

Modeling stochastic excitation of acoustic modes in stars: present status and perspectives

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Abstract

Solar-like oscillations have now been detected for more than ten years and their frequencies measured for a still growing number of stars with various characteristics (e.g. mass, chemical composition, evolutionary stage ...). Excitation of such oscillations is attributed to turbulent convection and takes place in the uppermost part of the convective envelope. Since the pioneering work of Goldreich & Keeley (1977), more sophisticated theoretical models of stochastic excitation were developed, which differ from each other both by the way turbulent convection is modeled and by the assumed sources of excitation. We briefly review here the different underlying approximations and assumptions of those models. A second part shows that computed mode excitation rates crucially depend on the way time-correlations between eddies are described but also on the surface metal abundance of the star.

Individual Objects: Sun, α Cen A, HD 49933

Introduction

Solar p -modes are known to have finite lifetimes (a few days) and a very low amplitude (a few cm/s in velocity and a few ppm in brightness). In the last decade, solar-like oscillations have been detected in numerous stars, in different evolutionary stages and with different metallicity (see recent review by Bedding & Kjeldsen, 2007). Their finite lifetimes are a consequence of several complex damping processes that are not clearly identified so far. Their excitation is attributed to turbulent convection and takes place in the uppermost part of the convective envelope, which is the place of vigorous turbulent motions.

Measuring the mode amplitude and the mode lifetime enables to infer the excitation rate, \mathcal{P} (the energy which is supplied per unit time to the mode). Deriving \mathcal{P} put constraints on the theoretical models of mode excitation by turbulent convection (Libbrecht, 1988).

Goldreich & Keeley (1977, GK hereafter) have proposed the first theoretical model of stochastic excitation of solar acoustic modes by the Reynolds stresses. Since this pioneering work, different improved models have been proposed (Dolginov & Muslimov, 1984; Balmforth, 1992; Goldreich et al., 1994; Samadi & Goupil, 2001; Chaplin et al., 2005; Samadi et al., 2003; Belkacem et al., 2006b, 2008). These approaches differ from each other either by the way turbulent convection is described, or by the excitation process. In the present paper, we briefly review the main assumptions and approximations on which the different theoretical models are based.

As shown by Samadi et al. (2003), the energy supplied per time unit to the modes by turbulent convection crucially depends on the way eddies are time-correlated. A realistic

modeling of the eddy time-correlation at various length scales is then an important issue, which is discussed in details below. The structure and the properties of the convective upper envelope also has an important impact on the mode driving. In particular, the surface metal abundance can significantly change the efficiency of the mode driving.

Theoretical models

Most of the theoretical models of stochastic excitation adopt GK's approach. This approach first consists in solving, with appropriate boundary conditions, the equation that governs the *adiabatic* wave propagation (also called the homogeneous wave equation). This provides the well-known adiabatic displacement eigenvectors ($\vec{\xi}(\vec{r}, t)$). Then, one includes in the wave equation turbulent sources of driving as well as a term of linear damping. The complete equation (so-called inhomogeneous wave equation) is then solved.

Among the sources of driving, the contribution of Reynolds stresses, which represents a mechanical source of driving, is considered. Goldreich et al. (1994, GMK hereafter) have proposed to include in addition the entropy fluctuations (also referred to as non-adiabatic gas pressure fluctuations). It is generally assumed that the entropy fluctuations behave as a passive scalar. In that case, Samadi & Goupil (2001, SG hereafter) have shown that the contribution of the *Lagrangian* entropy fluctuations vanishes. On the other hand, as shown by SG, the advection of the entropy fluctuations by the turbulent velocity field contributes efficiently to the mode driving in addition to the Reynolds stresses. This advective term corresponds to the buoyancy force associated with the entropy fluctuations. Since it involves the entropy fluctuations, it can be considered as a thermal source of driving.

