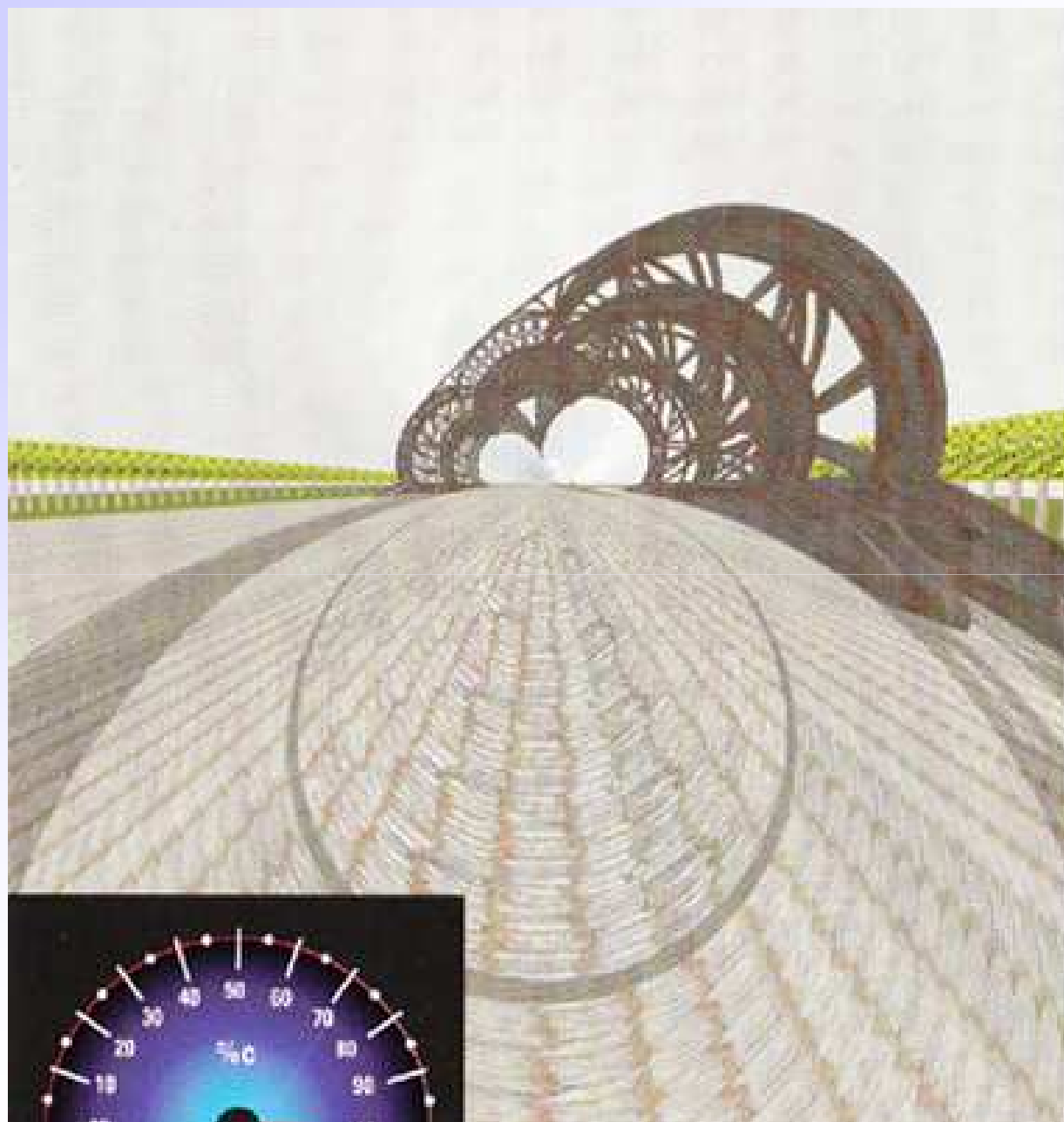
