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Signature of main sequence internal structure in post-main sequence stars

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Abstract

Post Main Sequence (post MS) stars keep a trace of their past main sequence history. Typically, the presence or not of an Intermediate Convective Zone (ICZ) above the H-burning shell of massive stars critically depends on the details of the Main Sequence (MS) phase modeling (convection criterion, mass loss, ...). We show here how the excitation of g-modes in blue supergiant stars is closely related to this ICZ, allowing to constrain different associated physical processes.

Introduction

Based on Hipparcos data, about 30 periodically variable supergiant stars have been detected and analysed (Waelkens et al. 1998; Aerts et al. 1999; Mathias et al. 2001; Lefever et al. 2007). From observations by the MOST satellite, Saio et al. (2006) (S06) also reported the discovery of p and g-mode pulsations (about 48 frequencies) in the B supergiant star HD 163899. The presence of g-mode pulsations in a supergiant star is quite challenging but a theoretical explanation has been first proposed by S06. After a review of some important characteristics of the deep interior of these stars, we show how such seismic observations allow to constrain these regions.

Internal structure of blue supergiants and relation with MS phase

B supergiants are post MS massive stars that have a hydrogen burning shell and have (or not have) begun core helium burning. This contracting high density core is surrounded by an expanding diluted envelope. The characteristics of the deep regions around the H-burning shell strongly depend on the details of the MS phase. In particular, depending on this past history, an ICZ can be present just above the H-burning shell. During the MS phase of massive stars, the radiative and the adiabatic temperature gradients decrease in the central regions: the first because of the decreasing opacity due to decreasing X, and the second because of the increasing radiative pressure. As a consequence, the convective core either grows or slowly receeds and a region where $\nabla_{\rm rad} \sim \nabla_{\rm ad}$ can develop above it. Afterwards during the post MS phase, hydrogen burning takes place in a shell around the core; so L/m increases and thus $\nabla_{\rm rad}$ increases. As a result, the region in which the temperature gradients are close to each other easily becomes convective since it just needs a slight increase in the radiative temperature gradient, and an ICZ is formed.

However, the appearance of this ICZ critically depends on other physical processes. If a large enough mass loss rate is taken into account during the MS phase, the central temperature increases less quickly. Hence, the adiabatic gradient no longer decreases significantly, the convective core receeds more quickly and does not leave behind a region where $\nabla_{\rm rad} \sim \nabla_{\rm ad}$ (Chiosi & Maeder 1986). Therefore, with significant mass loss, no ICZ appears during the post MS phase (Godart et al. 2009). Overshooting during MS can also prevent the formation of an ICZ during the post MS phase. Finally, depending on the criterion used to define the boundaries of the convective zones: Schwarzschild $\nabla_{\rm rad} = \nabla_{\rm ad}$ or Ledoux $\nabla_{\rm rad} = \nabla_{\rm ad} + \beta/(4-3\beta)\nabla_\mu$, ICZ of very different kinds can appear (Lebreton et al. 2009).

Non-radial modes in blue supergiants

The structure of a supergiant star is characterized by a high density contrast between the small size contracting core and the low density expanding envelope. This particular structure has important consequences on the physics of non-radial oscillations. In the deep very dense radiative regions, the Brunt-Vaïsälä (BV) and Lamb frequencies, respectively N and L_ℓ take huge values compared to those reached in the envelope. The direct consequence is that modes of different types (p- and g-modes) in the envelope have all a g-mode character in the deep interior where $\sigma^2 < N^2$, L_ℓ^2.

A first consequence of the large buoyancy restoring force in the core is that the eigenfunctions present a huge number of nodes there:
$n \approx \sqrt{\ell(\ell+1)} \int N/r\, dr\; P\,/\,(2\pi^2)$, where P is the pulsation period. For a β Cep type mode with period around the fundamental radial one, $n \approx 100$ and for a typical SPB type mode with larger period, $n \approx 1000$! This implies that the frequency spectrum is extremely dense, the number of non-radial modes being multiplied by about 50 compared to what is obtained in an envelope model without the core. This is the first bad news since it seems that there is no hope to identify the individual non-radial modes and use their frequencies to constrain the internal physics.