The solution of the inhomogeneous wave equation corresponds to the forced mode displacement, $\delta\vec{r}$ (or equivalently the mode velocity $\vec{v}_{\text{osc}} = d\delta\vec{r}/dt$). A detailed derivation of the solution for *radial* acoustic modes can be found in Samadi & Goupil (2001, SG hereafter) or in Chaplin et al. (2005, CHE hereafter). It can be written as

$$\langle || \vec{v}_{\text{osc}} ||^2 \rangle(\omega_0) = || \vec{\xi} ||^2 \frac{\mathcal{P}}{2\eta I} = \frac{|| \vec{\xi} ||^2}{16\eta I^2} (C_R^2 + C_S^2), \quad (1)$$

where ω_0 is the mode frequency, \mathcal{P} is the mode excitation rate (the rate at which energy is supplied to the mode), η the mode damping rate (which can be derived from seismic data), I the mode inertia, and finally C_R^2 and C_S^2 the contribution of the Reynolds stress and the entropy fluctuations, respectively. The expressions for C_R^2 and C_S^2 are

$$C_R^2 = 4\pi^3 \mathcal{G} \int_0^M dm \rho_0 \left(\frac{d\xi_r}{dr} \right)^2 S_R(r, \omega_0) \quad (2)$$

$$C_S^2 = \frac{4\pi^3 \mathcal{H}}{\omega_0^2} \int_0^M dm \frac{\alpha_s^2}{\rho_0} g_r S_S(r, \omega_0) \quad (3)$$

where $\alpha_s = (\partial P_g / \partial s)_\rho$, P_g is the gas pressure, s the entropy, ρ_0 the mean density, \mathcal{G} and \mathcal{H} are two anisotropic factors (see their expression in SG), S_R and S_S are the “source terms” associated with the Reynolds stresses and entropy fluctuations respectively, ξ_r the adiabatic mode radial eigen-displacement, and finally $g_r(\xi_r, r)$ a function that involves the first and second derivatives of ξ_r (see its expression in SG).

The source functions, S_R and S_S , involve the dynamic properties of the turbulent medium. The expression for S_R is (see SG and Samadi et al. (2005)):

$$S_R(r, \omega_0) = \int_0^\infty dk \int_{-\infty}^{+\infty} d\omega \frac{E^2(k)}{k^2} \chi_k(\omega_0 + \omega) \chi_k(\omega) \quad (4)$$

where k is the wavenumber, $E(k)$ the time averaged kinetic energy spectrum, $\chi_k(\omega)$ is the frequency component of $E(k, \omega)$ (the kinetic energy spectrum as a function of k and ω). A similar expression is derived for S_S (see SG). Note that the function $\chi_k(\omega)$ can be viewed as a measure in the Fourier domain of the time-correlation between eddies. This function is generally referred as 'eddy time-correlation' function.

The derivation of Eqs. (1)-(4) is based on several assumptions and approximations. Among them, the main ones are:

- *quasi-Normal approximation (QNA)*: the fourth-order moments involving the turbulent velocity are decomposed in terms of second-order ones assuming the QNA, i.e. assuming that turbulent quantities are distributed according to a Normal distribution with zero mean. However, as shown recently by Belkacem et al. (2006a), the presence of plumes causes a severe departure from this approximation. Belkacem et al. (2006a) have proposed an improved closure model that takes the asymmetry between plumes and granules as well as the turbulence inside the plumes into account. As shown by Belkacem et al. (2006b), this improved closure model reduces the discrepancy between theoretical calculations and the helioseismic constraints.
- *isotropic, homogeneous, incompressible turbulence*: the turbulent medium is assumed to be isotropic, homogeneous, and incompressible at the length scale associated with the contributing eddies. This assumption is justified for low turbulent Mach number, M_t (see SG). However, when the anisotropy is small, it is possible to apply a correction that takes the departure from isotropy into account. Such corrections have been proposed by SG and CHE in two different ways. However, in both formalisms, the exact domain over which these corrections are valid is unknown and remains to be specified.
- *radial formalism*: radial modes are usually considered. However, generalizations to non-radial modes have been proposed by Dolginov & Muslimov (1984), GMK and Belkacem et al. (2008).
- *passive scalar assumption*: as pointed out above the entropy fluctuations are supposed to behave as a passive scalar (see the discussion).
- *length-scale separation*: eddies contributing to the driving are supposed to have a characteristic length scale smaller than the mode wavelength. This assumption is justified for low M_t (see SG and the discussion).

