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Theoretical instability strips for δ Scuti and γ Doradus stars

M.-A. Dupret[1], A. Grigahcène[1], R. Garrido[1], M. Gabriel[2], and R. Scuflaire[2]

[1] Instituto de Astrofísica de Andalucía-CSIC, Apartado 3004, 18080 Granada, Spain
[2] Institut d'Astrophysique et de Géophysique de l'Université de Liège, Belgium

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Abstract. New theoretical instability strips for δ Sct and γ Dor stars are presented. These results have been obtained taking into account the perturbation of the convective flux following the treatment of Gabriel (1996). For the first time, the red edge of the δ Sct instability strip for non-radial modes is obtained. The influence of this time-dependent convection (TDC) on the driving of the γ Dor gravity modes is investigated. The results obtained for different values of the mixing-length parameter α are compared for the γ Dor models. A good agreement with observations is found for models with α between 1.8 and 2.0.

Key words. stars: oscillations – stars: interiors – stars: variables: general

1. Introduction

The determination of theoretical instability strips is of great interest in the study of variable stars, because their confrontation to observations enables to test our knowledge of stellar interiors and our understanding of the driving mechanisms of pulsating stars. We consider in this paper the case of δ Sct and γ Dor stars.

The theoretical blue edge of the δ Sct instability strip has been determined by many authors, for radial as well as for non-radial modes (e.g. Pamyatnykh 2000). But the determination of the theoretical red edge is a more difficult matter, because it requires a non-adiabatic treatment of the interaction between convection and pulsation. Xiong et al. (2001) and Xu et al. (2002) succeeded to obtain a theoretical red edge for radial modes, using the non-local time-dependent convection theory of Xiong et al. (1998), and Houdek (2000) studied the convective effects on radial p-mode stability in δ Sct stars, using the time-dependent convection treatment of Gough (1977). In this paper, we present the first theoretical blue and red edges of the δ Sct instability strip obtained for non-radial modes as well, following the time-dependent convection treatment developed by Gabriel (1996).

γ Dor stars are a recently discovered class of variable stars. These stars are located in a region of the HR diagram that is bounded by ~7200−7550 K on the zero-age main sequence (ZAMS) and by ~6900−7400 K near the end of the main sequence phases (Handler & Shobbrook 2002a). Using frozen convection (FC) models, Guzik et al. (2000) showed that the driving of the γ Dor gravity modes can be explained by a convective flux blocking mechanism at the base of their convective envelope. A first theoretical instability strip has been obtained by Warner et al. (2003), following this approach. However, in a significant part of the convective envelope, the FC approximation is not valid, because the life-time of the convective elements becomes shorter than the pulsation period. In this paper, we show that the driving of the γ Dor g-modes is successfully explained by our TDC models.

A brief explanation of our TDC treatment is given in Sect. 2. The theoretical instability strips obtained for δ Sct and γ Dor stars are then presented in Sects. 3 and 4 respectively.

2. Time-dependent convection (TDC) treatment

In this paper, we follow the TDC treatment of Gabriel (1974, 1996, 1998, 2000). We split the variables in two parts, describing respectively the average model and the convection. Therefore we write: $y = \overline{y} + \Delta y$, $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{V}$, where y is any scalar variable and $\boldsymbol{v}$ is the velocity vector. $\overline{y}$ and $\boldsymbol{u}$ are the average values, while Δy and $\boldsymbol{V}$ describe the convection.

The mean equations of mass, momentum and energy conservations are respectively:

$$\frac{\partial\overline{\rho}}{\partial t} + \nabla\cdot(\overline{\rho}\boldsymbol{u}) = 0\,, \tag{1}$$

$$\frac{\partial\left(\overline{\rho}\boldsymbol{u}\right)}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}\boldsymbol{u}) + \nabla\cdot\left(\overline{\rho\boldsymbol{V}\boldsymbol{V}}\right) = -\overline{\rho}\,\nabla\overline{\Phi} - \nabla\cdot\overline{\mathsf{P}}\,, \tag{2}$$

