Determination of the complete bifurcation behaviour of aeroelastic systems with freeplay

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Introduction

- Aeroelastic systems are different to structural dynamic systems in terms of damping:
  - Damping plays a much more important role
  - Damping is non-proportional and, hence, the modes are complex
  - Damping is parameter-dependent (i.e. changes with flight condition)
  - At a critical flight condition the net damping of one of the modes can become zero, leading to self-excited oscillations

Nonlinear Aeroelasticity

- Nonlinearity in aeroelastic systems can have several sources:
  - Aerodynamic: separated flows, buffeting, moving shock waves
  - Structural: large deformations, concentrated nonlinearities in bearings, engine mounts, external store mounts
  - Control system: control surface limiters, control surface rate limiters, nonlinear control laws

Freeplay

- Freeplay is one of the most omnipresent structural nonlinearities
- There are very strict airworthiness guidelines concerning the amount of freeplay permitted on control surface bearings, especially all-moving ones
- These guidelines increase maintenance costs by a significant amount, especially for military aircraft
- It is important to be able to determine the effect of freeplay on aeroelastic behaviour – in this way maybe the airworthiness requirements can be relaxed.

Objective

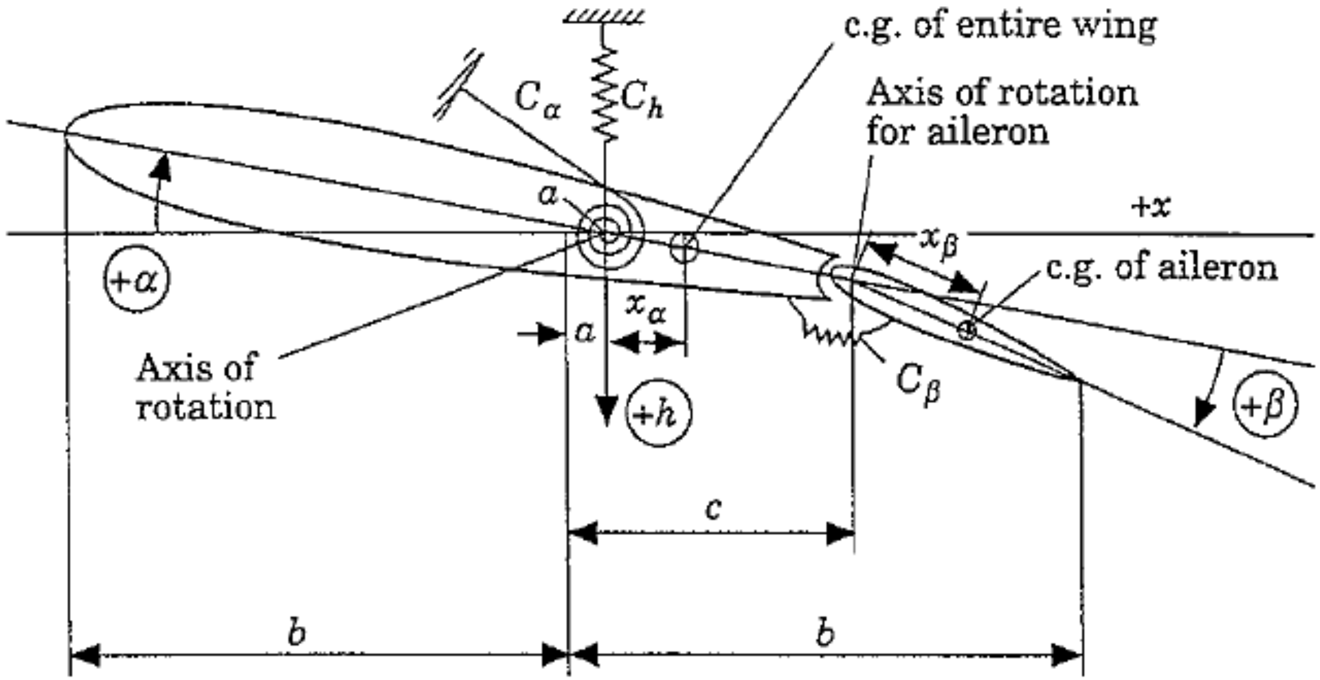
- There has been a lot of research on simple aeroelastic systems with freeplay
- Such systems can undergo self-sustained oscillations known as Limit Cycle Oscillations (LCOs) at a wide range of flight conditions
- The LCOs can jump abruptly in amplitude with small changes in flight condition; they can also become aperiodic
- Nobody has managed to observe and explain the complete bifurcation of such systems as yet.

Aeroelatic system

- 2D flat plate airfoil with pitch, plunge and control surface degrees of freedom.

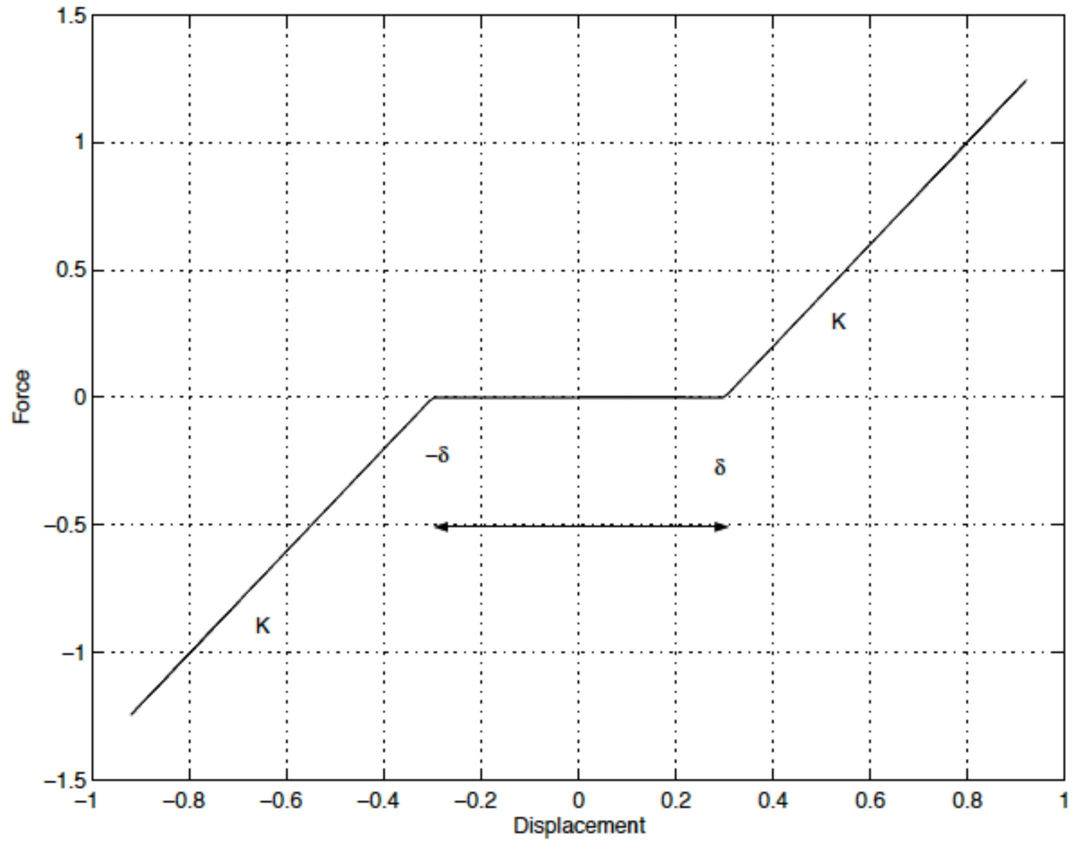
The three degrees of freedom are constrained by three springs.