The second bad news concerns the energetic aspects of the oscillations. In a radiative g-mode cavity with short wavelength oscillations, radiative damping always occurs. Later we will give a simple expression for the mechanical energy lost per length unit by the mode during each pulsation cycle, the dominating term is of the order of:

$$\mathrm{d}W_{\rm rad}/\mathrm{d}r \approx \frac{L}{\sigma\,\mathrm{d}\ln T/\mathrm{d}r}\,\frac{\delta T}{T}\,\frac{\mathrm{d}^2(\delta T/T)}{\mathrm{d}r^2} \;=\; \ell(\ell+1)\,\frac{N^2}{\sigma^3}\,\frac{T^4}{\kappa\rho}\,\left|\frac{\delta T}{T}\right|^2\,\frac{16\pi ac}{3}. \qquad (1)$$

This radiative damping of non-radial modes is thus very large due to the huge values of N^2 in the core, and it seems that there is no hope to observe them in post MS stars.

However, in the reasoning above we have assumed that the eigenfunction modulus $|\delta T/T|^2$ is not negligible, and this is not always the case due to the presence of an ICZ. In a convection zone $N^2 < 0$ and g-modes are evanescent (left panel of Fig. 1). Therefore, the ICZ can act as a potential barrier: some modes can cross it, others are reflected. This is illustrated in Fig. 1 (right panel). For the mode that crosses the convective barrier (grey), the amplitudes of short wavelength oscillations are significant in the radiative core and strong radiative damping ensues (bottom left panel of Fig. 2): this mode cannot be observed. But for another mode with close frequency, the amplitudes are small in the radiative core (black in the right panel of Fig. 1). Hence, the radiative damping remains small compared to the κ-mechanism occurring in the iron opacity bump near the surface, as shown in top left panel of Fig 2; this mode is unstable and could be observed. So this is a first good news: in the dense spectrum of non-radial modes, some reflect on the convective shell and can be excited and observed. It is useful to note that, if only reflected modes are considered, the mode propagation cavity is similar to a MS B star: because of the reflection, the radiative core can be somehow forgotten

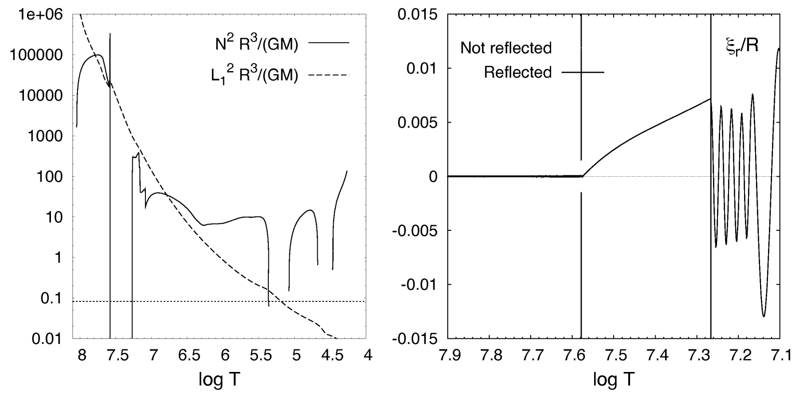

Figure 1: Left: Propagation diagram showing the dimensionless squared BV and Lamb frequencies, in a model with an ICZ and $M = 13M_\odot$, $T_{\rm eff} = 18325\,K$, $\log(L/L_\odot) = 4.5$ and $\log g = 3.06$. Right: Radial displacement ξ_r/R for a reflected and a non reflected $\ell = 1$ mode with dimensionless frequency $\omega = \sigma\sqrt{R^3/GM} = 0.29$.

and the bottom of the cavity is a convective region. As a consequence, the frequencies of reflected modes behave as they do in a MS star. This is illustrated in the right panel of Fig. 2 where we give the typical evolution of the reflected $\ell = 1$ frequencies during the post MS phase of $13M_\odot$ models. So this is the second good news: when restricting to the reflected modes, the frequency pattern is sparse enough and asteroseismology of blue supergiants becomes possible. The third good news is of course that such non-radial modes are observed, as mentioned above.