$$\overline{\rho}\,\overline{T}\,\frac{\mathrm{d}\overline{s}}{\mathrm{d}t} = -\nabla\cdot\overline{\boldsymbol{F}}_{\mathrm{R}} - \nabla\cdot\overline{\boldsymbol{F}}_{\mathrm{C}} + \overline{\rho\varepsilon} + \overline{\rho\varepsilon_2} + \overline{\boldsymbol{V}.\nabla p}\,, \tag{3}$$

where Φ is the gravitational potential, P is the gas + radiation stress tensor, $\overline{\rho\boldsymbol{V}\boldsymbol{V}}$ is the Reynolds stress tensor, s is the entropy, $\boldsymbol{F}_{\mathrm{R}}$ and $\boldsymbol{F}_{\mathrm{C}}$ are the radiative and convective flux vectors, ε is the energy generation rate by nuclear reactions, ε_2 is the rate of dissipation of turbulent kinetic energy into heat and $\overline{\boldsymbol{V}.\boldsymbol{\nabla}p}$ is the work of the pressure gradient.

Subtracting Eqs. (1)–(3) from the corresponding non-averaged ones gives the equations for convection. They are then simplified in such a way that their stationary solutions lead to the classical mixing length theory of Böhm-Vitense (1958). This procedure ensures the compatibility between the theories used to compute our equilibrium models and to evaluate their vibrational stability. The continuity, motion and energy equations for convection take then the following form:

$$\nabla \cdot \boldsymbol{V} = 0, \tag{4}$$

$$\overline{\rho}\,\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} + \rho\boldsymbol{V}\cdot\nabla\boldsymbol{u} = \frac{\Delta\rho}{\rho}\nabla\overline{p} - \nabla\Delta p - \frac{8\rho\boldsymbol{V}}{3\tau_{\mathrm{C}}}, \tag{5}$$

$$\frac{\mathrm{d}\Delta s}{\mathrm{d}t} + \frac{\Delta(\rho T)}{\overline{\rho T}}\frac{\mathrm{d}\overline{s}}{\mathrm{d}t} = -\boldsymbol{V}\cdot\nabla\overline{s} - \frac{\Gamma^{-1}+1}{\tau_{\mathrm{C}}}\,\Delta s, \tag{6}$$

where $\Gamma = \tau_{\mathrm{R}}/\tau_{\mathrm{C}}$ is the convective efficiency, τ_{C} is the life-time of the convective elements and τ_{R} is the cooling characteristic time of turbulent eddies due to radiative losses.

The perturbation of Eqs. (1)–(3) gives the linear pulsation equations, where new terms appear such as the perturbation of the convective flux and the Reynolds stress tensor. Of these terms, only the perturbation of the convective flux vector is taken into account in this paper and it is given by:

$$\delta\boldsymbol{F}_{\mathrm{C}} = \overline{\boldsymbol{F}}_{\mathrm{C}}\left(\frac{\delta\rho}{\overline{\rho}} + \frac{\delta T}{\overline{T}}\right) + \overline{\rho T}\left(\overline{\delta\Delta s\boldsymbol{V}} + \overline{\Delta s\delta\boldsymbol{V}}\right). \tag{7}$$

The unknown correlation terms in Eq. (7) can be obtained from the fluctuation equations. More precisely, we perturb Eqs. (4)–(6) and we search for solutions of the form $\delta(\Delta X) = \delta(\Delta X)_{\boldsymbol{k}}\ \mathrm{e}^{\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{r}}\ \mathrm{e}^{\mathrm{i}\sigma t}$. Then we integrate these particular solutions over all values of $\boldsymbol{k}_\theta$ and $\boldsymbol{k}_\phi$ such that $\boldsymbol{k}_\theta^2+\boldsymbol{k}_\phi^2 = A\,\boldsymbol{k}_r^2$ assuming A as a constant ($A = 1/2$ for an isotropic turbulence). Then the horizontal averages are computed. Finally, the perturbed convective flux takes the following form:

$$\delta\boldsymbol{F}_{\mathrm{C}} = \delta F_{\mathrm{Cr}}(r)\ Y_l^m(\theta,\phi)\ \boldsymbol{e}_r + \delta F_{\mathrm{Ch}}(r)\left(r\nabla_h Y_l^m(\theta,\phi)\right) \tag{8}$$