The control surface spring contains a freeplay nonlinearity



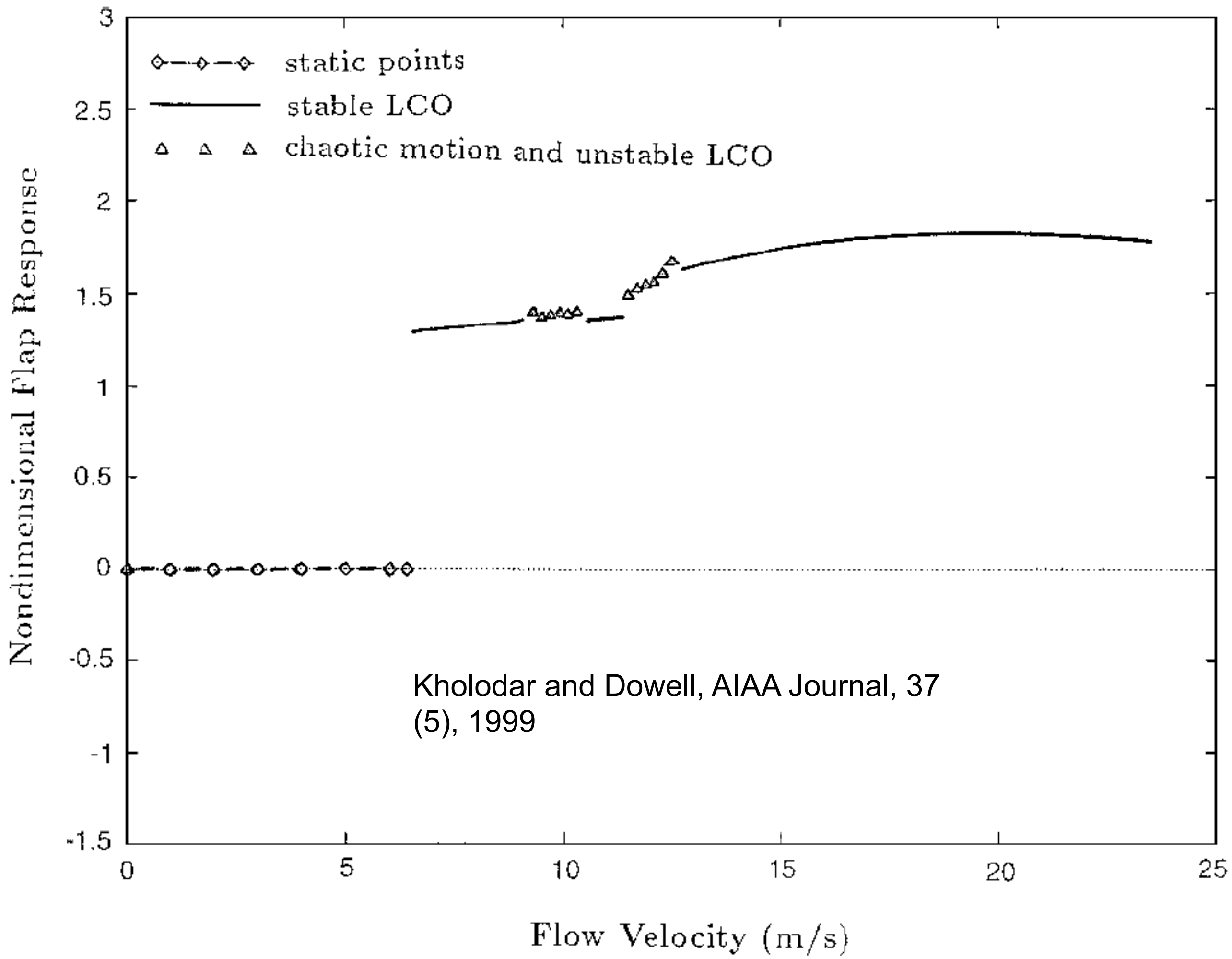
Freeplay

- Freeplay is an interesting nonlinearity:
  - The nonlinearity appears abruptly when the response amplitude exits the deadzone
  - The modes also change abruptly. Can we talk of nonlinear normal modes?
  - There are essentially two nonlinearities:
    - Change in stiffness
    - Jump in equilibrium position



LCO amplitude

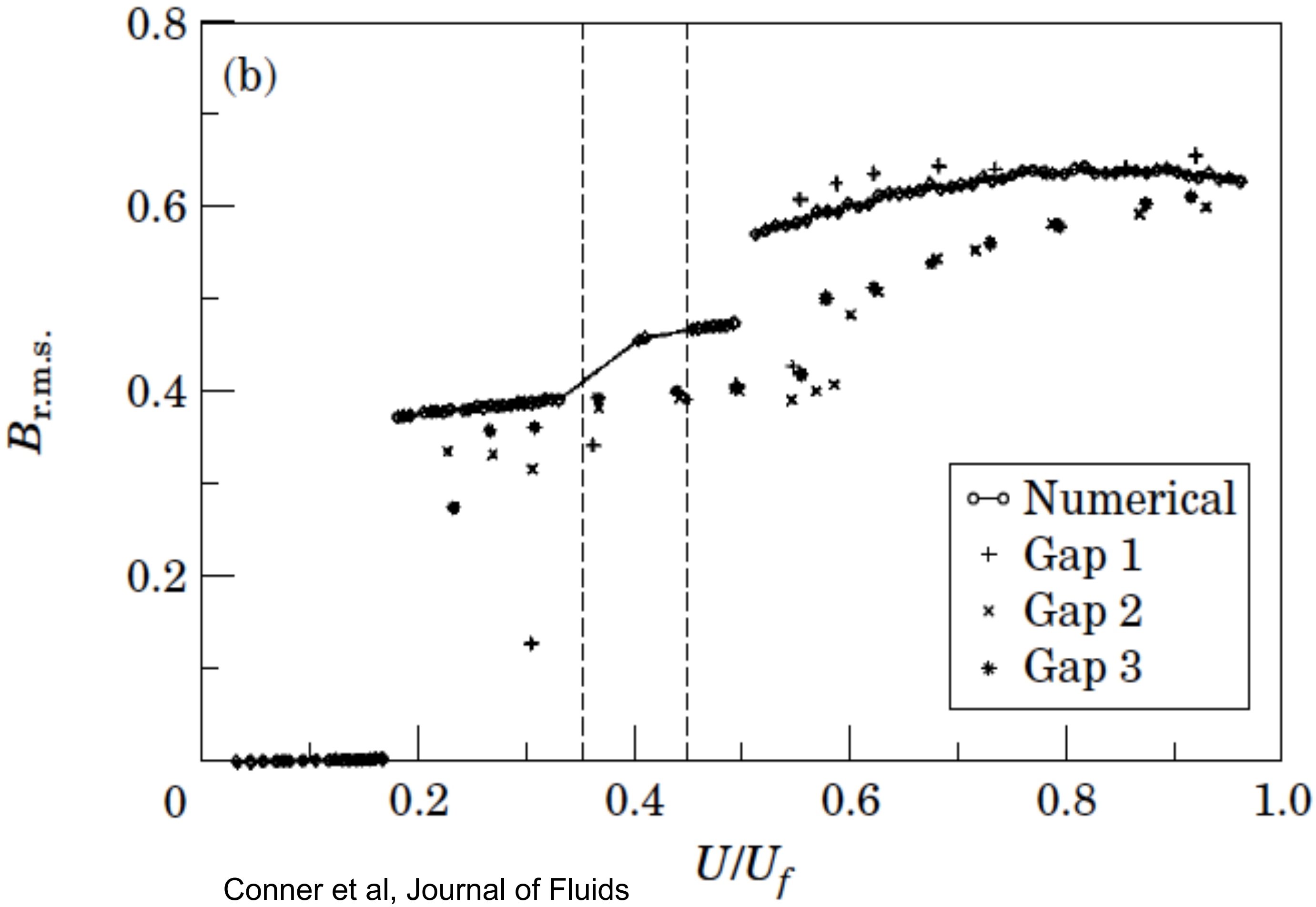
- Several researchers have shown that this system can undergo chaotic motion for some airspeed values
- The reason for the appearance of chaos has not yet been discovered.