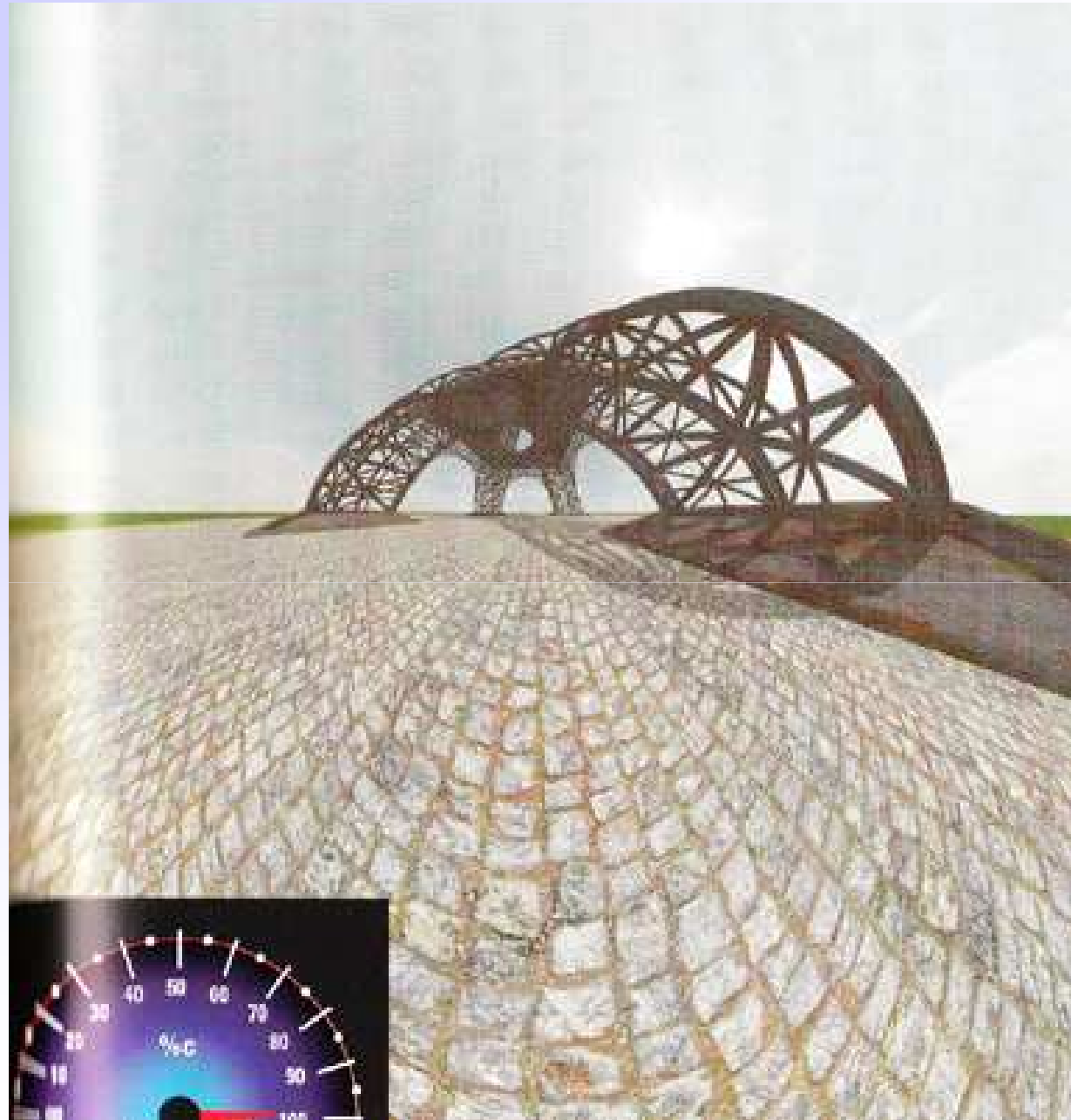