and the problem is naturally separated in spherical harmonics. $\delta F_{\mathrm{Cr}}(r)$ and $\delta F_{\mathrm{Ch}}(r)$ are related to the perturbed mean quantities by first order differential equations. Proceeding so, the timescales of pulsation and convection coming respectively from the perturbation of the left hand side and right hand side of Eqs. (5) and (6) are both taken into account (for more details, we refer to the role played by the coefficients B and D defined at p. 239 of Gabriel 1996). Therefore, our treatment does not assume instantaneous adaptation of convection to pulsation nor frozen convection.

The main source of uncertainty in any MLT treatment of convection-pulsation interaction comes from the expression which is adopted for the perturbation of the mixing-length $l = \alpha H_{\mathrm{p}}$. In this paper, we adopt the following formula:

$$\frac{\delta l}{l} = \frac{1}{1+(\sigma\tau_{\mathrm{C}})^2}\ \frac{\delta H_{\mathrm{p}}}{H_{\mathrm{p}}}, \tag{9}$$

with the two limit cases $\delta l/l \to \delta H_{\mathrm{p}}/H_{\mathrm{p}}$ when $\sigma\tau_{\mathrm{C}} \ll 1$ and $\delta l/l \to 0$ when $\sigma\tau_{\mathrm{C}} \gg 1$, where H_{p} is the pressure scale height.

Our equilibrium models are computed using a local mixing-length approach. For reasons of consistency between the equilibrium and perturbed models, our TDC treatment is also local.

3. δ Sct instability strip

We have implemented the perturbation of the convective flux in the linear non-adiabatic code MAD (Dupret et al. 2002). In order to determine the theoretical instability strips, a large number of evolutionary tracks were computed by the code CLÉS (Liège), with masses going from 1.4 to 2.2 $M_\odot$, with different values of the MLT parameter α, with core overshooting $\alpha_{\mathrm{ov}} = 0.2$ and with solar metallicity. Then, we studied the stability of the modes in the appropriate frequency range. Contrary to the calculations with FC, with our TDC treatment we are able to reproduce the red edge of the δ Sct instability strip, for radial as well as for non-radial modes.

In Fig. 1, we present the theoretical instability strip obtained for radial modes, for models with the solar calibrated value $\alpha = 1.8$. Each curve corresponds to the blue or red edge of a mode of given radial order n. Labels enable to identify the modes (e.g. $p_{4\mathrm{R}}$ for the red edge of the p_4 mode). As the radial order of the modes increases, the blue and red edges are displaced towards higher T_{eff}. For the sake of clarity of the figure, we only give the results for modes from p_1 to p_4. However we stress that, on the blue side of the instability strip, we find unstable modes up to p_7. The small points correspond to the position of observed δ Sct stars, as taken from the catalogue of Rodriguez et al. (2000), using the calibrations of Moon & Dworetsky (1985). We have only presented the results obtained for $\alpha = 1.8$. We note that the theoretical red edges are displaced towards lower effective temperatures when α decreases. The physical reason is that the red edge corresponds to models with a given size of the convective envelope; this size increases when α increases or T_{eff} decreases. As comparison, we also give in Fig. 1 the position of the red edge for the fundamental radial mode as obtained by Xiong et al. (2001) ("$\odot$") and by Houdek (2000) ("$\bullet$").

In Fig. 2, we present the theoretical instability strip obtained for $\ell = 2$ modes, for models with $\alpha = 1.8$. Again we see that, as the radial order of the modes increases, the blue and red edges are displaced towards higher T_{eff}. For the sake of clarity of the figure, we only give the results for modes from g_3 to p_4. For non-radial modes, the shapes of the blue and red edges are not as straight as for radial modes, because of the avoided crossings.