Kholodar and Dowell, AIAA Journal, 37 (5), 1999

Experimental results

- This kind of LCO behaviour was also measured experimentally
- Instead of chaos, high noise responses were obtained.



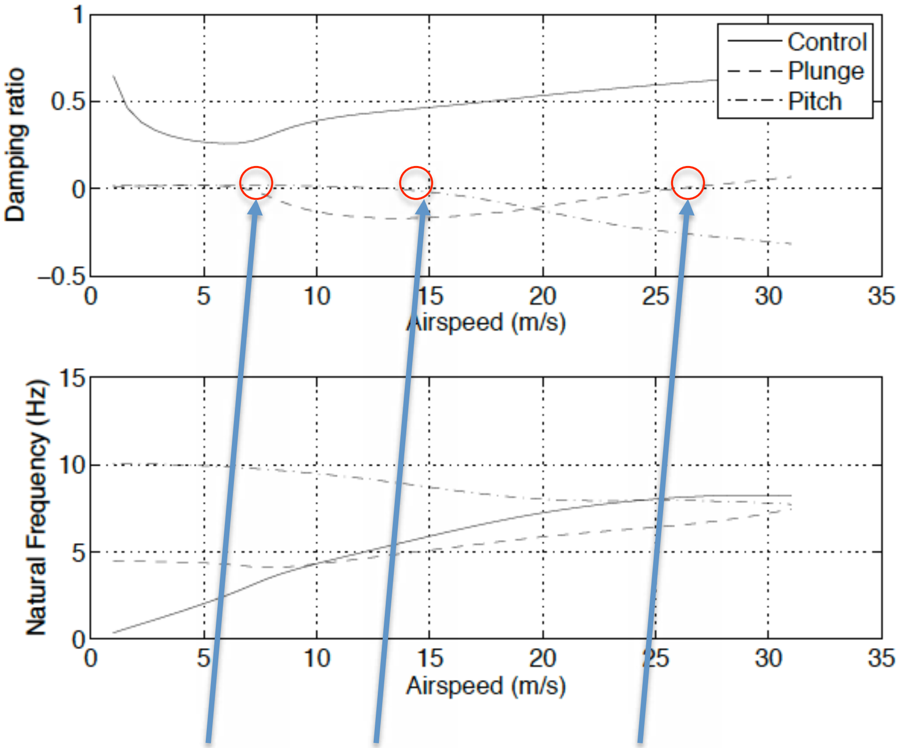
Conner et al, Journal of Fluids and Structures, vol 11, 89-109, 1997

Bifurcation prediction

- The bifurcation prediction is carried out here in four parts:
  - Stability analysis of underlying linear systems
    - Modal parameter variation with flight condition for both inner and outer linear systems
  - Equivalent linearization (first order Harmonic Balance analysis)
    - First approximation of the all the LCO branches
  - Shooting procedure from the equivalent linearized results
    - Exact calculation of discrete points on the bifurcation diagram
  - Numerical continuation from the shooting results
    - Obtain a complete picture of the full bifurcation behaviour

