratio Method (FRM) for an estimation of mode identification, and 3) Time Dependent Convection (TDC) for instabilities and non-adiabatic studies. In the present work, we summarise the main characteristics of the FRM and present a self-consistent procedure for modelling these stars. This procedure is discussed using the case of the γ Doradus star 9 Aurigae (Zerbi et al. 1997).

2 The FRM

The FRM makes use of the asymptotic analytical form of the g-mode frequencies to estimate the radial order n and spherical degree ℓ of the observed frequencies (Moya et al. 2005; Suárez et al. 2005). If we have an oscillation frequency in this regime, non-rotating asymptotic theory gives the following analytical dependences (Smeyers & Moya 2007):

$$\sigma = \frac{\sqrt{\ell(\ell+1)}}{\pi(n+1/3)} \int_{r_\mathrm{b}}^{r_\mathrm{t}} \frac{N}{r} \mathrm{d}r, \qquad (1)$$

where r_b and r_t are the bottom and top limits of the stellar radiative zone respectively, and N is the Brunt-Väisälä frequency. This equation has been obtained taking into ac-

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count the discontinuities of variables in the transition between convective and radiative zones in the stellar interiors.

With two oscillation frequencies, we can take their ratio. If we assume some information about their ℓ (we know their value or we assume them to be the same for both modes), this ratio is the direct ratio of the radial orders plus a factor (1/3 in this case), since the integral is constant for a given model. This provides a first estimation of the radial orders of these frequencies: the couple of integers whose ratio is similar to the one observed.

If we have a third oscillation frequency, and we repeat this procedure, the number of triplets fulfilling the observed ratios is drastically reduced. This is a requirement for the FRM: to have at least three observational frequencies. A second step in the method is the estimation of a new observable, the Brunt-Väisälä integral $I = \int_{r_b}^{r_t} \frac{N}{r} dr$. Once we have a small set of triplets of possible radial order identifications, we can use Eq. (1) to assign a value of I to each triplet, for a given ℓ, since we also know σ (the observed frequencies).

This is the final goal of the Frequency Ratio Method, to provide a small set of possible observables and mode identifications to be fulfilled by the numerical models. The FRM is completed when these numerical models are found. One of the sources of uncertainty of this method is stellar rotation. A complete study of the influence of rotation in the FRM, its limits of validity and the range where the method is robust is given in Suárez et al. (2005) and Suárez et al. (2008). Finally we want to remark that any additional information about the ℓ of the modes can be exploited with the FRM through Eq. (6) in Moya et al. (2005).

3 Time-Dependent Convection (TDC)

The driving mechanism of gravity modes in γ Doradus stars has been a matter of debate for the last decade. The theory developed by Guzik et al. (2000) ascribes the origin of pulsation to convection. In particular, pulsations are considered to be driven by a periodic flux-blocking at the base of the convective envelope. The balance between this flux-blocking driving and radiative damping in the g-mode cavity explains the location of their instability strip. The approximation of Frozen Convection (FC) considered by Guzik et al. (2000) in their non-adiabatic modelling holds true only in a very narrow region near the bottom of the convective zone. In that region, the lifetime of the convective elements is larger than the pulsation periods. However, this is not the case in the rest of the convective zone. This problem has been recently solved by the Time-Dependent Convection theory developed by Gabriel (1996) and Grigahcène et al. (2005). The new TDC models confirm the periodic flux blocking at the base of the convective envelope as being responsible for the driving of g modes. Moreover, the balance between this flux and radiative damping in the g-mode cavity explains the location of the instability strip for these stars (Dupret et al. 2005, 2004).

Table 1 Photometric physical characteristics of 9 Aurigae. $T_{\rm eff}$ is given in Kelvin, gravity and metallicity in dex, rotational velocity in km/s and frequencies in c/d (Zerbi et al. 1997).

$T_{\rm eff}$	$\log g$	[Fe/H]	$v \sin i$
6990 ± 150	4.17 ± 0.1	-0.18 ± 0.1	18
f_1	f_2	f_3	
0.7948	0.7679	0.3429	

Table 2 Amplitude ratios and phase-lags (in °) of the observed frequencies obtained with the Strömgren photometric filters. We have defined $f(x) = \phi(x - y)$, that is, the phase-lag of the observations with the filter x as compared with those of the filter y.

Freq.	$f(u)$	$f(v)$	$f(b)$	u/y	v/y	b/y
0.7948	0.2 ±3.2	−0.5 ±2.8	−0.4 ±2.8	0.972 ±0.056	1.516 ±0.076	1.331 ±0.066
0.3429	0.1 ±6.1	0.1 ±5.5	0.0 ±5.4	0.983 ±0.104	1.434 ±0.125	1.278 ±0.117
0.7679	−8.5 ±6.1	−3.7 ±5.5	−4.0 ±5.5	1.064 ±0.112	1.525 ±0.134	1.353 ±0.126

Table 3 Possible mode identifications of the observed frequencies provided by the FRM. $I_{\rm obs}$ is the observational Brunt-Väisälä frequency (see text for details).

n_1	n_2	n_3	ℓ	$I_{\rm obs}$
33	34	77	1	681.14
57	59	133	2	678.24

On the other hand, this treatment provides improved non-adiabatic observables $\delta T_{\rm eff}/T_{\rm eff}$ and phase-lag (the phase between the effective temperature variations and the radial displacement). This allows us to use multicolour photometric mode identification for an estimate of the spherical degree ℓ of the observed modes (Garrido et al. 1990).

