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Theoretical aspects of g-mode pulsations in γ Doradus stars

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Abstract

γ Dor stars are main sequence variable A-F stars whose long periods (between 0.35 and 3 days) correspond to high-order gravity mode pulsation. First, we present some aspects of their internal physics and evolutionary status. Second, we consider the potential of the g modes as a probe of these internal physics. In particular, we consider the effect of sharp features present near the convective core top on the g-mode period pattern. Third, we analyse the driving mechanism of the γ Dor g modes, we stress the role of Time-Dependent Convection (TDC) and for the first time we study the role played by turbulent viscosity variations in this frame. Finally, we consider the important problem of mode identification. We show that the theoretical multi-colour photometric amplitude ratios and the phase differences between the light and velocity curves predicted by TDC models much better agree with observations than Frozen Convection (FC) models. Hence, a more secure photometric mode identification is possible with TDC models.

Internal physics and evolutionary status of γ Dor stars

As detailed by Kaye (2007), γ Dor stars are intermediate-mass main-sequence stars pulsating in high-order gravity modes. For $Z = 0.02$, their masses range typically from 1.5 to 1.7 $M_\odot$. They are located in a narrow region at the red side of the δ Sct instability strip. In this particular region of the HR diagram, the thin convective zones associated with the partial ionization of He and H begin to merge and form a single larger convective envelope (CE). As we are going to show, this is important for the understanding of their driving mechanism. The exact extension of this CE is subject to theoretical uncertainties and depends on the treatment of convection adopted. We refer to the paper of Montalbán et al. (2007) for more details about this aspect.

Concerning the central regions, in the γ Dor mass domain, the main energy source changes from PP-chain nuclear reactions to CNO cycle ones, and because of the high sensitivity to the temperature of the latter, a convective core (CC) appears. The evolution of this CC depends on the mass of the star, as can be seen in Fig. 1, left panel. For higher masses it shrinks, while for lower masses it grows. In simple models, a growing CC is expected to create a discontinuity of chemical composition at its upper boundary. Hence, in a thin region above it, the radiative gradient can be again larger than the adiabatic gradient, which could lead to partial mixing. This phenomenon called semi-convection (Gabriel & Noels 1977; Crowe & Matalas 1982) is still a matter of debate.

In the two cases of a shrinking and growing CC, the determination of its exact extension is subject to large uncertainties (as for the CE). There is for example the well known overshooting parameter widely used in stellar evolution codes and which just reflects our lack of knowledge at this level.