Linear systems

Inner system

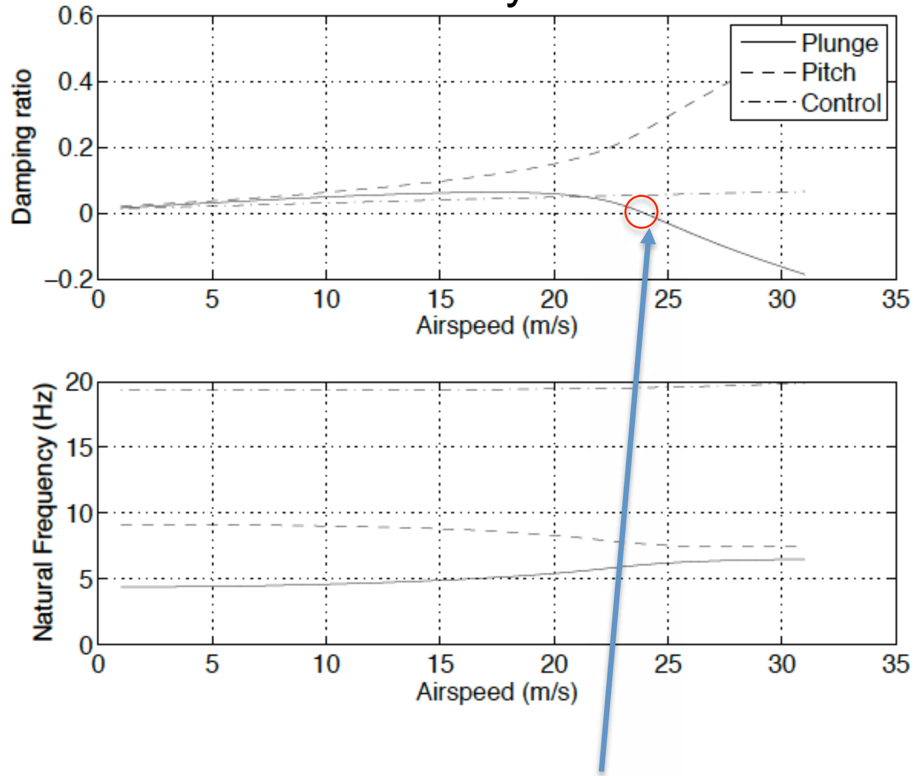


VF1 flutter event
V=6.85m/s

VF2 flutter event
V=13.11m/s

VF3 Stabilization event
V=25.91m/s

Outer system



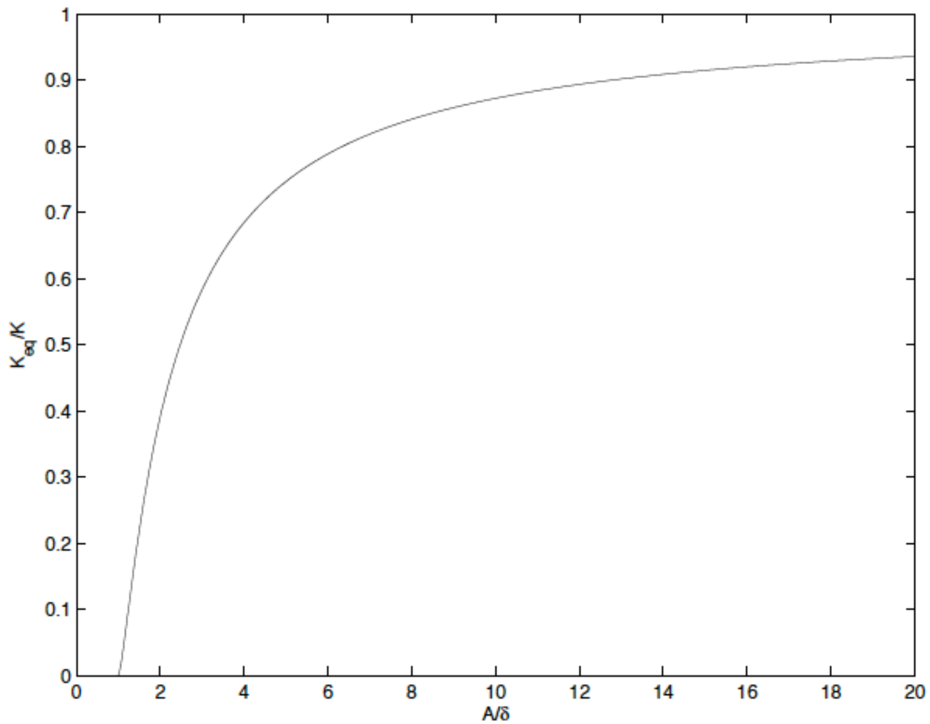
VFlin Flutter event
V=23.95m/s

Flutter

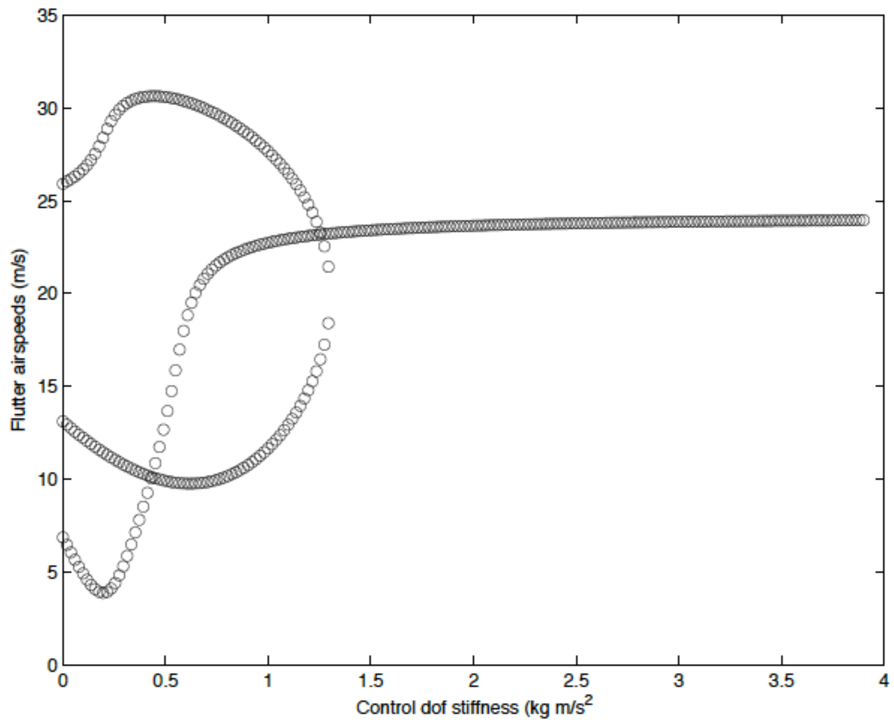
- Mathematically, flutter is described as a Degenerate Hopf Bifurcation
- At the flutter condition, the system's phase plane contains no attractors, only centres. At higher airspeeds the response becomes sinusoidally divergent (unstable focal point)
- It is important to note that at, say 10m/s, only the inner system flutters; the outer system is stable. Therefores, LCOs are expected.
- Can there be LCOs at airspeeds lower than the first inner system flutter speed (6.85m/s)?

Equivalent linearization

Describing function for freeplay



Flutter speed variation with control surface stiffness

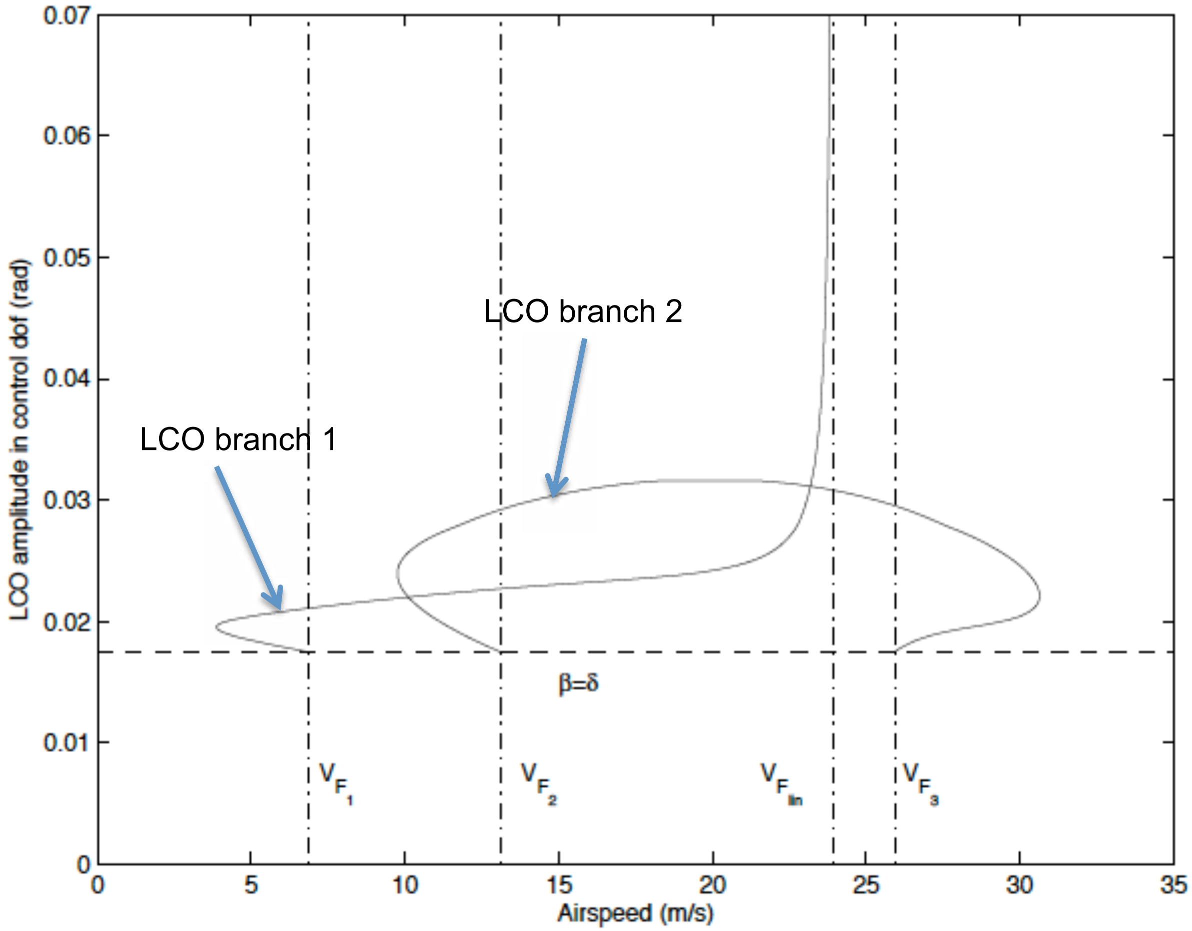


Equivalent linearization models the LCO of a nonlinear system by the flutter of a linear system. It also creates an equivalent linear stiffness depending on the response amplitude

Equivalent linearized LCOs

There are two LCO branches:
Branch 1: begins at VF1 as a subcritical Hopf, undergoes a fold and asymptotes towards Vflin.
Branch 2: begins at VF2 as a subcritical Hopf, undergoes two folds and ends at VF3 as a supercritical Hopf.