Eddy time-correlation

Most of the theoretical formulations explicitly or implicitly assume a Gaussian function for $\chi_k(\omega)$ (GK; Dolginov & Muslimov, 1984; GMK; Balmforth, 1992; CHE). However, hydrodynamical 3D simulations of the outer layers of the Sun show that, at the length associated with the energy bearing eddies, χ_k is rather Lorentzian (Samadi et al., 2003). As pointed out by CHE, a Lorentzian χ_k is also a result predicted for the largest, most-energetic eddies by the time-dependent mixing-length formulation of convection by Gough (1977). Therefore, there are some numerical and theoretical evidences that χ_k is rather Lorentzian at the length scale of the energy bearing eddies.

The excitation of the low-frequency modes ($\nu \lesssim 3$ mHz) is mainly due to the large scales. However, the higher the frequency the more important the contribution of the small scales. Solar 3D simulations show that, at small scales, χ_k is neither Lorentzian nor Gaussian (Georgobiani et al., 2006). Then, according to Georgobiani et al. (2006), it is impossible to separate the spatial component $E(k)$ from the temporal component at all scales with the same simple analytical functions. However, such results are obtained using Large Eddy

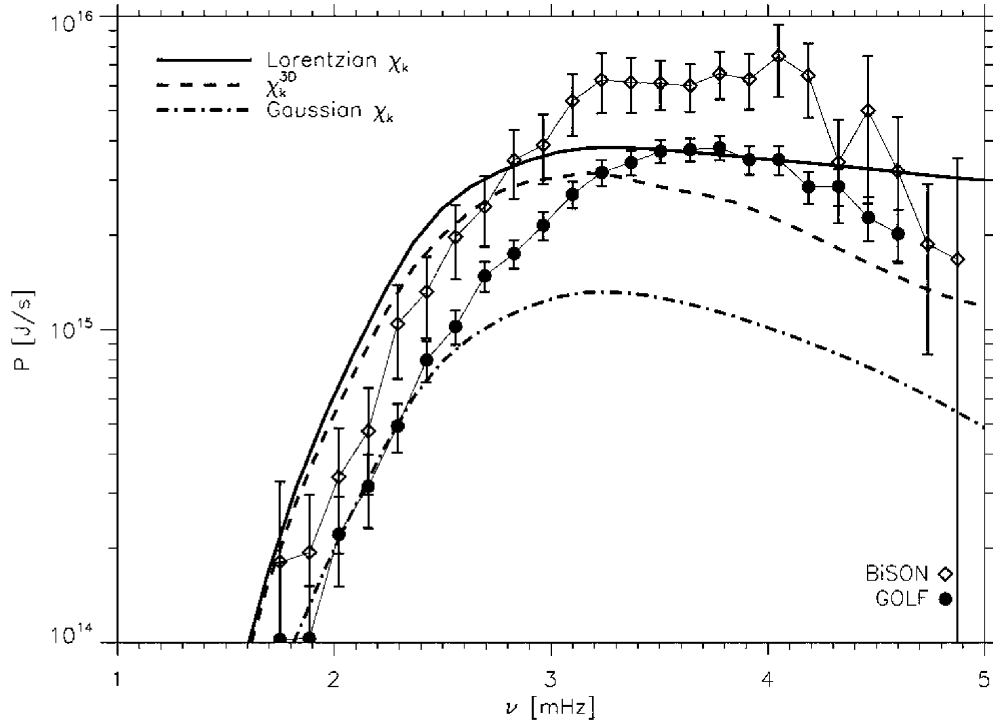


Figure 1: Solar p -mode excitation rates, \mathcal{P} , as a function of $\nu = \omega/2\pi$. The dots correspond to seismic data from SOHO/GOLF (filled circles) and from the BiSON network (diamonds). The thick lines correspond to semi-theoretical calculations based on different choices for χ_k : Lorentzian χ_k (solid line), χ_k^{3D} i.e. χ_k directly derived from the solar 3D simulation (dashed line), and a Gaussian χ_k (dot-dashed line).