In Fig. 3, we compare the work integral of the fundamental radial mode obtained with TDC and FC, for a model situated just at the red edge of the δ Sct instability strip. Regions where the work increases (resp. decreases) are driving (resp. damping) the oscillations. The surface value of the work integral is the dimensionless growth rate $(-\Im(\sigma)\,\tau_{\mathrm{dyn}})$. The vertical line indicates the base of the convective envelope. With a local TDC treatment, spatial oscillations are present at the base of the convective envelope, as first explained by Baker & Gough (1979). With a non-local treatment (Xiong et al. 2001) or with other specific treatments that we will present in a forthcoming paper, these spatial oscillations can be avoided and the stabilization of the δ Scuti p-modes still occurs at the same location in the HR diagram. Driving due to convective blocking and κ-mechanism occurs in the FC model but no longer plays a significant role in the TDC model.

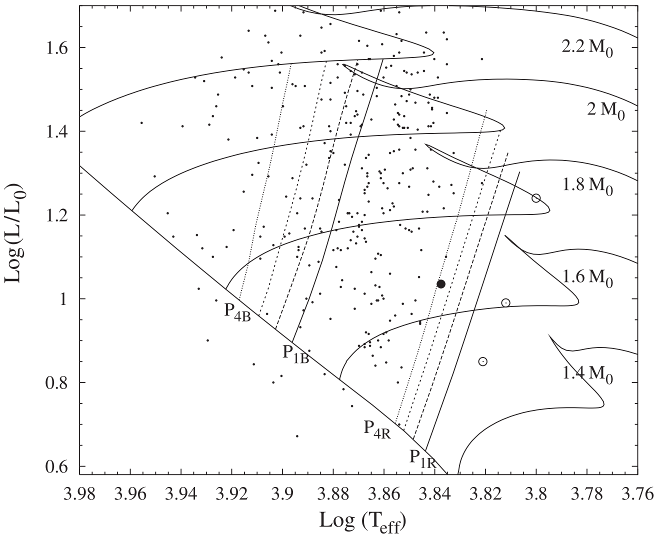

Fig. 1. Blue and red edges of the δ Sct theoretical instability strip. The lines are our TDC results for radial modes from p_1 to p_4, for models with $\alpha = 1.8$. The small points correspond to observations. As comparison, we give also the red edges for the fundamental radial mode obtained by Xiong et al. (2001) ("$\odot$") and by Houdek (2000) ("$\bullet$").

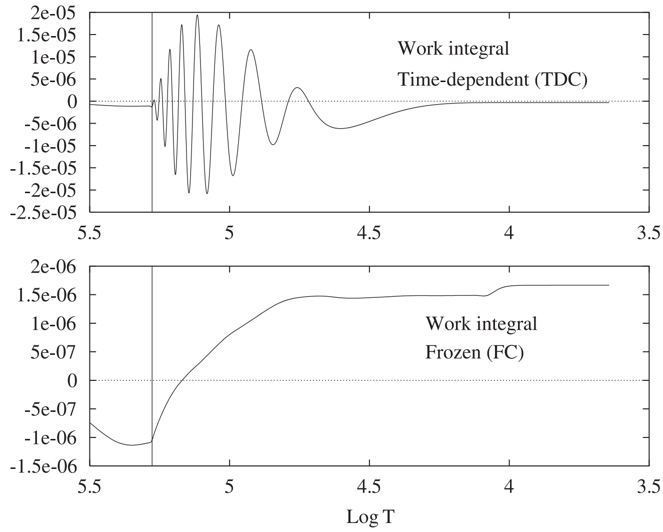

Fig. 3. Comparison between the work integral of the fundamental radial mode obtained with a TDC model (top panel) and a FC model (bottom panel). The vertical line indicates the base of the convective envelope. The characteristics of the equilibrium models are: $M = 1.6\ M_\odot$, $T_{\rm eff} = 6664.8$ K, $\log(L/L_\odot) = 0.9731$ and $\alpha = 1.8$.