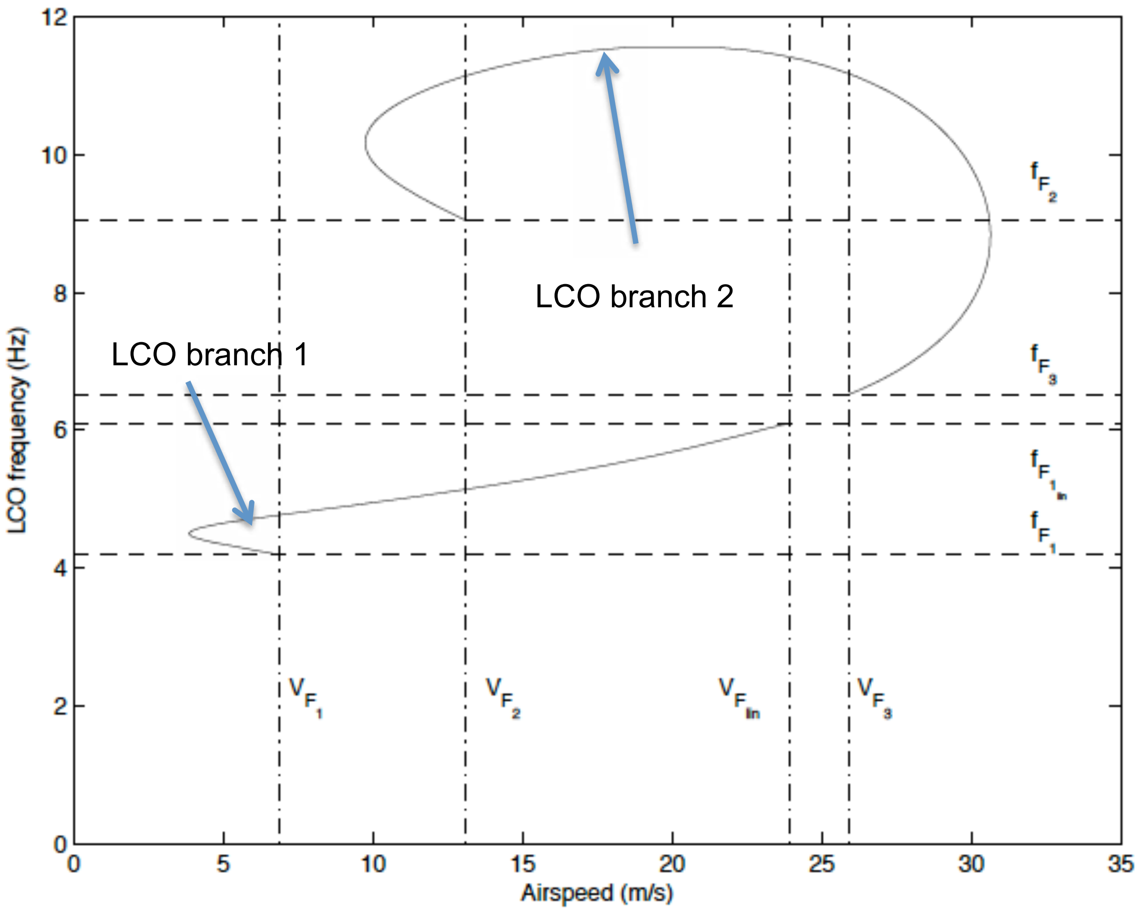
Between 10m/s and 25m/s two LCOs are possible. LCOs occur at airspeeds less than VF1.



LCO frequency

There two LCO branches have completely different frequencies, which are delimited by the flutter frequencies of the underlying linear systems.

In the case of branch 1, the LCO frequency asymptotes to the linear flutter frequency.



Shooting Approach

- Shooting attempts to calculate the exact periodic solutions of a nonlinear system
- If the system response is x(t), it is required that x(0)=x(T), where T is the period
- The difference x(0)-x(T) is calculated using a Runge-Kutta 4^{th}-5^{th} order scheme which pinpoints every crossing of the freeplay region
- The RK code is implemented as a C mex file that talks to Matlab
- A Newton system is used to minimize x(0)-x(T), using the equivalent linearized results as an initial guess.

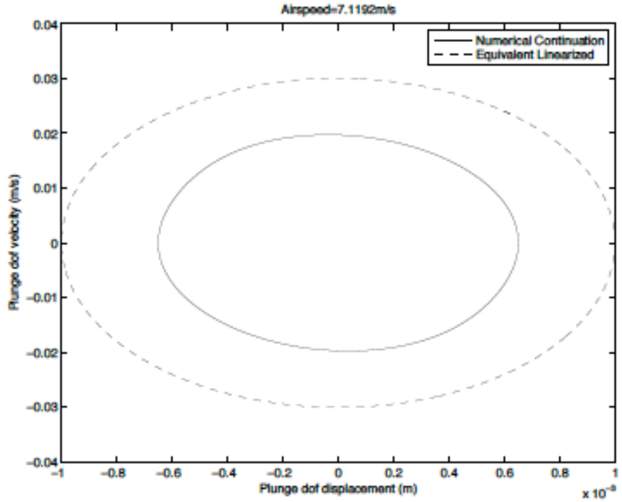
Shooting example

Phase-plane diagrams of the system's response.