8^{ème} cours de Mécanique Analytique (28/10/2010)









• 3.3 La transformation de Lorentz : approche standard

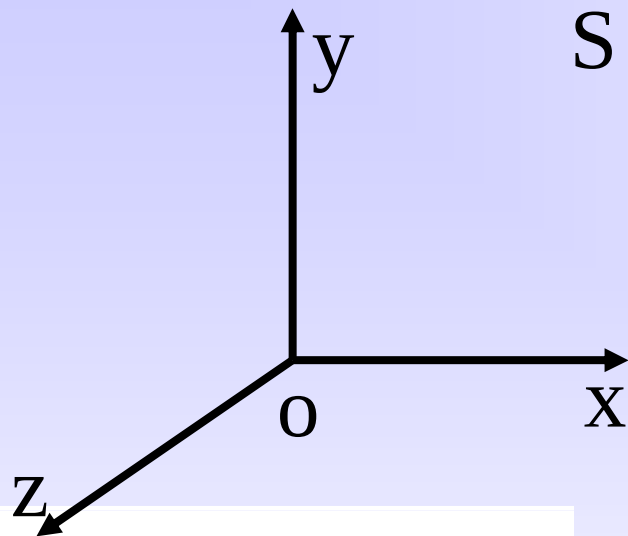
$$\left\{ \begin{array}{l} x = \frac{\bar{x} + V\bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y = \bar{y} \\ z = \bar{z} \\ t = \frac{\frac{V}{c^2}\bar{x} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{array} \right. \quad (3.6)$$

$$\vec{r} = \vec{OP} \text{ et } \vec{\bar{r}} = \vec{\bar{O}P}$$

$$\vec{r} = \vec{\bar{r}} + \frac{\left(1 - \sqrt{1 - \frac{V^2}{c^2}}\right) \frac{\vec{V} \cdot \vec{\bar{r}}}{V^2} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \vec{V}$$

$$t = \frac{\frac{\vec{V} \cdot \vec{\bar{r}}}{c^2} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

• 3.3 La transformation de Lorentz

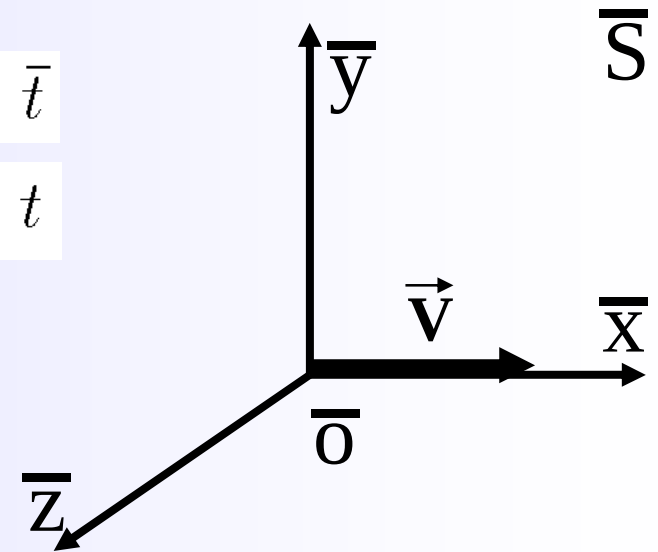


$$\left\{ \begin{array}{l} x = \frac{\bar{x} + V\bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y = \bar{y} \\ z = \bar{z} \\ t = \frac{\frac{V}{c^2}\bar{x} + \bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{array} \right. \quad (3.6)$$

•

$$\bar{x}, \bar{y}, \bar{z}, \bar{t}$$

$$x, y, z, t$$



$$v = \frac{\bar{v} + V}{1 + \frac{\bar{v}V}{c^2}} \quad (3.12)$$

$$\vec{\gamma} \neq \vec{\bar{\gamma}}$$

$$\bar{\gamma} = \gamma \left(\frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{vV}{c^2}} \right)^3$$

- 3.5 L'espace de Minkowski et la distance spatio-temporelle

$$x^\alpha = (ct, x, y, z) \quad (3.16)$$

Événement : point de E_4

Observateur : système de coordonnées de E_4

$$(x^0 = ct) \quad x^i (x^1 = x, x^2 = y, x^3 = z)$$

un événement E et deux observateurs S et $\bar{S}$

$$x^\alpha \text{ et } x^{\bar{\beta}}$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$x^\alpha = \sum_{\bar{\beta}=0}^3 \Lambda^\alpha_{\bar{\beta}} x^{\bar{\beta}} \quad (3.17)$$

$$x^\alpha = \Lambda^\alpha_{\bar{\beta}} x^{\bar{\beta}} \quad (3.18)$$

$$\Lambda^\alpha_{\bar{\beta}} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3.19)$$