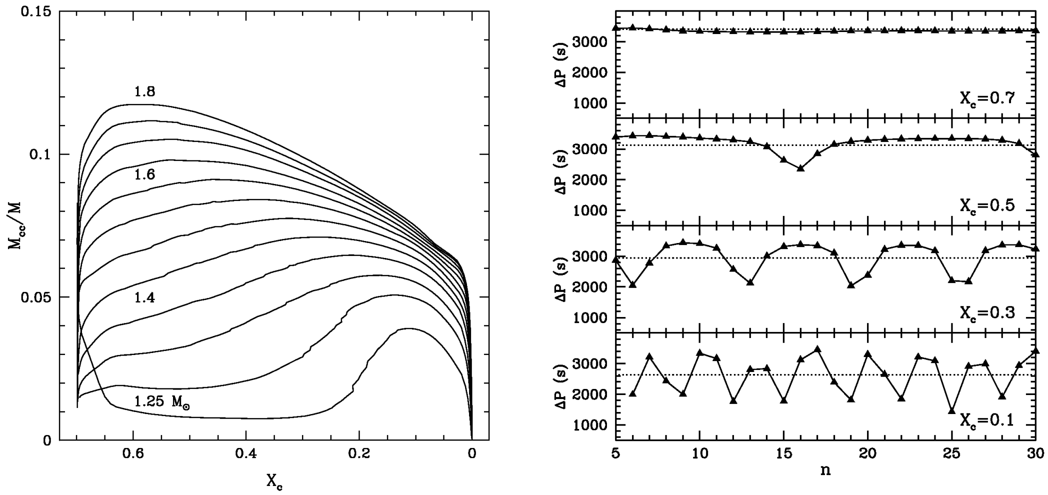

Figure 1: Left: Mass fraction of the convective core (M_{cc}/M) during the main sequence for models in a mass range 1.25-1.8 $M_\odot$. Each line describes the evolution of a model of a given mass. Right: Period spacing ΔP_n as a function of the radial order n of $\ell = 1$ g modes in 1.6 $M_\odot$ models with decreasing central hydrogen abundance. A constant period spacing, on which are superposed periodic components, describes the spectrum of g modes. Dotted lines represent the constant period spacing predicted by Eq. 1.

g-mode periods as a probe of the deep layers of γ Dor stars

The gravity modes of γ Dor stars have the highest inertia in the very deep layers near the top of the CC. Hence, they give a unique opportunity to probe these poorly known deep regions.

As shown by Tassoul (1980), in the asymptotic regime and if the effect of rotation is neglected, the periods of high-order gravity modes are approximately given by:

$$P_{n\ell} = \Pi_0 \, \frac{2\pi^2(n+1/2)}{\sqrt{\ell(\ell+1)}} \tag{1}$$

where ${\Pi_0}^{-1} = \int_{r_a}^{r_b} N/r \, \mathrm{d}r$ is the integral of the Brunt-Väisälä frequency N from the base to the top of the g-mode cavity (typically the radiative region between the CC and CE, for high-order g modes), we call it the buoyancy radius. Similarly to the dynamical time in the case of p modes, the buoyancy radius is the first quantity that can be deduced from the g-mode periods. It is closely related to the size of the CC and its determination allows to constrain it.

A method called the Frequency Ratio Method (FRM) based on this asymptotic relation was recently proposed by Moya et al. (2005). For any couple of modes with same ℓ, we have $\sigma_{n_1}/\sigma_{n_2} = (n_2+1/2)/(n_1+1/2)$. Different combinations of possible n can be determined by this way, and finally the constraints given by the buoyancy radius can be used to restrict the number of possible models for the star. The FRM has been applied to different γ Dor stars: HD 12901 (Moya et al. 2005), HD 218427 (Rodríguez et al. 2006a), HD 239276 (Rodríguez et al. 2006b) and 9 Aurigae (Moya et al. 2006). In this latter case, the FRM was part of a full consistent scheme including photometric mode identification and stability analysis based on TDC models (see next sections). This study allowed to constrain simultaneously the deep and superficial layers of 9 Aur. However, the FRM not always gives conclusive results, it has been applied only to stars with very few modes (3) and does not take into account the significant effect of rotation on the periods (see next section).

As shown for white dwarfs (e.g. Brassard et al. 1992), sharp variations in N affect the period spacing of g modes ($\Delta P_n = P_{n+1} - P_n$). We recall that the Brunt-Väisälä frequency can be written as:

$$N^2 = \frac{\rho g^2 \delta}{P}(\nabla_{\rm ad} - \nabla + \frac{\varphi}{\delta}\nabla_\mu), \quad (2)$$

where $\nabla_\mu = {\rm d}\ln\mu/{\rm d}\ln P$, $\delta = -\partial\ln\rho/\partial\ln T|_P$, $\varphi = \partial\ln\rho/\partial\ln\mu|_{P,T}$.