Physical constraints

As seismic interpretation of non-radial oscillations in Blue supergiants is possible, this opens the way to a probe of their very deep layers and the physical processes affecting them. As we have seen, the key point is the presence of an ICZ, so we can hope to constrain the physical processes related to its presence. First, we have mentioned above that significant mass loss can prevent the appearing of such ICZ. Godart et al. (2009) show how the bottom boundary of the instability strip is displaced towards higher luminosities (lower gravities) when significant mass loss is included. The observed instability strip of Slowly Pulsating B supergiants (SPBsg) gives thus an upper limit on the mass loss rate in this mass range. Overshooting can also prevent the formation of an ICZ during the post MS phase and extends the MS phase. Concerning for example the SPBsg HD 163899, we could imagine either that it is a MS star with large overshooting, or that it could be a post MS star with moderate overshooting during this phase. Godart et al. (2009) show that none of these two possibilities agree with the seismic observations; this gives limits on overshooting during MS and post MS phase.

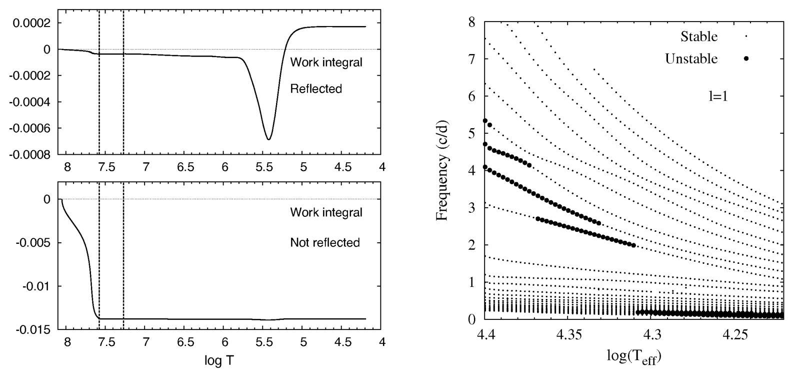

Figure 2: Left: Work integral for a reflected mode (top) and nonreflected mode (bottom), same modes and model as in Fig. 1. Right: Evolution of the frequencies of $\ell = 1$ reflected modes as a function of $\log T_{\rm eff}$, for models with $M = 13 M_{\odot}$.

Numerical recipe

It is not easy to solve the non-radial oscillation equations in post MS stars accurately and therefore it is useful to mention numerical recipes adequate in this context. First, it is possible to avoid the computation of the full dense spectrum of non-radial modes by preselecting the modes that are reflected at the bottom of the ICZ (and potentially excited). This is achieved by imposing a rigid boundary condition at the base of the ICZ: $\xi_r = 0$. This condition automatically leads to small amplitudes of all eigenfunctions in the radiative core, as we have verified by numerous full numerical computations. Moreover, we can justify this choice by considering the simple case of two cavities with constant wavenumbers k_1 and k_2 separated by an evanescent region of size h and zero wavenumber. Analytical solutions are easily obtained in this case, showing that in the limiting case where $k_1 >> h^{-1}$, the smallest amplitudes in cavity 1 are obtained for frequencies corresponding to a rigid boundary at the base of the evanescent region. Artificially imposing a boundary condition inside the star does not allow to get the real frequencies of the full star. But as the frequency spectrum of the full star is very dense, there always exists a mode with frequency close to the one corresponding to rigid reflexion and the error introduced by this approach is very small.