$$\beta = \frac{V}{c} \quad (3.20)$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$x^\alpha = \Lambda^\alpha_{\bar{\beta}} x^{\bar{\beta}}$$

$$x^\alpha = \Lambda^\alpha_{\bar{\beta}} x^{\bar{\beta}} + M^\alpha \quad (3.21)$$

$$x_i = A_{ij} \left[\bar{x}_j + \left[(\gamma - 1) \frac{V_k \bar{x}_k}{V^2} + \gamma \bar{t} \right] V_j + a_j \right]$$

$$t = \gamma \left[\frac{V_k \bar{x}_k}{c^2} + \bar{t} \right] + b \quad (3.9)$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \quad (3.10)$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$x^\alpha = \Lambda_{\bar{\beta}}^\alpha x^{\bar{\beta}} + M^\alpha \quad (3.21)$$

$$\Lambda_{\bar{\beta}}^\alpha = \begin{pmatrix} \gamma & \gamma \frac{V_j}{c} \\ \gamma A_{ik} \frac{V_k}{c} & A_{ij} + (\gamma - 1) A_{ik} \frac{V_k V_j}{V^2} \end{pmatrix} \quad (3.22)$$

$$\begin{aligned} \alpha &= 0, i \\ \bar{\beta} &= 0, j \\ i, j &= 1 \dots 3 \end{aligned}$$

$$M^\alpha = \begin{pmatrix} b c \\ A_{ik} a_k \end{pmatrix} \quad (3.23)$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$E_1 : x^\alpha(1), x^{\bar{\alpha}}(1)$$

$$E_2 : x^\alpha(2), x^{\bar{\alpha}}(2)$$

$$\begin{aligned}\Delta x^\alpha &= x^\alpha(2) - x^\alpha(1) \\ \Delta x^{\bar{\alpha}} &= x^{\bar{\alpha}}(2) - x^{\bar{\alpha}}(1)\end{aligned}\quad (3.24)$$

$$\text{4-vecteur } \overrightarrow{E_1 E_2}$$

$$x^\alpha(2) = \Lambda^\alpha_{\bar{\beta}} x^{\bar{\beta}}(2) + M^\alpha$$

$$x^\alpha(1) = \Lambda^\alpha_{\bar{\beta}} x^{\bar{\beta}}(1) + M^\alpha$$

$$\Delta x^\alpha = \Lambda^\alpha_{\bar{\beta}} \Delta x^{\bar{\beta}} \quad (3.25)$$