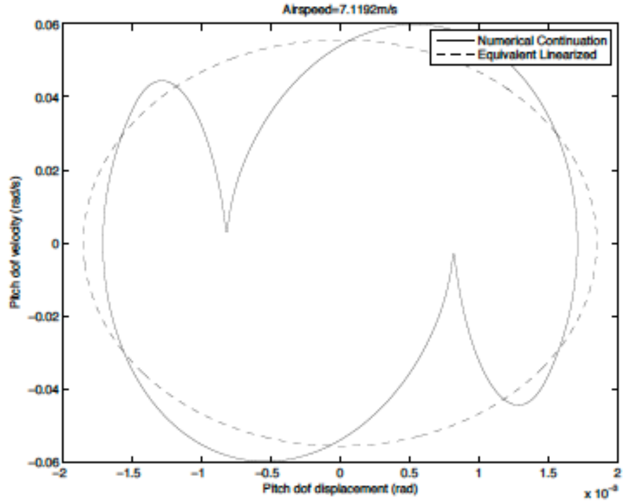
Dashed line: Equivalent linearization

Solid line: Exact response from shooting

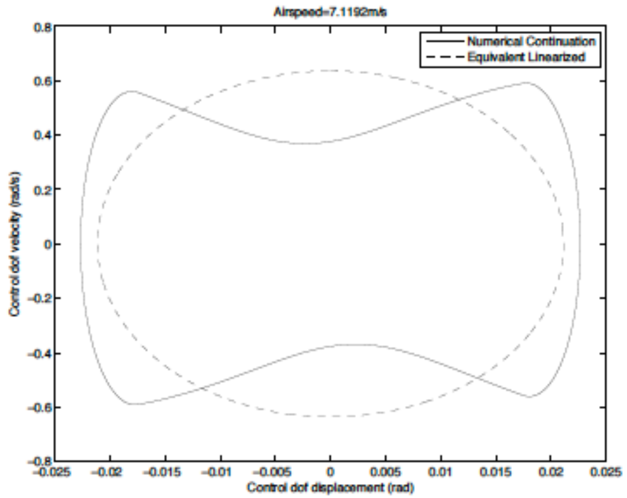
Airspeed: 7.12m/s



(a) $h - \dot{h}$

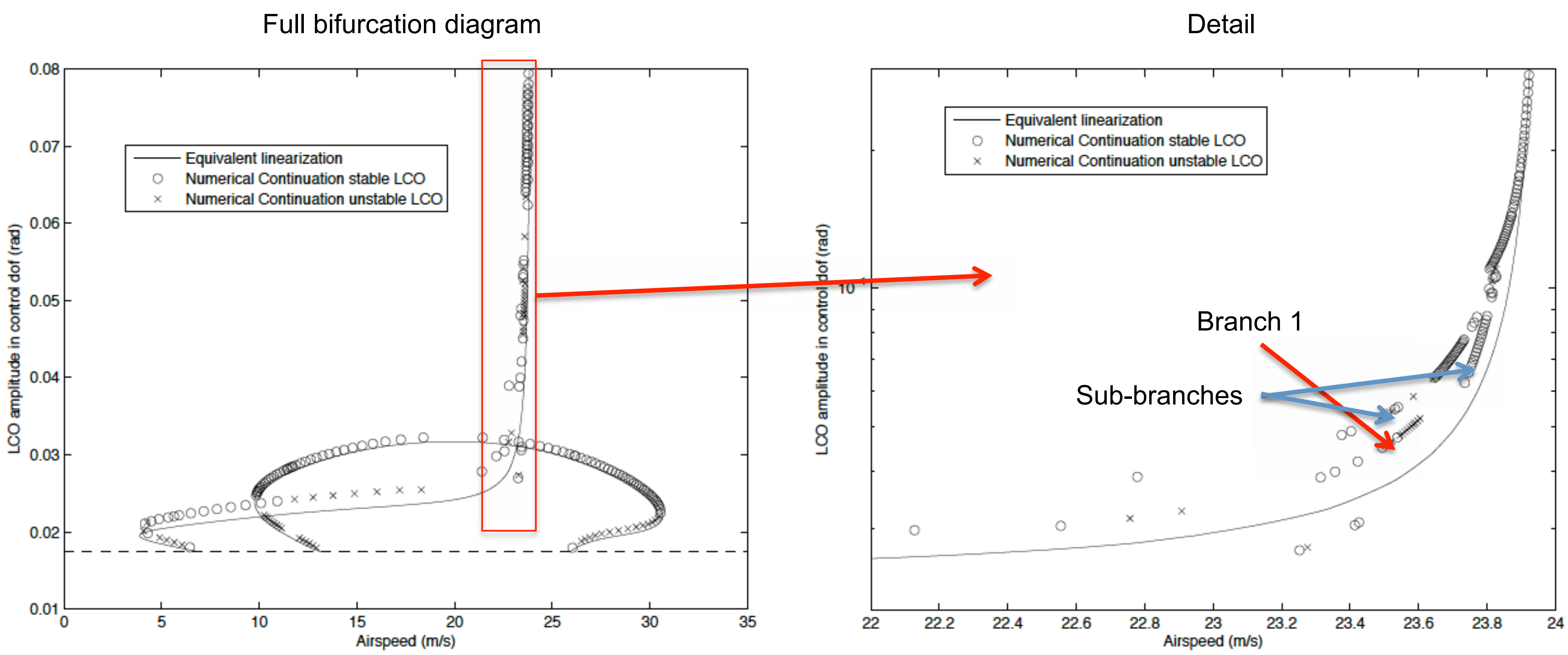


(b) $\alpha - \dot{\alpha}$



(c) $\beta - \dot{\beta}$

Improved bifurcation diagram



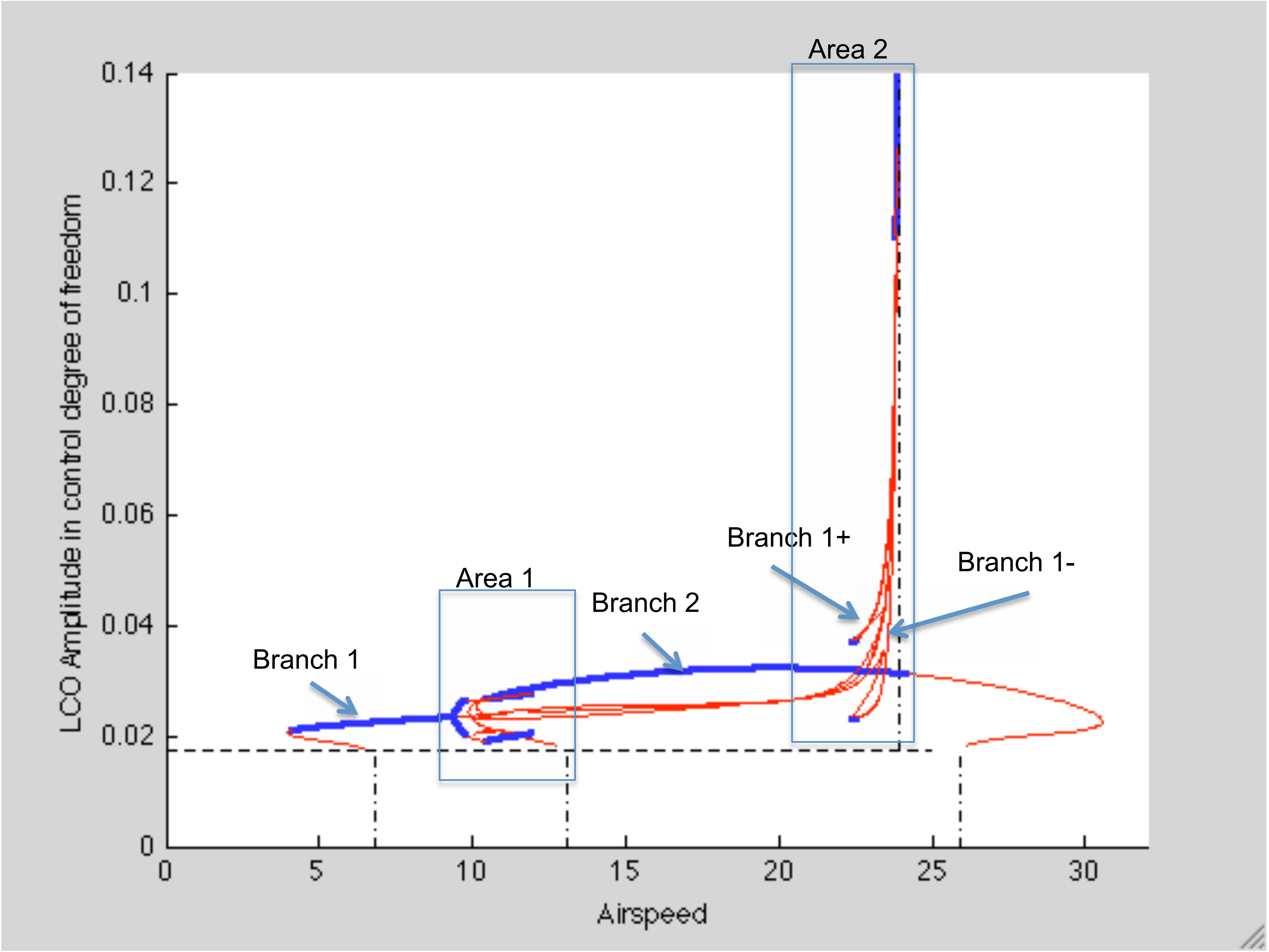
•A shooting calculation is launched from every equivalent linearized LCO. Some calculations fail to converge, others tend to group around specific regions.
•The shooting calculations have revealed the existence of sub-branches of branch 1, termed 1+ and 1-.