4 Modelling asymptotic g-mode pulsators

With the two theoretical tools presented above a complete self-consistent procedure for the modelling of asymptotic g-mode pulsators can be devised. This procedure is illustrated by the example of the γ Doradus star 9 Aurigae. The available photometric observations for this star provide three oscillation frequencies. Its observational physical parameters are summarised in Tables 1 and 2. The largest pulsational period is still controversial, and the final results of this example must be taken with all due caution. However, these three frequencies are still useful for the purpose of presenting our procedure. The final result depends on the observational information, and the accuracy of this information is not one of the aims of the present work.

The frequency ratios of the observed modes are $f_2/f_1 = 0.966$, $f_3/f_1 = 0.431$, $f_3/f_2 = 0.447$, with an er-

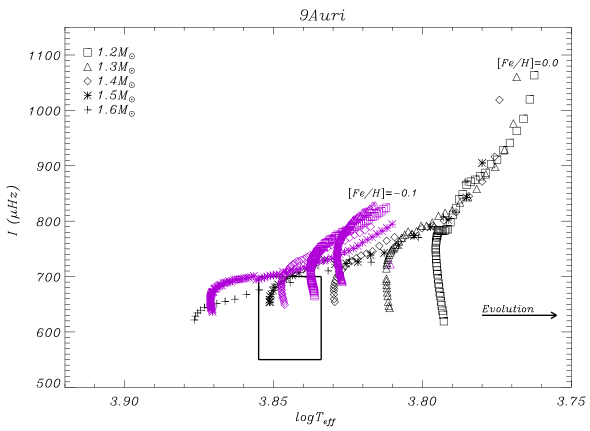

Fig. 1 (online colour at: www.an-journal.org) I as a function of the effective temperature for the representative models of 9 Aurigae, with masses in the range of $M = 1.2$–$1.6\ \mathrm{M}_\odot$, for two metallicities $[\mathrm{Fe/H}] = 0$ and -0.1. The box represent the uncertainty on $(I_{\mathrm{obs}}, T_{\mathrm{eff}})$ of all the sets of mode identifications obtained with the FRM.

Table 4 Main physical and theoretical characteristics of the model fulfilling all the observational constraints of 9 Aurigae.

$M/M_\odot$	T_{eff}	$\log g$	[Fe/H]	$\log L/L_\odot$
1.4	7006	4.28	−0.1	0.63
$R/R_\odot$	Age	I_{th}	α_{ov}	α_{MLT}
1.41	600	681.5	0.3	1.6

ror ± 0.010 (Moya et al. 2005; Suárez et al. 2005). There are only two possible mode identifications for these ratios, as listed in Table 3, together with the corresponding Brunt-Väisälä integral (I_{obs}). This makes it possible to construct a plot comparing the observed effective temperature and I_{obs} with those obtained using theoretical models. This plot is shown in Fig. 1, where different evolutionary tracks of different masses and metallicities are used. The box is the range where the observed I are located. We see that only models at the beginning or the middle of their evolution give the observed I at the observed temperatures.

We searched for models giving the observed frequencies with the mode identification obtained, I and T_{eff}. In this case, we only found one model fulfilling the constraints. The main physical characteristics of this model are shown in Table 4.

As far as the multicolour photometric observations and TDC are concerned, the previously selected model does not discriminate between the two possible ℓ values. We therefore need the amplitude ratio/phase differences diagnostic diagrams (Garrido et al. 1990) to distinguish the correct ℓ of the observed modes. For the theoretical predictions, the non-adiabatic observables $\delta T_{\mathrm{eff}}/T_{\mathrm{eff}}$, $\delta g_{\mathrm{eff}}/g_{\mathrm{eff}}$ and phase lag, given by TDC, are necessary. Figure 2 shows the observed multicolour observables obtained using the Strömgren filters, normalised to the filter y. Only predictions for $\ell = 2$ fulfill all the observations. Unfortunately, the large