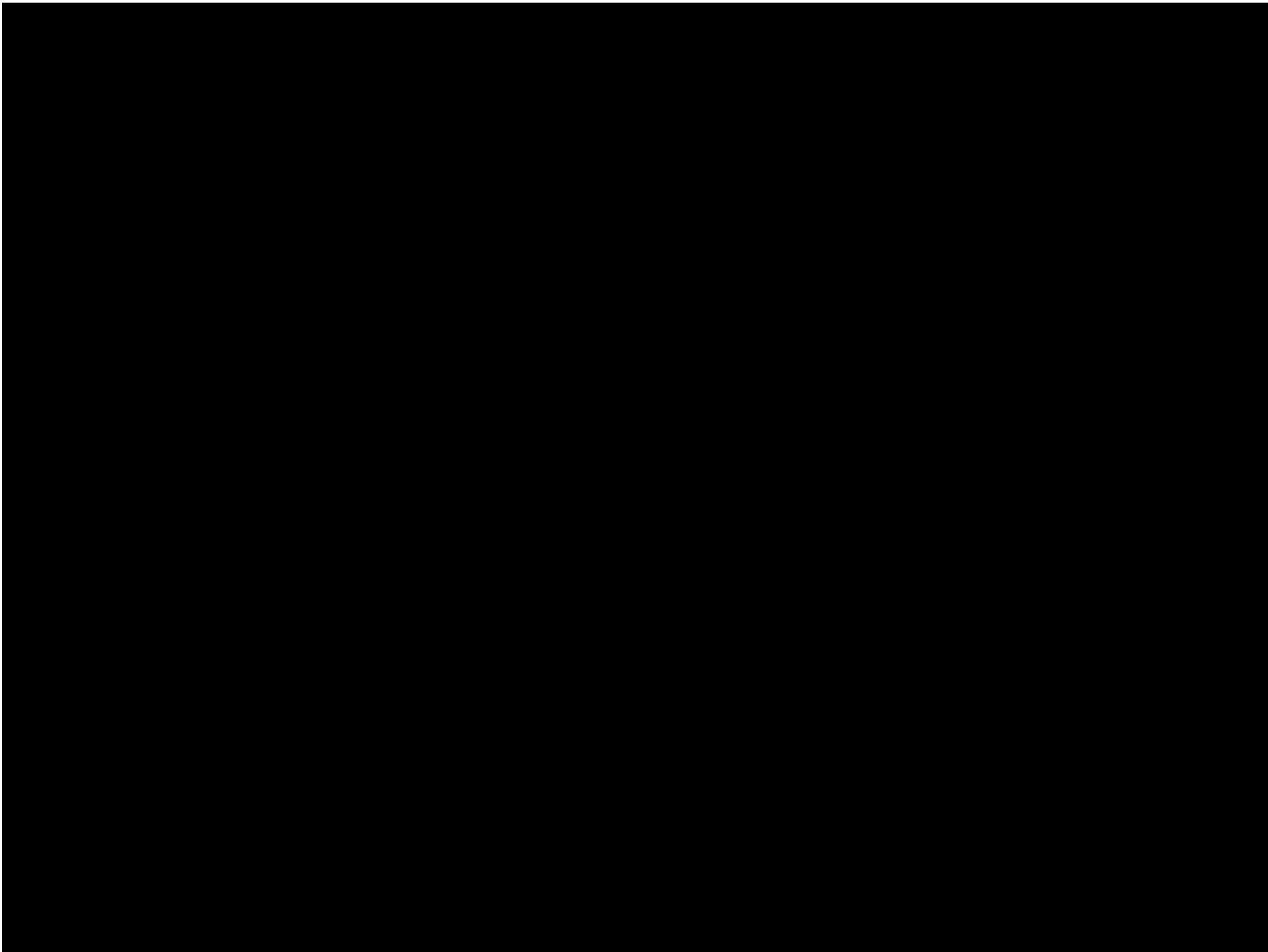
- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\Delta x^\alpha = \Lambda^\alpha_{\bar{\beta}} \Delta x^{\bar{\beta}} \quad (3.25)$$

$$\Delta s^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta = \eta_{\alpha\beta} \Lambda^\alpha_{\bar{\mu}} \Lambda^\beta_{\bar{\nu}} \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} \quad (3.26)$$

$$\begin{aligned} \eta_{oo} &= -1 \\ \eta_{oi} &= 0 \\ \eta_{ij} &= \delta_{ij} \end{aligned} \quad (3.27)$$

$$\Delta s^2 = -c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \quad (3.28)$$



- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\eta_{\alpha\beta} \Lambda_{\bar{\mu}}^{\alpha} \Lambda_{\bar{\nu}}^{\beta} \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}}$$

$$-c^2(\Delta \bar{t})^2 + (\Delta \bar{x})^2 + (\Delta \bar{y})^2 + (\Delta \bar{z})^2$$

$$\begin{aligned} \eta_{\alpha\beta} \Lambda_{\bar{\mu}}^{\alpha} \Lambda_{\bar{\nu}}^{\beta} \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} &= \eta_{00} \Lambda_{\bar{\mu}}^0 \Lambda_{\bar{\nu}}^0 \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} + \eta_{i0} \Lambda_{\bar{\mu}}^i \Lambda_{\bar{\nu}}^0 \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} \\ &+ \eta_{0j} \Lambda_{\bar{\mu}}^0 \Lambda_{\bar{\nu}}^j \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} + \eta_{ij} \Lambda_{\bar{\mu}}^i \Lambda_{\bar{\nu}}^j \Delta x^{\bar{\mu}} \Delta x^{\bar{\nu}} \\ &= - [\gamma^2 c^2 (\Delta \bar{t})^2 + 2\gamma^2 \beta c \Delta \bar{t} \Delta \bar{x} + \gamma^2 \beta^2 (\Delta \bar{x})^2] \\ &+ [\gamma^2 \beta^2 c^2 (\Delta \bar{t})^2 + 2\gamma^2 \beta c \Delta \bar{t} \Delta \bar{x} + \gamma^2 (\Delta \bar{x})^2] \\ &+ (\Delta \bar{y})^2 + (\Delta \bar{z})^2 \\ &= - c^2 (\Delta \bar{t})^2 + (\Delta \bar{x})^2 + (\Delta \bar{y})^2 + (\Delta \bar{z})^2 \end{aligned}$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$(\Delta t)^2 \quad (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2.$$