LCO branch continuation

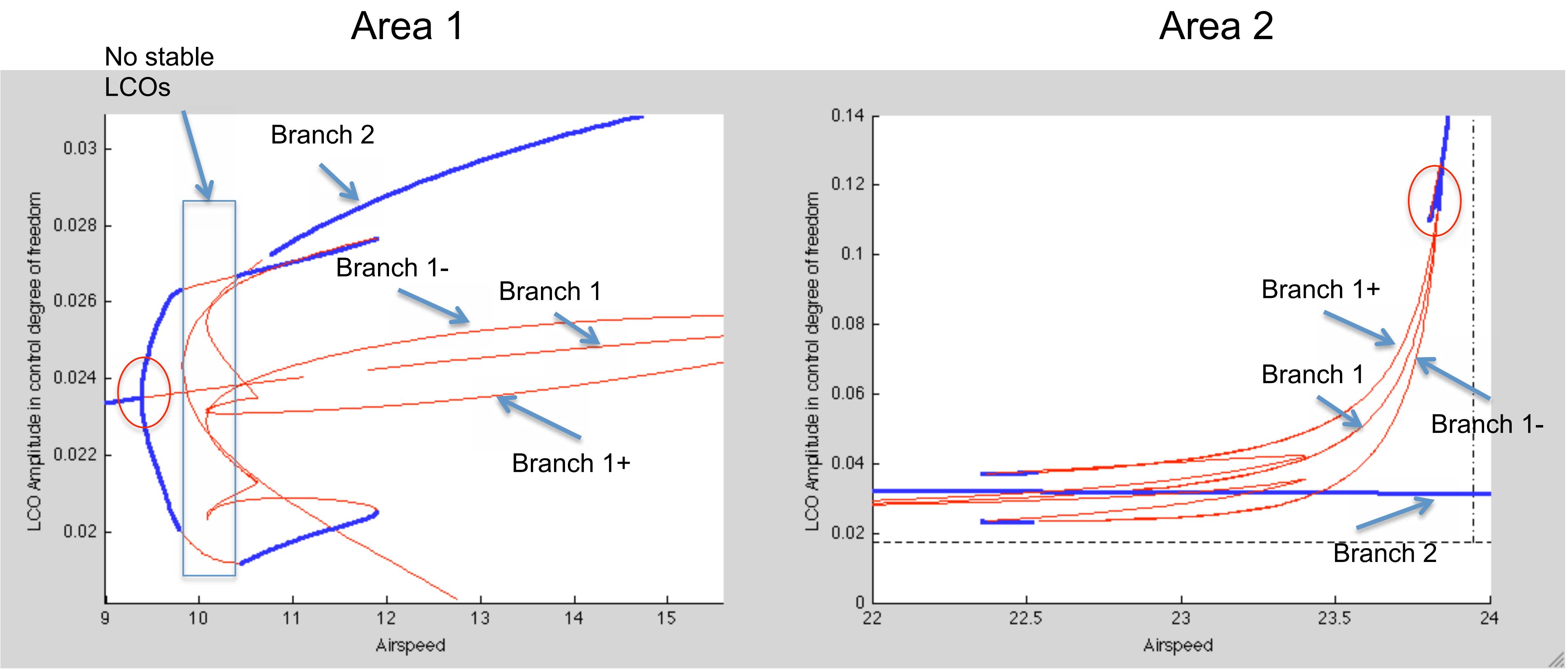
- In order to calculate the complete bifurcation behaviour a numerical continuation approach is used
- It is applied to both branches and the sub-branches
- The continuation method is similar to the shooting approach used earlier but with an additional path following component.
- The path following is a pseudo-arclength procedure.

Full bifurcation

- Blue lines denote stable LCOs, red lines unstable LCOs.
- In areas 1 and 2 there are many coexisting LCOs.
- In area 2 there are no stable low amplitude LCOs on branch 1.