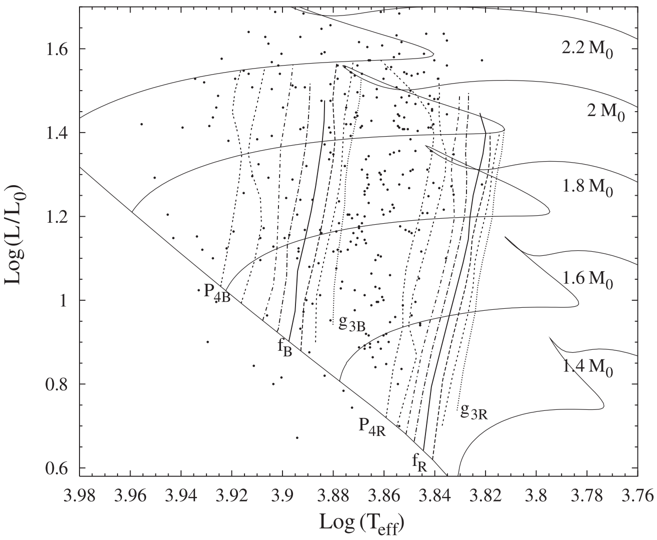

Fig. 2. Blue and red edges of the δ Sct theoretical instability strip for models with $\alpha = 1.8$ and $\ell = 2$ modes from g_3 to p_4, as obtained with our TDC treatment.

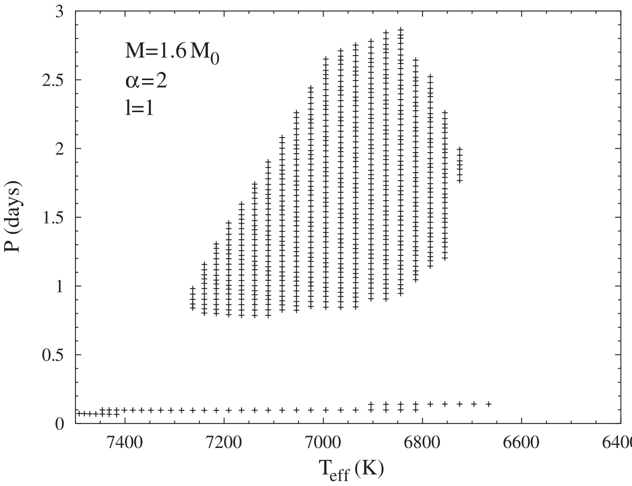

Fig. 4. Periods (in days) of all the unstable $\ell = 1$ gravity modes obtained for models of 1.6 $M_\odot$ with $\alpha = 2$, as a function of the effective temperature. Each cross corresponds to a given g-mode.

4. γ Dor instability strip

The driving of high order gravity modes in γ Dor stars can also be explained by our TDC models. In Fig. 4, we show the periods of all the unstable $\ell = 1$ gravity modes obtained for models of 1.6 $M_\odot$ with $\alpha = 2$, as a function of the effective temperature. Each cross corresponds to an unstable mode. As can be seen, the periods of those modes correspond to the typical observed periods of γ Dor stars. Moreover, we see at the bottom of this figure that our models have also unstable p-modes. The existence of stars pulsating with both δ Sct p-modes and γ Dor high order g-modes would thus be explained by our theoretical models. We refer to Handler & Shobbrook (2002a) for an observational research (however not yet conclusive) of such hybrid δ Sct - γ Dor stars. In Fig. 5, we show the theoretical instability strips of γ Dor $\ell = 1$ modes obtained with our TDC treatment (thick lines), for three families of models with different values of the MLT parameter α: 1, 1.5 and 2. As comparison, we give in the same figure the FC results of Warner et al. (2003) obtained with $\alpha = 1.87$ (thin dashed lines). In this case, we give global instability strips and not individual ones for each mode. For any model inside the instability strip, at least one unstable high order g-mode is found; outside it all are stable. The small circles correspond to the observed positions of bona fide γ Dor stars from the catalogue of Handler (2002b); their effective temperatures are taken from Kaye et al. (1999) who used the calibrations of Villa (1998). Some evolutionary tracks for models with $\alpha = 1.5$ are also given in this figure. As for δ Sct stars, we see that the theoretical predictions are