Once the non-adiabatic pulsation equations are solved in the envelope, we cannot completely disregard the radiative core because reflexion is only partial. In the radiative core, we have seen that the eigenfunctions have a huge number of nodes (up to $n \approx 1000$!). This makes the task of solving accurately the differential problem very difficult in this region. But at the same time, two approximations are valid with high accuracy in this region. First, the internal energy of these deep layers is very high, which justifies the so-called quasi-adiabatic approximation. This approximation can be presented in different ways (Dziembowski 1977, Van Hoolst et al. 1998, Unno et al. 1989), but leading always to the same final result. We can also present it as follows. First, the adiabatic problem is solved to get the eigenfunctions ξ_r, δP, ... Next, these adiabatic eigenfunctions are used to determine δL from the perturbed diffusion equation:

$$\frac{\delta L}{L} = 4\,\frac{\xi_r}{r} + 3\,\frac{\delta T}{T} - \frac{\delta \kappa}{\kappa} - \ell(\ell+1)\,\frac{\xi_h}{r} + \frac{\mathrm{d}\,\delta T/\mathrm{d}r}{\mathrm{d}T/\mathrm{d}r}\,. \quad (2)$$

Finally, with all these ingredients the work integral can be computed and divided by the inertia to get the damping rate of the modes, which gives in a pure radiative zone without nuclear reactions:

$$\eta = \frac{\int_0^M \frac{\delta T}{T}\left(\frac{\partial \delta L}{\partial m} - \frac{\ell(\ell+1)\,L}{4\pi\rho r^3}\frac{T'}{\mathrm{d}T/\mathrm{d}\ln r}\right)\mathrm{d}m}{2\,\sigma^2\int_0^M |\vec{\xi}|^2\,\mathrm{d}m}, \tag{3}$$

where σ (resp. η) are the real (resp. imaginary) parts of the angular frequency (time-dependence: $\exp(i\sigma t - \eta t)$). The quasi-adiabatic treatment is used for the integration in the radiative core ($r < r_0$) and the full non-adiabatic eigenfunctions are used for the envelope ($r \geq r_0$). Moreover, another important simplification is to use the asymptotic theory, which applies perfectly in the radiative core where the wavelength of the eigenfunctions is by far smaller than the scale heights of different equilibrium quantities. A full non-adiabatic asymptotic treatment was derived by Dziembowski (1977). Here we use instead the standard adiabatic asymptotic theory, which gives the following expressions for the radial (ξ_r) and transversal (ξ_h) components of the displacement far from the edges of a g-mode cavity ($\sigma^2 << L_\ell^2,\ N^2$):

$$\frac{\xi_r}{r} = K\,\frac{[\ell(\ell+1)]^{1/4}}{\sqrt{\sigma\, r^5\,\rho\, N}}\,\sin\left[\int_{\mathrm{r_0}}^{\mathrm{r}} \mathrm{k_r\,dr}\right] \tag{4}$$

$$\frac{\xi_h}{r} = \frac{P'}{\sigma^2 r^2 \rho} = \frac{K}{[\ell(\ell+1)]^{1/4}}\sqrt{\frac{N}{\sigma^3 r^5 \rho}}\,\cos\left[\int_{\mathrm{r_0}}^{\mathrm{r}} \mathrm{k_r\,dr}\right], \tag{5}$$

where the local radial wavenumber is given by $k_r = \sqrt{\ell(\ell+1)}\,N\,/\,(\sigma\, r)$. The K constant is obtained by applying the continuity of δP at the bottom of the ICZ (r_0), which is equivalent to the continuity of P' because of our rigid boundary condition. We note that in this asymptotic limit, we also have $|\delta P/P| >> |P'/P|$ so that $\delta P/P \simeq (d\ln P/dr)\,\xi_r$ and near incompressibility: $|\delta\rho/\rho| << \ell(\ell+1)\,|\xi_h/r|$, so that $d\xi_r/dr \simeq \ell(\ell+1)\,\xi_h/r$.