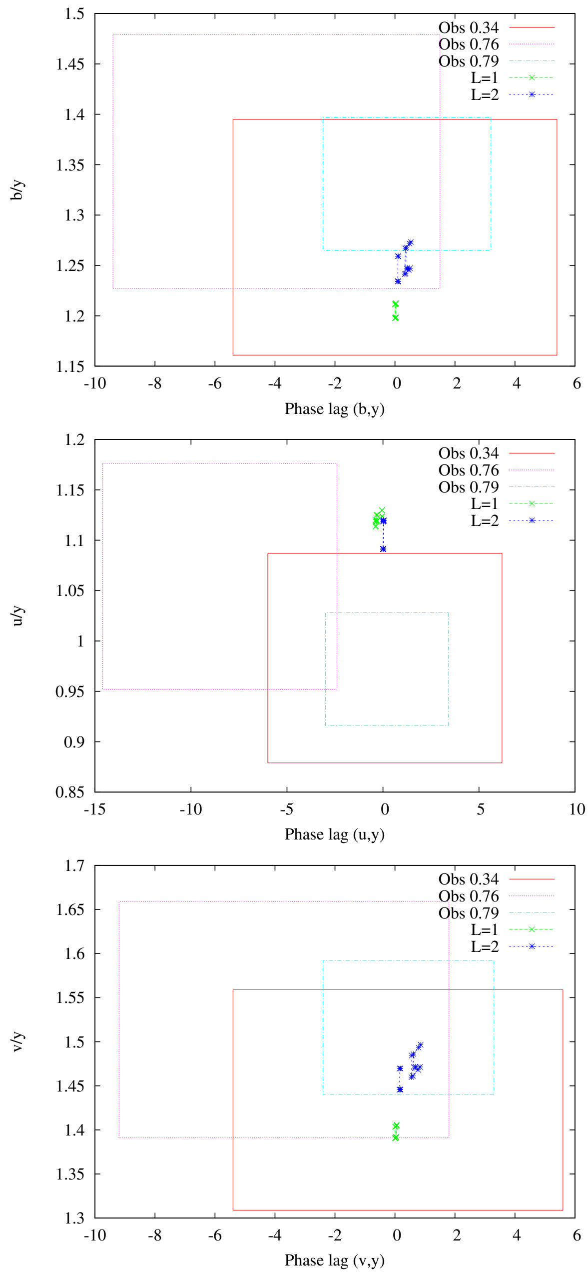

Fig. 2 (online colour at: www.an-journal.org) Observed phase-lags vs. amplitude ratios of the three oscillation frequencies of 9 Aurigae, obtained with the Strömgren filters. The theoretical predictions of TDC for $\ell = 1$ and 2 are also shown.

observational errors make this study inconclusive, but nonetheless useful for the purpose of presenting the procedure.

Finally, the Mixing Length parameter α is fixed by studying the instability of the modes of the model selected above. Figure 3 shows the imaginary part of the frequency of the modes (Dupret et al. 2005) as a function of their real part. When the imaginary part of the frequency is positive, the mode is stable and cannot be observed. Therefore, the observed modes can only be the overstable ones (imaginary part negative). In this figure, we see how the controversial longest period frequency is never predicted overstable. The

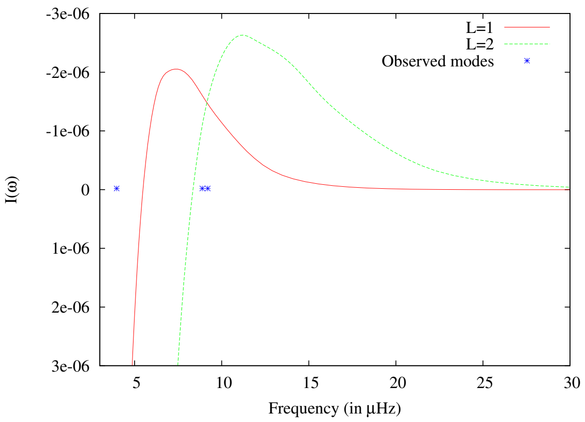

Fig. 3 (online colour at: www.an-journal.org) Imaginary part of the non-adiabatic frequencies as a function of its real part (in μHz), for frequencies with $\ell = 1$ and 2. The observed frequencies are also shown.

same happens for all the $\alpha_{\rm MLT}$ studied. The other two frequencies are predicted as overstable only for $\alpha_{\rm MLT} = 1.6$. This is the value provided by this study.

5 Conclusions

With the help of the most recent improvements in the modelling of asymptotic g-mode pulsators, in particular γ Doradus stars, a procedure for identifying modes and constraining models of these stars is presented. The γ Doradus star 9 Aurigae has been used as an example to illustrate this procedure. For estimating the mode identification, the Frequency Ratio Method has been used. With at least three non-consecutive oscillation frequencies a small set of possible mode identifications can be obtained. In addition, the observational value of the Brunt-Väisälä integral can also be given. The comparison of multicolour observations with the theoretical predictions together with the instability study closes the procedure and makes it self-consistent. The recent development of Time Dependent Convection provides highly accurate theoretical predictions of the non-adiabatic observables and instability ranges of these stars. By comparing such predictions with the observations, only for the models given by the FRM, we finally obtain a model fulfilling all of the observations.

The minimal observational requirements to apply this procedure are at least three oscillation frequencies observed in multicolour photometry. Any additional information provided by spectroscopic observations, or more observed frequencies, can easily be included in the procedure, making the final result more consistent. Finally, we wish to point out that this is the only procedure available nowadays for mode identification and modelling of asymptotic g-mode pulsators in the MS, when the only observational information is photometry, and a small number of non-consecutive pulsational modes are observed.

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