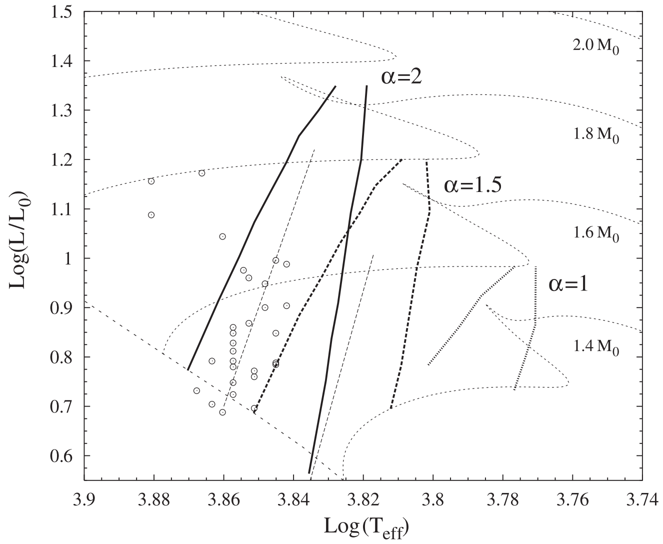

Fig. 5. γ Dor theoretical instability strips for $\ell = 1$ modes, for three families of models with different values of α: 1, 1.5 and 2 obtained with our TDC treatment (thick lines), compared to the FC results of Warner et al. (2003) (thin dashed lines, $\alpha = 1.87$). The small circles correspond to observations of bona fide γ Dor stars.

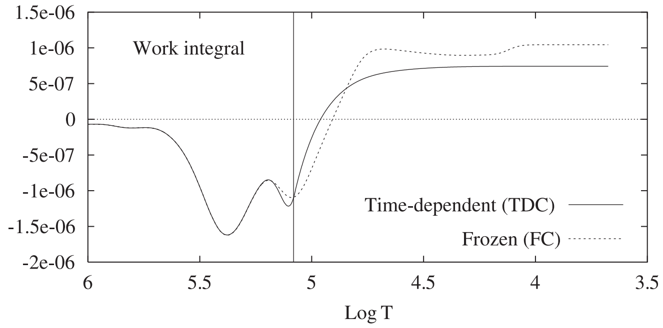

Fig. 6. Comparison between the work integral of the $\ell = 1$, g_{47} mode obtained with a TDC model (solid line) and a FC model (dashed line). The vertical line indicates the base of the convective envelope. The characteristics of the equilibrium model are: $M = 1.6\ M_\odot$, $T_{\rm eff}$ = 6934.8 K, $\log(L/L_\odot) = 0.9564$ and $\alpha = 2$.

very sensitive to α. The depth of the convective envelope plays the major role in the driving of γ Dor g-modes. This explains the high sensitivity of our results to α. In Fig. 6, we compare the work integral of the $\ell = 1$, g_{47} mode obtained with TDC and FC, for a model situated in the middle of the γ Dor instability strip. In both TDC and FC models, the main driving occurs at the base of the convective envelope (in agreement with Guzik et al. 2000). Small κ-driving in the H ionization zone occurs in the FC model but no longer in the TDC model.

5. Conclusion

Including the perturbation of the convective flux, following the Gabriel's treatment, we obtained theoretical instability strips for δ Sct and γ Dor stars. For δ Sct stars, we succeed to reproduce both the blue and red edges, for radial as well as for non-radial modes. With the solar calibrated value $\alpha = 1.8$, a good agreement with observations is found. We obtained also theoretical instability strips for the γ Dor g-modes. The theoretical instability strips of γ Dor stars are very sensitive to the value of the MLT parameter α. We get a good agreement with observations for models with $\alpha = 2$. We note that the calibration used to go from observational to theoretical HR diagrams is subject to uncertainties, however these are expected to be small compared to the effect of changing α in our models. In this paper, we only emphasized the main results. More details about the theory and the physical interpretation of the results will be presented in a forthcoming paper (Grigahcène et al. in preparation). Houdek (2000) pointed out that the perturbation of the turbulent pressure (not yet included in our study) can also play a significant role in the stabilization of the δ Sct p-modes. Our future prospect is to include the perturbation of the full Reynolds stress tensor (Gabriel 1987) and the dissipation of turbulent kinetic energy in our non-adiabatic code and study their influence on the driving of δ Sct and γ Dor radial and non-radial modes.

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