Simulation (LES). The way the small scales are treated in LES can affect our description of turbulence. Indeed, He et al. (2002) have shown that LES results in a $\chi_k(\omega)$ that decreases at all resolved scales too rapidly with ω with respect to direct numerical simulations (DNS). Moreover, Jacoutot et al. (2008a) found that computed mode excitation rates significantly depend on the adopted sub-grid model. Furthermore, at a given length scale, Samadi et al. (2007) have shown that χ_k tends toward a Gaussian when the spatial resolution is decreased. As a conclusion the numerical resolution or the sub-grid model can substantially affect our description of the small scales.

As shown by Samadi et al. (2003), calculation of the mode excitation rates based on a Gaussian χ_k results for the Sun in a significant under-estimation of the maximum of \mathcal{P} whereas a better agreement with the observations is found when a Lorentzian χ_k is used (see Fig. 1). A similar conclusion is reached by Samadi et al. (2008a) in the case of the star α Cen A.

Up to now, only analytical functions were assumed for $\chi_k(\omega)$. We have here implemented, for the calculation of \mathcal{P} , the eddy time-correlation function derived *directly* from long time series of 3D simulation realizations with an intermediate horizontal resolution ($\simeq 50$ km). As shown in Fig. 1, the mode excitation rates, \mathcal{P} , obtained from χ_k^{3D} , are found comparable to that obtained assuming a Lorentzian one, except at high frequency. This is obviously the direct consequence of the fact that a Lorentzian χ_k reproduces rather well χ_k^{3D} , except at high frequency where χ_k^{3D} decreases more rapidly than the Lorentzian function. At high frequency, calculations based on a Lorentzian χ_k result in larger \mathcal{P} and reproduce better the helioseismic constraints than those based on χ_k^{3D} . This indicates perhaps that χ_k^{3D} decreases too rapidly

with frequency than it should do. This is consistent with He et al. (2002)'s results who found that LES predicts a too rapid decrease with ω compared to the DNS (see above).

CHE also found that the use of a Gaussian χ_k severely under-estimates the observed solar mode excitation rates. However, in contrast with Samadi et al. (2003), they mention that a Lorentzian χ_k results in an over-estimation for the low-frequency modes. They explain this by the fact that, at a given frequency, a Lorentzian χ_k decreases too slowly with depth compared to a Gaussian χ_k . Consequently, for the low-frequency modes, a substantial fraction of the integrand of Eq. (2) arises from large eddies situated deep in the Sun. This might suggest that, in the deep layers, the eddies that contribute efficiently have rather a Gaussian χ_k (see the discussion below).

Impact of the surface metal abundance

We have computed two 3D hydrodynamical simulations representative – in effective temperature and gravity – of the surface layers of HD 49933, a star which is rather metal poor compared to the Sun. One 3D simulation (hereafter labeled as S0) has a solar metal abundance and one other (hereafter labeled as S1) has a surface iron-to-hydrogen abundance, $[\text{Fe}/\text{H}]$, ten times smaller. For each 3D simulation we match in the manner of Samadi et al. (2008a) an associated global 1D model and we compute the associated acoustic modes.

The rates \mathcal{P} at which energy is supplied into the acoustic modes associated with S1 are found about three times smaller than those associated with S0. This difference is related to the fact that a low surface metallicity implies surface layers with a higher mean surface density. In turn, higher mean surface density implies smaller convective velocity and hence a less efficient driving of the acoustic modes (for details see Samadi et al., 2008b). This result illustrates the importance of taking the metallicity of the star into account when computing \mathcal{P} . This conclusion is qualitatively consistent with that by Houdek et al. (1999) who – on the basis of a mixing-length approach – also found that the mode amplitudes decrease with decreasing metal abundance.