We see that sharp variations of N can come from the behaviour of the superadiabatic gradient ($\nabla - \nabla_{\rm ad}$) and from the mean molecular weight gradient ∇_μ. In the case of main-sequence models with a convective core, sharp variations of N are built near the top of the CC by the combined action of convective mixing, nuclear burning and by the displacement of the CC border. The signatures of such sharp variations in the g-mode period pattern are presented in the right panel of Fig. 1 in the case of 1.6 $M_\odot$ main-sequence models. ΔP_n can be described as a superposition of the uniform period spacing predicted by Eq. 1, expected for a model without sharp variations in N, and a periodic component resulting from the sharp variation of N near the stellar core. In analogy with the case of white dwarfs, it can be shown (see Miglio 2006) that the periodicity and amplitude of such a periodic component can be related, respectively, to the location and sharpness of the variation in N. As an example we show in Fig. 1 that, as a star evolves on the main sequence and the edge of the convective core is displaced, the periodicity of the components in ΔP_n changes. These periodic components represent very sensitive probes of the location and sharpness of the chemical composition gradient in the core of γ Dor stars, nonetheless, whether these signatures could be detected and correctly interpreted given the effects of rotation on g-mode periods (see next section) needs further investigations. In present ground based observations, only a few modes are observed and it is clearly not possible to detect such signatures. Theoretical models predict however all the modes to be excited in a determined range of periods (see next sections). With future space observations for example with COROT, many more modes should be detected at lower amplitude, and we could observe maybe signatures of non-equidistant spacing.

Rotation-oscillation interactions

Rotation can affect significantly stellar oscillations in two ways. First, when the centrifugal acceleration is not negligible compared to gravity, the spherical symmetry is broken. Second, when the pulsation periods are of the same order as the rotation period, the Coriolis acceleration plays a major role in the movement pulsation equation. The latter case occurs typically in γ Dor stars because of their long periods. Hence, for a correct modelling of these stars we would have to include the terms associated with rotation in the pulsation equations.

Perturbative and non-perturbative approaches can be followed in modelling the rotation-oscillation interactions. The non-perturbative ones are much more appropriate in the case of γ Dor stars. Non-perturbative theories have been derived by Lee & Saio (1987) (LS), Dintrans & Rieutord (2000) (DR), Lignières et al. (2006) and Reese et al. (2006) (RL). DR applied their theory to a typical γ Dor model, showing that the second order perturbative theory reaches its limits for $\sigma_{\rm rot}/\sigma_{\rm pul} \simeq 0.1$.

Whatever the treatment adopted, it is evident that rotation affects strongly the pulsation frequency pattern of γ Dor stars. The rotational splittings are larger than the frequency spacing of consecutive modes ($|\sigma_{l,n,m+1} - \sigma_{l,n,m}| > |\sigma_{l,n+1,m} - \sigma_{l,n,m}|$) and they are not equidistant at all ($\sigma_{l,n,m+1} - \sigma_{l,n,m} \neq \sigma_{l,n,m} - \sigma_{l,n,m-1}$), which makes it impossible to detect them without mode identification based on other observables (see last section). Moreover, if we consider the evolution of the theoretical frequencies as a function of the rotation frequency, a lot of avoided crossings occur between consecutive modes, which complicates a lot the pulsation frequency pattern. For these reasons, we must not be too optimistic when trying to interpret the observed frequencies with simple models.

