

Problems and prospects in stellar physics

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Abstract

Main difficulties and uncertainties in stellar physics originate in the modelling of convection, microscopic and macroscopic transport mechanisms and rotation. I consider each of these physical aspects, with a particular attention to their possible probe by asteroseismology. Another important question about opacities is considered by Montalbán & Miglio (2008).

Convection

Convection is the most difficult physical process to model in stellar interiors. First, because of its non-local character: the convective flux cannot realistically be related to local gradients. Second, because of its spectral nature with space scales going from the dissipation scale of centimeters to stellar sizes. And third, because of its extremely turbulent character, as shown by the huge values of the Reynolds number.

Convective envelopes

Deep enough in the star, the high efficiency of convection leads to a quasi-adiabatic stratification. But this is not the case in the upper part of convective envelopes where the description of the super-adiabatic stratification is subject to large uncertainties. There are two approaches to model this region: using an analytical theory or performing 2D or 3D hydrodynamic simulations following the Large Eddies Simulation (LES) approach.

The simplest and crudest analytical theory is the Mixing-Length Theory (MLT) which neglects the non-local and spectral nature of convection, the whole spectrum being reduced to a single representative scale: the mixing-length, generally parametrized as $l = \alpha H_p$. Simple modeling of the action of buoyancy over one mixing-length leads to the well-known cubic equation whose solution gives the required quantities.

Another more realistic analytical theory is the Full Spectrum of Turbulence (FST) approach (Canuto & Mazzitelli 1991, Canuto et al. 1996). The main limitation of this theory is its *local* character, the convective flux being expressed as: $F_c = (4acT^4)/(3\kappa\rho) H_p^{-1} (\nabla - \nabla_{ad}) \phi$. To obtain the function $\phi(\Gamma)$, the authors solved once and for all the Navier & Stokes equations in the Fourier space of turbulence for different convective efficiencies Γ , using the eddy damped quasi-normal Markovian approximation (Lesieur 1987) that avoids getting negative energy at relevant wavelengths. This gives the energy spectrum and by a simple integration ϕ . A modified version of the cubic equation is then obtained, which is explicitly designed for a very simple implementation in a stellar evolution code, like the MLT.

A third analytical approach corresponds to the so-called Reynolds stress models, where the classical approach of turbulence splitting physical quantities and equations into an average and fluctuating contribution is followed. The average equations introduce second order correlation

terms: the convective flux, the Reynolds stress tensor and the dissipation of turbulent kinetic energy into heat. To obtain them, the fluctuation equations are multiplied by appropriate quantities and averaged, but this introduces unknown higher order correlation terms up to the fourth order. To get as much equations as unknowns, a closure model must be included, expressing the fourth order correlations in term of the second order ones. The simplest closure model is the quasi-normal approximation, but the resulting equations can present singularities and unphysical wave behaviours. New closure models have been proposed avoiding such singularities (e.g. Cheng et al. 2005). The main strength of the Reynolds models is that they naturally hold the non-local character of turbulence as exhibited by the advection terms appearing explicitly in the equations.

The other very important approach to model convection is to perform full 2D or 3D hydrodynamic simulations. The main limitation is that the smallest scales at the bottom of the turbulent cascade are by far smaller than the scales reached by these simulations. The basics of the LES approach is to introduce a so-called subgrid scale model taking somehow into account the self similar dissipation of unresolved smaller scales into heat. This is done typically by the addition of a kind of eddy viscosity. 3D LES codes widely used in the context of stellar physics are e.g. the code of Stein & Nordlund (1998), CO⁵BOLD (Wedemeyer et al. 2004) and ASH (Miesch et al. 2000). The first two use a finite difference approach in a small cartesian box, while ASH uses a spectral approach in a global spherical domain.

An important question is how analytical models compare with the mean stratifications from LES. First, concerning solar-type convective envelopes, the super-adiabatic gradient from LES is slightly less picked than in MLT and FST local models. The turbulent pressures and velocities from LES exhibit clearly the non-local character of turbulence with a more or less exponential decrease in the subadiabatic atmosphere. But the largest differences are found in the thin superficial convective zones of A-type stars localized in the H and He partial ionization zones (Steffen 2006). The convective flux fraction from LES appears to be very small there ($F_c/F < 0.05$ in the H zone and negligible in He zone). MLT models do not reproduce this aspect (except with negligible α) but FST models do. Moreover, a striking feature in LES is that the H and He convective zones join in terms of the convective velocities, which is also found in Reynolds models (Kupka & Montgomery 2002). Hence, the whole region would have to be mixed when including microscopic and macroscopic chemical transport (see next sections). It is generally told that LES simulations cannot be included in stellar evolution computations, because the time spanned by LES are by far too small. But why not thinking about building a grid of simulations (with different T_{eff} and $\log g$) and interpolate, as it is generally done with stellar atmospheres?