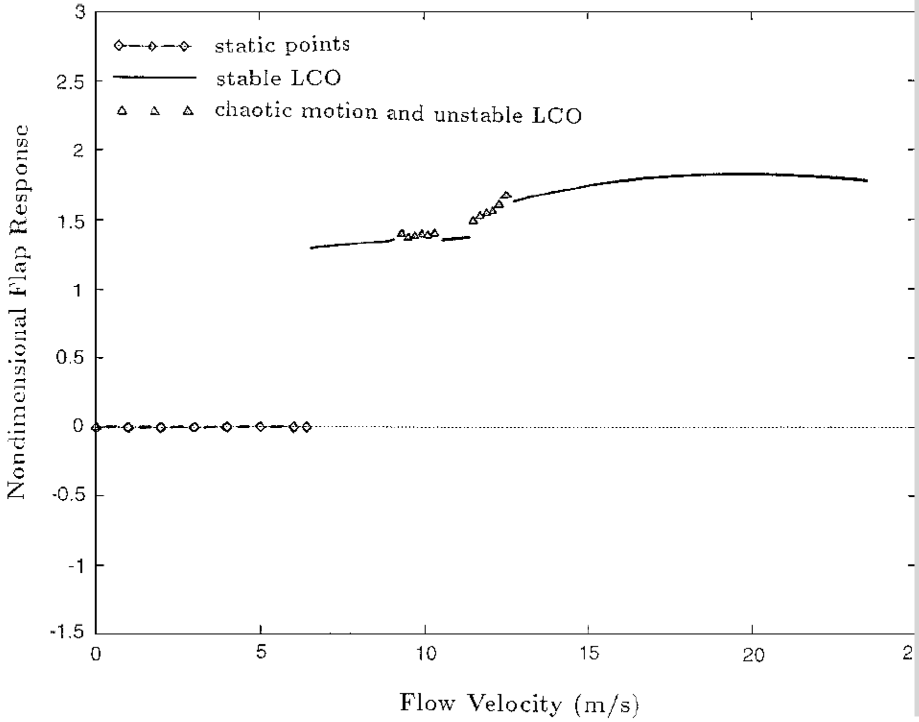
Areas 1 and 2

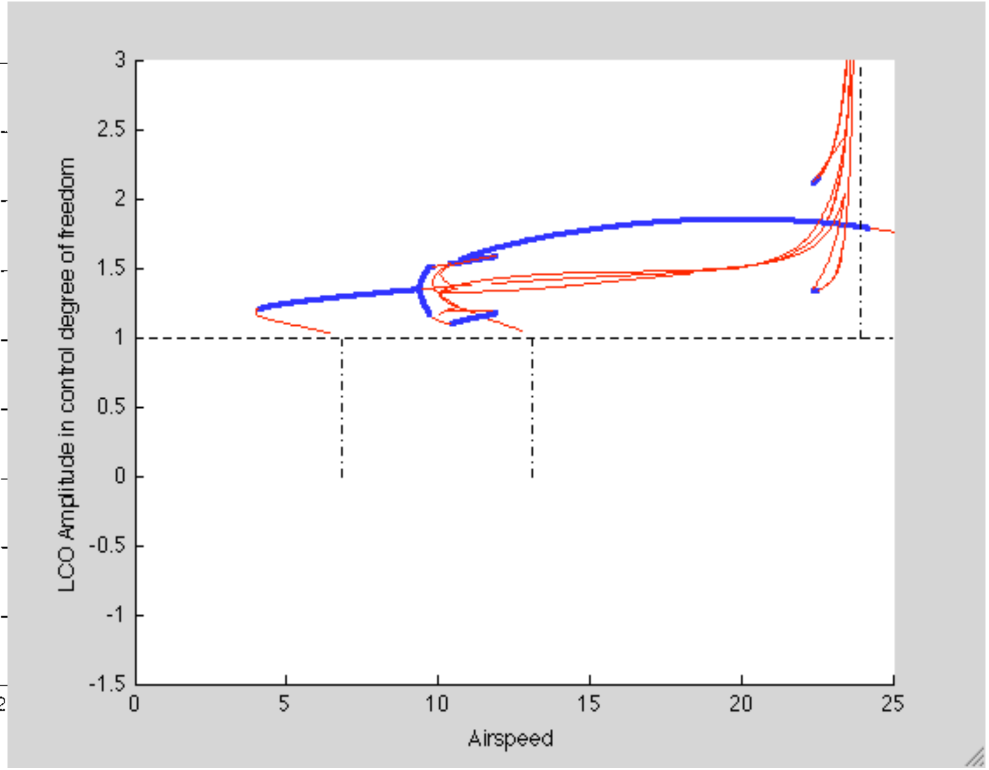


•Sub-branches split from branch 1 in area 1, undergo many fold bifurcations and changes in stability and eventually join branch 1 again in area 2.

•In area 1 there is an airspeed range in which no stable LCOs are possible!

Comparison

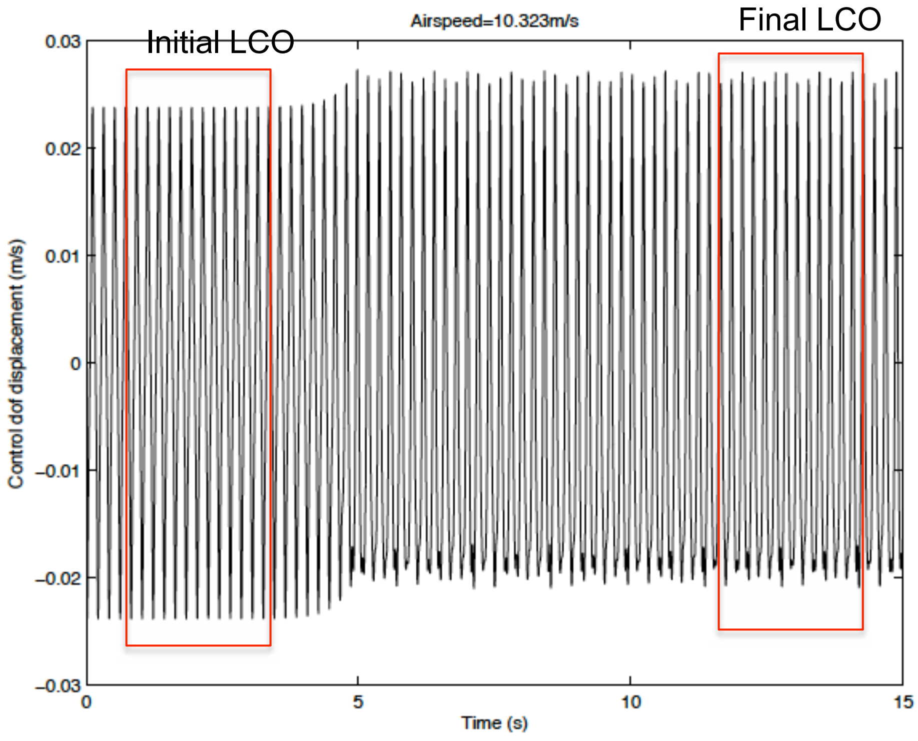




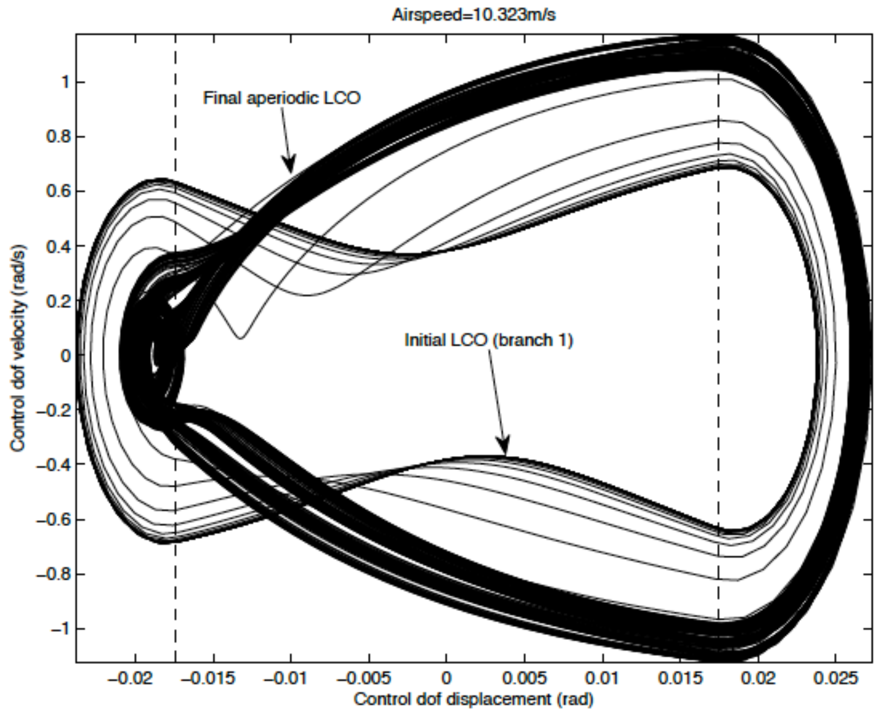
- The left plot was obtained by Kholodar and Dowell using straight numerical integration. The right plot was obtained using the present method.
- The chaotic motion regions correspond to the area where no stable LCOs coexist or where many LCOs coexist.
- Higher amplitude stable LCOs were missed by numerical integration.

Aperiodic LCOs

- In regions where many LCOs are possible, aperiodic LCOs can occur:



(a) Time response



(b) Phase space β-$\dot{\beta}$

Conclusions

- Using the present compound approach it is possible to obtain a full picture of the bifurcation behaviour of the system.
- The compound approach is necessary because of the complexity of the bifurcation.
- The occurrence of aperiodic motion was explained in terms of the coexistence of many possible LCOs.
- If this aperiodic motion is truly chaotic then the repeated looping of LCO sub-branches is a root to chaos.
- Notice also that, despite the symmetry in the system, the aperiodic LCOs can be asymmetric.