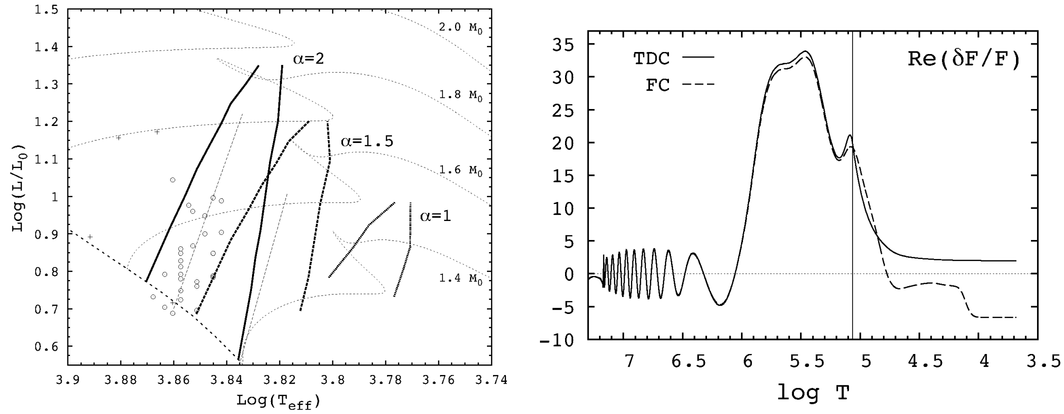

Figure 2: Left: γ Dor theoretical IS for $\ell = 1$ modes, for three families of models with different values of α: 1, 1.5 and 2 obtained with TDC treatment (Dupret et al. 2005a; thick lines), compared to the FC results of Warner et al. (2003) (thin dashed lines, $\alpha = 1.87$). The small circles correspond to observations. Right: $\Re\{\delta F/F\}$ (relative variation of the radial component of the total flux) as a function of $\log T$, obtained with TDC and FC treatments, for the mode $\ell = 1$, g_{22} ($f = 1.192$ c/d). Model with $M = 1.55\,M_\odot$, $T_{\rm eff} = 7020$ K, $\log(L/L_\odot) = 0.872$, $\alpha = 2$.

Driving mechanism of the g modes

To understand the driving mechanism of the γ Dor gravity modes, we have to consider more closely the transition region where the pulsation periods are of the same order as the thermal relaxation time. The important point is that, for γ Dor stars, this transition region is near the bottom of the Convective Envelope (CE). This lead Guzik et al. (2000) (G00) to explain the driving of the γ Dor g modes as follows. The radiative luminosity drops suddenly at the bottom of the convection zone. Therefore, at the hot phase of pulsation, the increasing energy coming from below the convection zone cannot be transported by radiation inside it. If we admit that the convective flux does not adapt immediately to the changes due to oscillations, the energy is thus periodically blocked and transformed in mechanical work like in a heat engine, leading to the oscillations.

G00 used a Frozen Convection (FC) treatment in their non-adiabatic modelling; but this approximation is not justified in most of the convection envelope. We have implemented in our linear non-radial non-adiabatic pulsation code the more realistic Time-Dependent Convection (TDC) treatment of Gabriel (1996) and Grigahcène et al. (2005). This treatment takes the time-variations of the convective flux ($\delta\vec{F}_{\rm c}$), the turbulent pressure ($\delta p_{\rm t}$) and the dissipation rate of turbulent kinetic energy ($\delta\epsilon_2$) into account. The results presented in Dupret et al. (2004, 2005a) show that these TDC terms do not affect much the driving of the g modes; this supports the driving mechanism proposed by G00.

We show in the left panel of Fig. 2 the theoretical instability strips for the γ Dor g modes obtained by Dupret et al. (2004, 2005a) with TDC treatment and by Warner et al. (2003) with FC treatment. A good agreement with the observed instability strip can be obtained for $\alpha \simeq 2$ (near the solar calibrated value). The theoretical instability strip is displaced towards lower effective temperatures when we decrease α, simply because the size of the convective envelope (key point for the driving) is directly related to α.

The TDC formalism of Gabriel allows also the determination of the variations of the non-diagonal components of the Reynolds stress tensor (turbulent viscosity) (Gabriel 1987). We have recently implemented for the first time the corresponding terms and equations in a non-adiabatic pulsation code. Preliminary results indicate that they can play a significant role in the driving and damping of high order g modes. However, the equations become singular at the convective envelope boundaries when these terms are included, which leads to serious numerical problems. Further work is required to solve them.

The stabilization of the high-order g modes at the red side of the γ Dor instability strip is explained by a radiative damping mechanism occurring in the g-mode cavity. We refer to Dupret et al. (2005a) for more details about this damping mechanism. We note also that TDC models predict the existence of stars having simultaneously unstable high-order gravity modes of γ Dor type and unstable low-order p-g modes of δ Sct type. The detection of stars with such hybrid behaviour would present a very high interest for asteroseismology: their high order g modes would enable us to probe the very deep layers of the star and their low-order p-g modes would enable us to probe the intermediate and superficial layers. Much observational effort has been performed to detect such hybrid stars and two have been discovered: HD 209295 (Handler et al. 2002) and HD 8801 (Henry & Fekel 2005). We refer to Grigahcène et al. (2006) for more details about this aspect.