Convective penetration and overshooting

Because of their inertia, convective elements are expected to penetrate slightly in the stable subadiabatic layers, defining the so-called overshooting region whose description is quite uncertain. Two possible pictures can arise. If the Peclet number is large, inertia dominates over the radiative losses. Hence, the convective blobs keep their heat content when penetrating the stable region. There, they are quickly braked by buoyancy and heat exchanges lead to a nearly adiabatic stratification. According to Zahn (1991), we can call this *convective penetration*. For small Peclet numbers, the picture is very different: the turbulent motions are unable to keep their temperature and density contrast in the subadiabatic layers, they can thus proceed deeper, do not affect the temperature stratification but just extend the mixing region. We can call this *overshooting*. Considering the values of the Peclet number, we should expect that convective penetration occurs above convective cores, while overshooting occurs only in the thin superficial convective zones of A stars. However, mixing associated with differential rotation can somehow mimick extended core overshooting.

It is useful to consider what numerical simulations tell us about the vicinity of convective zones. I consider here mainly the results obtained by Browning et al. (2004) using the code ASH and by Brummell et al. (2002) in a smaller cartesian box. A general feature appearing in 3D simulations is that the flows are highly time-dependent, with complex and intermittent features in space and time entering the subadiabatic region. The non-local character of the phenomenon is evident. Negative values of the convective flux are found, in agreement with the above picture of convective penetration. However, the Peclet and Reynolds numbers of these simulations are by far smaller than reality and the simulation times are much smaller than the thermal time scale, which inhibits using them to predict the resulting mean stratification. Including rotation in the simulations appears to affect the results significantly. The overall scales are reduced and, from a dynamic point of view, the Coriolis force deflects the trajectories, with momentum transfer from radial to horizontal, particularly near the equator. An unexpected consequence is that global simulations predict a prolate form of the adiabatic part of the convective core. Finally, a general tendency is that increasing the Reynolds number of the simulations decreases the size of the overshooting region, as a consequence of the decreasing filling factor of the plumes.

Semi-convection

A problem which is often forgot is semi-convection. It typically occurs when the convective core grows in mass. As an immediate consequence, a discontinuity is created in X , and thus in the opacity ($\kappa \propto (1 + X)$). Hence $\nabla_{\text{rad}} > \nabla_{\text{ad}}$ above the convective core and there is an intrinsic ambiguity in the definition of its boundary. The only solution is to assume a region of partial mixing above the convective core, but the question arises which condition characterizes this “semi-convective” region: the Schwarzschild criterium $\nabla_{\text{rad}} = \nabla_{\text{ad}}$ or the Ledoux criterium $\nabla_{\text{rad}} = \nabla_{\text{ad}} + \beta/(4 - 3\beta)\nabla_{\mu} \equiv \nabla_{\text{L}}$ nullifying the buoyancy force? Semi-convection can have also other origins, but it is always characterized by an initial situation where the steep molecular weight gradient implies $\nabla_{\text{ad}} < \nabla < \nabla_{\text{L}}$. As $\nabla < \nabla_{\text{L}}$, the region is dynamically stable, the buoyancy acting as a restoring force. But $\nabla > \nabla_{\text{ad}}$ so that high ℓ gravity modes trapped in this region are locally driven (Kato 1966). It is expected that such vibrational instability will affect the chemical distribution, but how? We could hope that hydrodynamic simulations give us clues, but they are not numerous in the stellar context and only in 2D, as e.g. those of Merryfield (1995). These simulations show that the final state strongly depends on the super-adiabatic gradient $\nabla - \nabla_{\text{ad}}$ imposed as initial condition. If it is large, the initial disturbances evolve into large-amplitude standing waves that break rapidly, leading to disorganized regions of overturning, so that rapid global mixing ensues. But if it is low, such dramatic overturning does not occur and no significant mixing ensues. The question is which is the critical initial gradient delimitating these two scenarios? Unfortunately, present simulations do not provide us with an answer.

Transport processes

Microscopic diffusion

Different forces act segregatively on different particles and can modify progressively the repartition of chemicals in stellar radiative zones. On the one hand, the gravitational force pushes down the heavy particles, and on the other hand, the radiative forces push up the absorbant particles. Two formalisms are commonly used to derive the diffusion velocities of different elements: the Chapman-Enskog’s theory (Chapman & Cowling 1970) and the Burgers (1969) theory. The two are based on a perturbative treatment of the Boltzmann transport equation. The main source of uncertainties lies in the calculation of the collision integrals appearing in this equation. They diverge when a pure Coulomb potential is used, so that the shielding

by electrons must be taken into account. But the classical Debye-Hückel theory for weakly coupled plasmas is not accurate enough here as the electrostatic potential is hardly smaller than kT in stellar interiors. Paquette et al. (1986) did numerical computations using a so-called modified Debye-Hückel potential which has the quality of agreeing with other rigorous simulations in the two limiting cases of dilute and dense plasmas. Fitting of these results by Michaud & Proffitt (1993) are now widely used. Other improvements are to include quantum effects, as done by Schlattl & Salaris (2003), and dynamic shielding.