$$\Delta s^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta$$

$$\boxed{-c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 = -c^2(\Delta \bar{t})^2 + (\Delta \bar{x})^2 + (\Delta \bar{y})^2 + (\Delta \bar{z})^2} \quad (3.29)$$

$$\Delta s^2 = \delta_{ij} \Delta x^i \Delta x^j = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \quad (3.30)$$

$$\Delta \vec{x} \quad (\Delta x, \Delta y, \Delta z)$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

1) $\Delta s^2 < 0$ (intervalle Δs temporel)

le quadri-vecteur $\overrightarrow{E_1 E_2}$ est du genre "temps"

$$\Delta s^2 = |\Delta \vec{r}|^2 - c^2 \Delta \vec{t}^2 < 0 \quad (3.31)$$

$$\Delta \vec{r} = 0$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\Delta \vec{r} = 0$$

$$\vec{r} = \vec{r} + \left[(\gamma - 1) \frac{\vec{V} \cdot \vec{r}}{V^2} + \gamma \bar{t} \right] \vec{V} + \vec{a}$$

$$t = \gamma \left(\frac{\vec{V} \cdot \vec{r}}{c^2} + \bar{t} \right) + b \quad (3.8)$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \quad (3.10)$$

$$\Delta \vec{r} + \left[(\gamma - 1) \frac{\vec{V} \cdot \Delta \vec{r}}{V^2} + \gamma \Delta \bar{t} \right] \vec{V} = 0$$

$$V < c \quad \vec{V} = -\frac{\Delta \vec{r}}{\Delta \bar{t}} \quad (3.32) \quad V^2 = \frac{|\Delta \vec{r}|^2}{\Delta \bar{t}^2} < c^2$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\vec{r} = \vec{\bar{r}} + \left[(\gamma - 1) \frac{\vec{V} \cdot \vec{\bar{r}}}{V^2} + \gamma \bar{t} \right] \vec{V} + \vec{a}$$

$$t = \gamma \left(\frac{\vec{V} \cdot \vec{\bar{r}}}{c^2} + \bar{t} \right) + b \quad (3.8)$$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \quad (3.10)$$

$$\vec{V} = -\frac{\Delta \vec{\bar{r}}}{\Delta \bar{t}} \quad (3.32)$$

$$\Delta t = \gamma \left[\frac{\vec{V} \cdot \Delta \vec{\bar{r}}}{c^2} + \Delta \bar{t} \right] = \frac{\Delta \bar{t}}{\gamma} \quad (3.33)$$

l'ordre temporel de ces événements est donc absolu.

deux événements en relation causale ont un Δs^2 négatif.

- 3.5 L'espace de Minkowski et la distance sp.-temp.

2) $\Delta s^2 = 0$ (intervalle Δs lumineux ou nul)

3) $\Delta s^2 > 0$ (intervalle Δs spatial)

Le quadri-vecteur $\overrightarrow{E_1 E_2}$ est dit du "genre espace"

$$\Delta s^2 = |\Delta \vec{r}|^2 - c^2 \Delta \bar{t}^2 > 0$$

$$\Delta t = \gamma \left(\frac{\vec{V} \cdot \Delta \vec{r}}{c^2} + \Delta \bar{t} \right) = 0 \quad \frac{\vec{V} \cdot \Delta \vec{r}}{c \cdot c \Delta \bar{t}} = -1 \quad V < c$$

- 3.5 L'espace de Minkowski et la distance sp.-temp.