Mode identification

A crucial problem in asteroseismology is mode identification. This problem is particularly difficult for γ Dor stars, because of the combined effect of rotation and convection on the frequencies, the amplitudes, the phases and the surface geometry of the modes. As shown by Mathias et al. (2004), many γ Dor stars show line-profile variations. Hence, spectroscopic mode identification can often be performed for these stars (Balona et al. 1996; Aerts & Krisciunas 1996). Photometric mode identification methods, which are based on the analysis of the amplitude ratios and phase differences between different photometric passbands, can be also applied in γ Dor stars. These latter observables are particularly interesting from a theoretical point of view, because they are very sensitive to the non-adiabatic treatment of convection-pulsation interaction. Hence, comparison with observations enables us to constrain this treatment. We restrict the discussion to this last case.

An important result shown by Dupret et al. (2005b) is that TDC and FC non-adiabatic treatments give completely different predictions for the phase difference ψ_{T} between the flux variation and the displacement. The interpretation of these very different results can be deduced from the right panel of Fig. 2, where we give $\Re(\delta F/F)$, as obtained with TDC and FC treatment. We first note the drop of $\Re(\delta F/F)$ near the base of the CE ($\log T \simeq 5$, vertical line) present in both TDC and FC results. This corresponds to the flux blocking mechanism discussed above. In the FC case, the κ-mechanism occurs inside the convective envelope, in the partial ionization zones of He II ($\log T \simeq 4.8$) and H ($\log T \simeq 4.1$). These κ-mechanisms imply additional decreases of $\Re(\delta F/F)$ down to negative values, which explains the phase lags around 180° predicted by the FC models with high α. In contrast, these κ-mechanisms inside the CE are not allowed by TDC models, because they would lead to too high superadiabatic gradients. Therefore, $\delta F/F$ remains flat and positive after the flux blocking drop and its phase remains near 0°. In Dupret et al. (2005b), the application to specific γ Dor stars is considered. These authors show that TDC results much better agree with the observed photometric amplitude ratios between different passbands, allowing a better identification of the degree ℓ of the modes.

Finally, we stress that the photometric amplitude ratios and phase differences and the line-profile variations are expected to be strongly affected by rotation. Following the approach of Lee & Saio (1987), Townsend (1997, 2003) determined the effect of rotation on these observables which are widely used for mode identification. The main surface effect of rotation is to concentrate the oscillations along equatorial waveguides. This effect is expected to be significant in γ Dor stars and it would be important to take it into account in spectroscopic and photometric mode identification methods.

Conclusions

The analysis of the gravity-mode oscillations in γ Dor stars gives a unique opportunity to probe the deep layers near the CC edge in young intermediate mass stars. However, the effect of rotation complicates strongly the interpretation of their frequency pattern, and much work must still be done at this level. TDC models confirm that the driving of these g modes is due to a periodic flux blocking at the base of the convective envelope. The balance between this flux blocking driving and the radiative damping in the g-mode cavity explains the location of their instability strip. TDC models are required for a secure photometric mode identification in these stars. Comparison with the observed amplitude ratios and phase differences strongly constrains these models in the convective envelope.

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DISCUSSION

Roxburgh: is there any way of trying to test your time-dependent convection model for instance by comparing them to numerical simulations of convection?

Dupret: at this time, I think it would be difficult. We would have to isolate the high-order gravity modes in the numerical simulations, which is a problem. I agree it would be interesting but as far as I know, it hasn't even been tried yet.

Marc-Antoine Dupret thoroughly involved in an entertaining discussion.

Dennis Stello, Don Kurtz, Alosha Pamyatnykh and Wojtek Dziembowski concentrated on a talk.

Conny Aerts and Michel Breger.