Radiative forces

The segregative action of radiative forces on particles merits a particular emphasis. The main difficulty is in the computation of the monochromatic cross sections. Next, simple integrations and use of the formalisms cited above lead to the diffusion velocities. Significant numerical difficulties occur for the implementation in stellar evolution codes, mainly because of the very broad range of time-scales associated with this process and the large amount of data to handle. A parametric approach, as derived by Leblanc & Alecian (2004) can be extremely useful to decrease the computation time. In some cases, e.g. in subdwarf B stars, it is argued that, as the diffusion time scales are smaller than the stellar evolution one, equilibrium chemical profiles are reached and can be assumed; this simplifies of course a lot the problem.

Macroscopic transport due to differential rotation

As detailed by Zahn (2008), rotation is at the origin of two macroscopic transport processes of very distinct natures in radiative zones. First, the Von Zeipel theorem shows that rotation must generate a meridional circulation. After a transient phase, this circulation settles in a quasi-stationary regime allowing to advect angular momentum (and at the same time chemicals) towards the surface where it is lost for example by winds. Second, the horizontal shear and other instabilities due to differential rotation generate turbulence. This turbulence is expected to be much more vigorous in the horizontal plane because of the stabilizing effect of buoyancy, justifying the so-called shellular approximation where differential rotation in latitude is assumed negligible. Turbulence in radiative zones acts as a *diffusive* transport mechanism, which is qualitatively very different from the *advection* by meridional circulation. The main source of uncertainties lies in the turbulent diffusion coefficients that must be used to model this process (Mathis et al. 2004).

Magnetic field and differential rotation certainly influence each other. In a full radiative star, the high conductivity tends to freeze deformation of the strength lines. Hence, the magnetic field acts against differential rotation. But the things become complicated when the field connects with a convective envelope in differential rotation like in the Sun, because in this case, differential rotation is transmitted along the field lines. So, magnetic field is certainly an important actor but it cannot explain fully the solid rotation of the Solar radiative core predicted by helioseismology.

Transport by internal gravity waves

Internal gravity waves emitted at the boundary of convective zones transport angular momentum in the radiative zone where they are dissipated. Prograde waves propagating downwards increase the angular momentum towards the center while retrograde wave do the contrary. Because of the Doppler effect, the radiative dissipation is stronger for the prograde waves, so that the retrograde waves are able to proceed deeper and extract more angular momentum. Simulations by Charbonnel & Talon (2005) show that this process succeeds to explain the solid rotation of the present Solar core. The turbulence generated by these processes allows also to explain the Lithium underabundance. The role of Coriolis force in this frame is discussed by Mathis (2008).

Rotation: geometrical effect

In moderate to rapid rotators, the centrifugal force breaks the spherical symmetry, so that 2D stellar models are required to describe them correctly. If the time aspect is disregarded, static 2D models can be computed more easily (Roxburgh 2006). Some 2D models show convective envelopes trapped along the equator (Espinosa & Rieutord 2007); this is an example of how our representation of wave propagation cavities could be modified when including rotation.

Helioseismology as a probe of Solar physics

The Solar case is the best illustration of the success of seismology to probe the above physical processes. The two main quantities obtained by helioseismic inversion are the sound speed and the rotation rate. As a first example, sharp features of the sound speed derivative, as they appear e.g. at the base of the convective envelope manifest as oscillatory components in the frequency separations. The corresponding wavelength gives the location of the convective zone boundary and its amplitude is related to the size of the overshooting region (Roxburgh & Vorontsov 1994, Christensen-Dalsgaard et al. 1995). However, the latter authors showed that present observations do not allow to discriminate between thin sharp convective penetration and broad smooth overshooting. Including microscopic diffusion affects also the sound speed in the radiative core, giving a better agreement with the results of seismic inversion. Finally, the rotation profile obtained by inversion appears to be solid in the radiative core, constraining the different macroscopic transport processes. Transport by internal gravity waves is required to explain it.

Asteroseismology as a probe of stellar physics

Pulsating stars are a real zoo, enlightening very different physical processes. As for solar-like oscillations observed in main sequence and red giant stars, not only the frequencies are a source of information, but also the amplitudes and line-widths give constraints on the upper part of the convective envelope (Samadi 2008, Houdek 2008). On the other side, we have the big family of auto-driven pulsators. As their frequencies correspond to mixed modes and/or gravity modes, they inform us about the very deep layer of the star. But the interpretation of their much more complex frequency pattern requires mode identification methods based on other observables (Handler 2008, Telting 2008). Moreover in many of these stars, the coupling between oscillations and rotation is expected to be very large, which complicates a lot the picture (Lee 2008, Lignières 2008). Forgetting these difficulties, the main quantity that can be constrained from g-mode periods is the Brunt-Väisälä frequency N . Sharp features in N manifest as oscillatory components in the period separations, holding clear constraints on overshooting, μ gradients and thus on the different transport processes (see e.g. Miglio et al. 2008). Finally, it is useful to remark that constraints on opacities, transport processes and convection can be obtained by a non-adiabatic seismic analysis: comparing theoretical and observed ranges of excited modes, instability strips, spectro-photometric amplitude ratios and phase-lags, even when the individual modes are not identified. Asteroseismology already shows successes for several types of pulsators (β Cep stars, white dwarfs, sdBs, ...), as shown in detail in this conference. And many more fascinating results are expected from present and future space missions: COROT, MOST, Kepler, BRITE-C ...

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