Discussion and perspectives

The way mode excitation by turbulent convection is modeled is still very simplified. Some approximations must be improved, some assumptions or hypothesis must be avoided:

- *length-scale separation*: This approximation is less valid in the super-adiabatic region where the turbulent Mach number is no longer small (for the Sun M_t is up to 0.3). This spatial separation can however be avoided if the kinetic energy spectrum associated with the turbulent elements ($E(k)$) is properly coupled with the spatial dependence of the modes
- *eddy time-correlation function, χ_k* : Current models assume that χ_k varies with ω in the same way at any length scales and in any parts of the convective zone (CZ). At the length scale of the energy bearing eddy and in the uppermost part of the CZ, there are some strong evidences that χ_k is Lorentzian rather than Gaussian. However, as discussed here, it is not yet clear what is the correct description for χ_k at the small scales and also deep in the CZ. Use of more realistic 3D simulations would be very helpful to depict the correct dynamical behavior of the small scales as well as in the deep CZ.
- *passive scalar assumption*: This is a *strong hypothesis* that probably is no longer valid in the super-adiabatic part of the convective zone where the driving by the entropy is important. Indeed, the super-adiabatic layer is the seat of important radiative losses

by the eddies. Assuming that the entropy behaves as a passive scalar is not correct. To avoid this assumption, one needs to include eddy radiative losses in the model of stochastic excitation.

Finally, we stress that some solar-like pulsators are young stars that show a strong activity (e.g. HD 49933) which is often linked to the presence of strong magnetic fields. A strong magnetic field can inhibit convection (see e.g. Proctor & Weiss, 1982; Vögler et al., 2005). Furthermore, using 3D solar simulations, Jacoutot et al. (2008b) have studied the influence of magnetic fields of various strength on the convective cells and on the excitation mechanism. They found that a strong magnetic field results in turbulent motions of smaller scales and higher frequencies than in the absence of magnetic field, and consequently in a less efficient mode driving. Further theoretical developments are required to take the effects of a magnetic field into account in the theoretical calculation of the mode excitation rates.

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DISCUSSION

Houdek: The scaling law $(L/M)^{1.5}$ in Houdek et al. (1999) refers to amplitudes at the photosphere. At the height of $h = 200$ km, we obtained the $(L/M)^{1.1}$ law.

Bruntt: You find that low metallicity implies low pulsation amplitudes for solar-like oscillations. Have you tried to include metallicity as a parameter in your scaling relation of amplitudes?

Samadi: The scaling law $(L/M)^{0.7}$ proposed in Samadi et al. (2007) was indeed derived using stellar 3D simulations with solar metal abundances. Part of the remaining discrepancies between the scaling law $(L/M)^{0.7}$ and the observations may be explained by the fact that some stars have a metal abundance significantly different to that of the Sun. This is particularly so for HD 49933.

Noels: You talked about “solar” abundances. What abundances and metallicity are you referring to?

Samadi: We have considered both the “old” and the “new” solar abundances. At fixed $[\text{Fe}/\text{H}]$, the “new” solar abundances result in a lower total metal abundance than the “old” ones and hence in lower \mathcal{P} . In the case of HD 49933, mode excitation rates, \mathcal{P} , computed by assuming the “new” solar abundances are found ~ 30 % smaller than those computed assuming the “old” solar abundances (see Samadi et al., 2008b).

Noels: Could we have some solar-like oscillations in stars as massive as β Cepheid stars if a convection zone appears near the surface due to an accumulation of iron ?

Samadi: As far as we have a surface convective envelope, it is potentially possible to excite p modes. However, the excitation is efficient when the characteristic eddy turn-over time is of the same order than the period of the p modes confined near the surface.



Günter Houdek enjoying the boat ride