$$\Delta t = \gamma \left(\frac{\vec{V} \cdot \Delta \vec{r}}{c^2} + \Delta \bar{t} \right) = 0$$

$$\Delta \bar{t} = \gamma \left(\Delta t - \frac{\vec{V} \Delta \vec{r}}{c^2} \right)$$

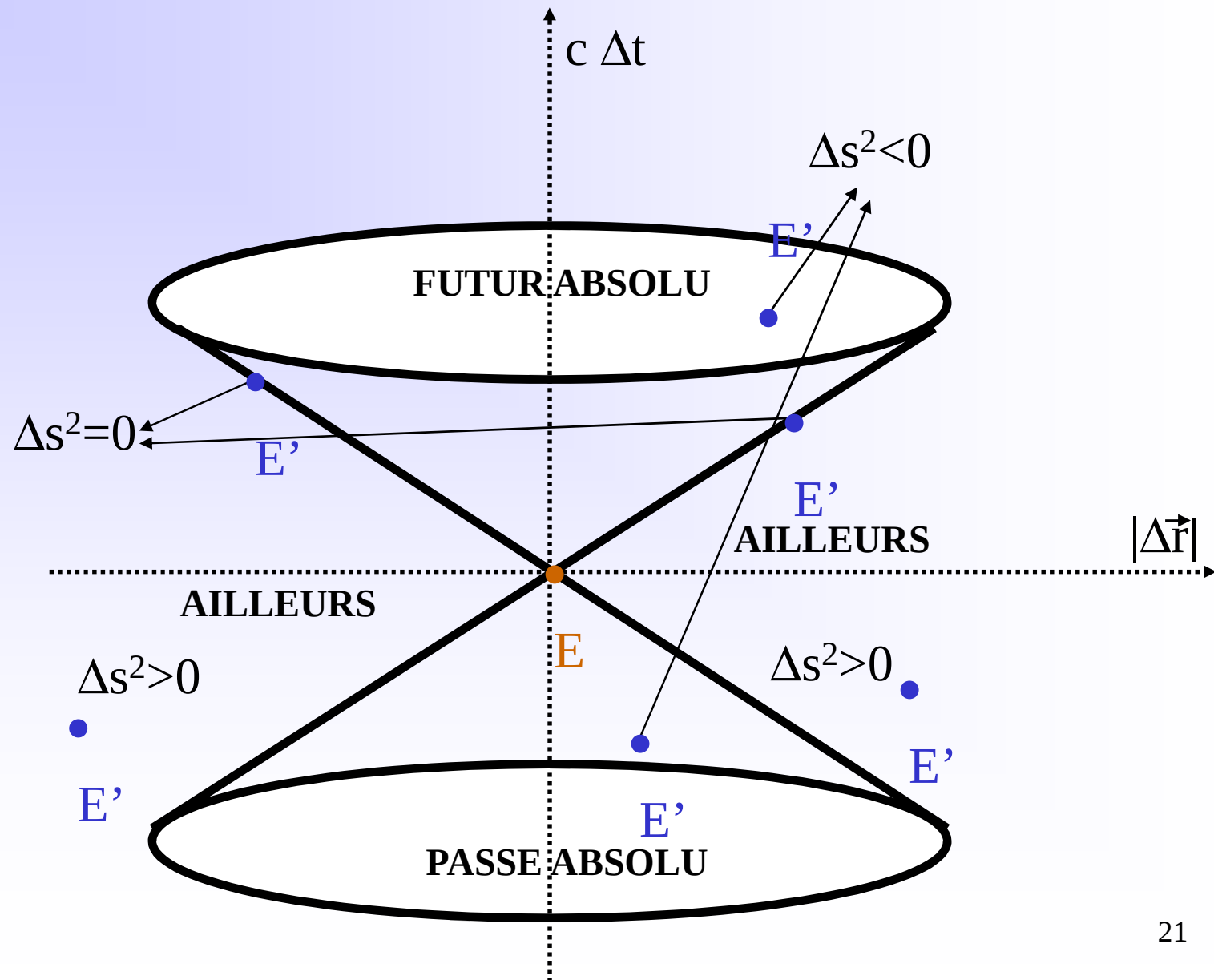
$$\Delta \bar{t} = - \frac{\vec{V} \Delta \vec{r}}{\sqrt{c^2 - V^2}} \cdot \frac{1}{c}$$