Substituting Eqs. 4 and 5 in Eq. 2 and keeping only the dominating terms in the asymptotic limit gives:

$$\frac{\delta L}{L} \simeq \frac{\mathrm{d}\,(\delta T/T)}{\mathrm{d}\ln T} - \ell(\ell+1)\,\frac{\xi_h}{r} \simeq \ell(\ell+1)\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - 1\right)\frac{\xi_h}{r}\,. \tag{6}$$

We obtain then from the equation of energy conservation and neglecting the transversal component of the divergence of the flux:

$$\begin{aligned} i\sigma T\delta s &\simeq -\frac{d\delta L}{dm} \\ &\simeq K\,\frac{[\ell(\ell+1)]^{5/4}}{4\pi}\,L\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - 1\right)\sqrt{\frac{N^3}{\rho^3\sigma^5 r^{11}}}\,\sin\left[\int_{\mathrm{r_0}}^{\mathrm{r}} \mathrm{k_r\,dr}\right] \\ &\simeq \frac{\ell(\ell+1)}{4\pi}\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - 1\right)\frac{N^2\,L}{\sigma^2 r^3\rho}\,\frac{\xi_r}{r}\,. \end{aligned} \tag{7}$$

The contribution of the radiative core to the numerator of Eq. (3) is thus simply given by:

$$\begin{aligned} \int_0^{m_0} \frac{\delta T}{T}\frac{\partial\delta L}{\partial m}\,\mathrm{d}m &\simeq \frac{\ell(\ell+1)}{\sigma^2}\int_0^{r_0}\frac{\rho g}{P}\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - 1\right)\nabla_{\mathrm{ad}} N^2 L\left(\frac{\xi_r}{r}\right)^2 \mathrm{d}r \\ &\simeq \frac{K^2\,[\ell(\ell+1)]^{3/2}}{2\sigma^3}\int_0^{r_0}\left(\frac{\nabla_{\mathrm{ad}}}{\nabla} - 1\right)\frac{\nabla_{\mathrm{ad}}\,N\,g\,L}{P\,r^5}\,\mathrm{d}r\,, \end{aligned} \tag{8}$$

and for the denominator:

$$2\sigma^2 \int_0^{m_0} |\vec{\xi}|^2 \, \mathrm{d}m \;\simeq\; 4\pi\, K^2 \int_0^{r_0} k_r \, \mathrm{d}r \,. \qquad (9)$$

These equations are perfectly compatible with those given in Dziembowski et al. (2001). Finally, it is important to emphasize that using Lagrangian or Eulerian formalisms can lead to different numerical results. In our first computations, we used a Lagrangian formalism, but it did not lead to the appropriate evanescent behaviour of the eigenfunctions in the ICZ; instead they showed a large wavelength oscillation. This comes from the fact that no control of the BV frequency is possible in a Lagrangian formalism. Because of numerical truncation errors, the finite difference scheme does not know the real value of N^2, nor its sign, when it is slightly negative like it is in an ICZ. With an Eulerian formalism instead, N^2 appears explicitely in the movement equation, and the correct value can be attributed to it. As was already pointed out by Dziembowski years ago, we thus emphasize that it is much better to use an Eulerian formalism for the finite difference scheme inside a deep convection zone.

Conclusions

Significant radiative damping affects most non-radial modes of the very dense spectrum of B supergiants. But if an ICZ is present above the H-burning shell, some modes can reflect on it and are not damped. Asteroseismology of B supergiants is thus possible, allowing to constrain different physical aspects of the MS and post MS history of these stars (mass loss, convection, ...).

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DISCUSSION

Dziembowski: While interpreting data on supergiant pulsators one should take into account the fact that the post-MS phase is very short in comparison with the core helium burning phase.

Dupret: I agree. The question is whether these stars are helium burning or not, as most of them have too small log g values to be on the main sequence. Concerning the possible excitation of g-modes, it depends on the presence or not of an intermediate convective zone but not so much on a possible He-burning convective core as it is too deep to change significantly the radiative damping.

Jerzykiewicz: If the MOST SPB variable is really a post-MS star, the periods will increase so fast as the star evolves through the Hertzsprung gap that the effect should be observable in a few years.

Dupret: That is a good idea, but I fear it would be difficult from an observational point of view. The spectrum of g-modes is dense and we have to be sure that we follow the time variation of the frequency of the *same* mode, and with enough precision.