Le cône de lumière

$$(x' - x)^2 + (y' - y)^2 + (z' - z)^2 - c^2(t' - t)^2 = 0$$

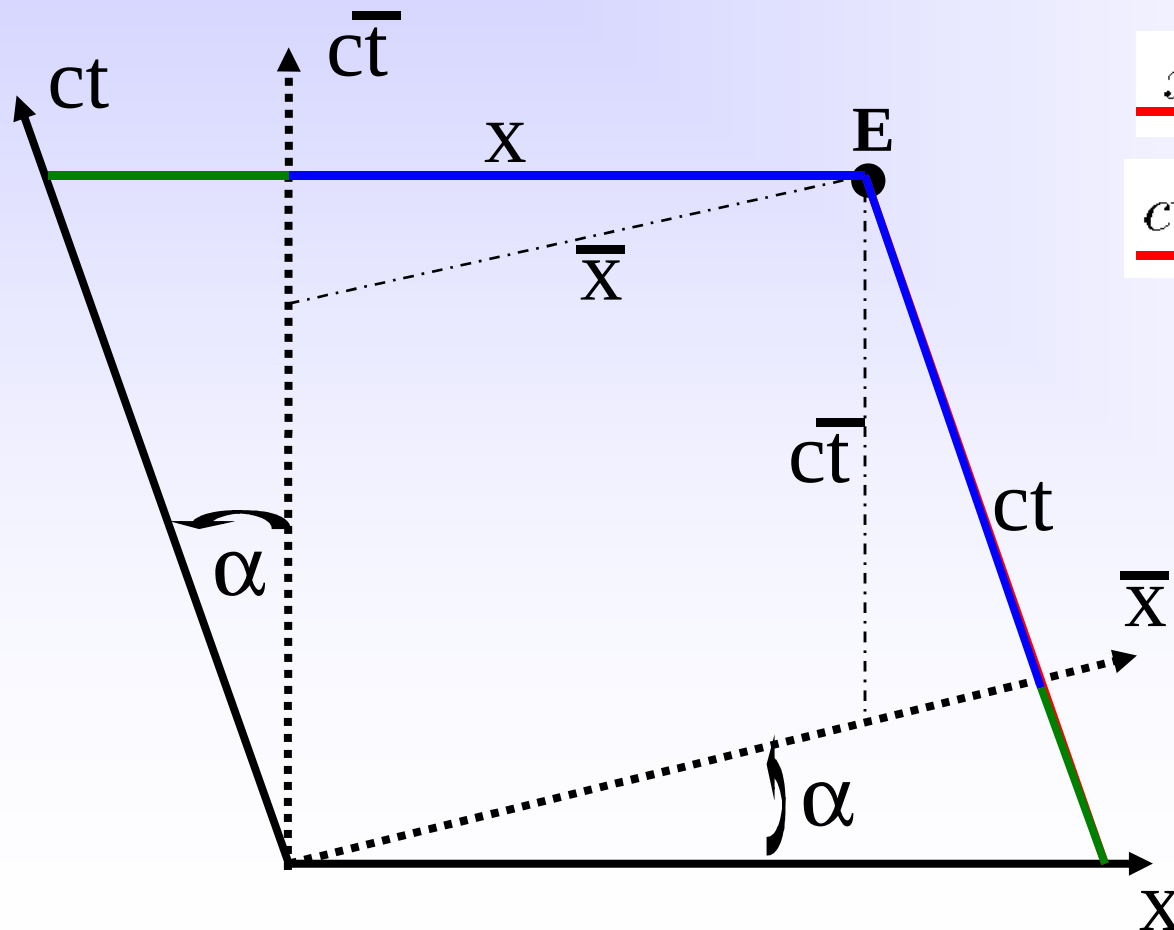
(3.34)

- 3.5 L'espace de Minkowski et la distance sp.-temp.



- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 1 :



$$\underline{x} = \underline{\cos \alpha \bar{x}} + \underline{\sin \alpha ct}$$

$$\underline{ct} = \underline{\sin \alpha x} + \underline{\cos \alpha \bar{ct}}$$

$$\sin(\alpha) = V/c$$

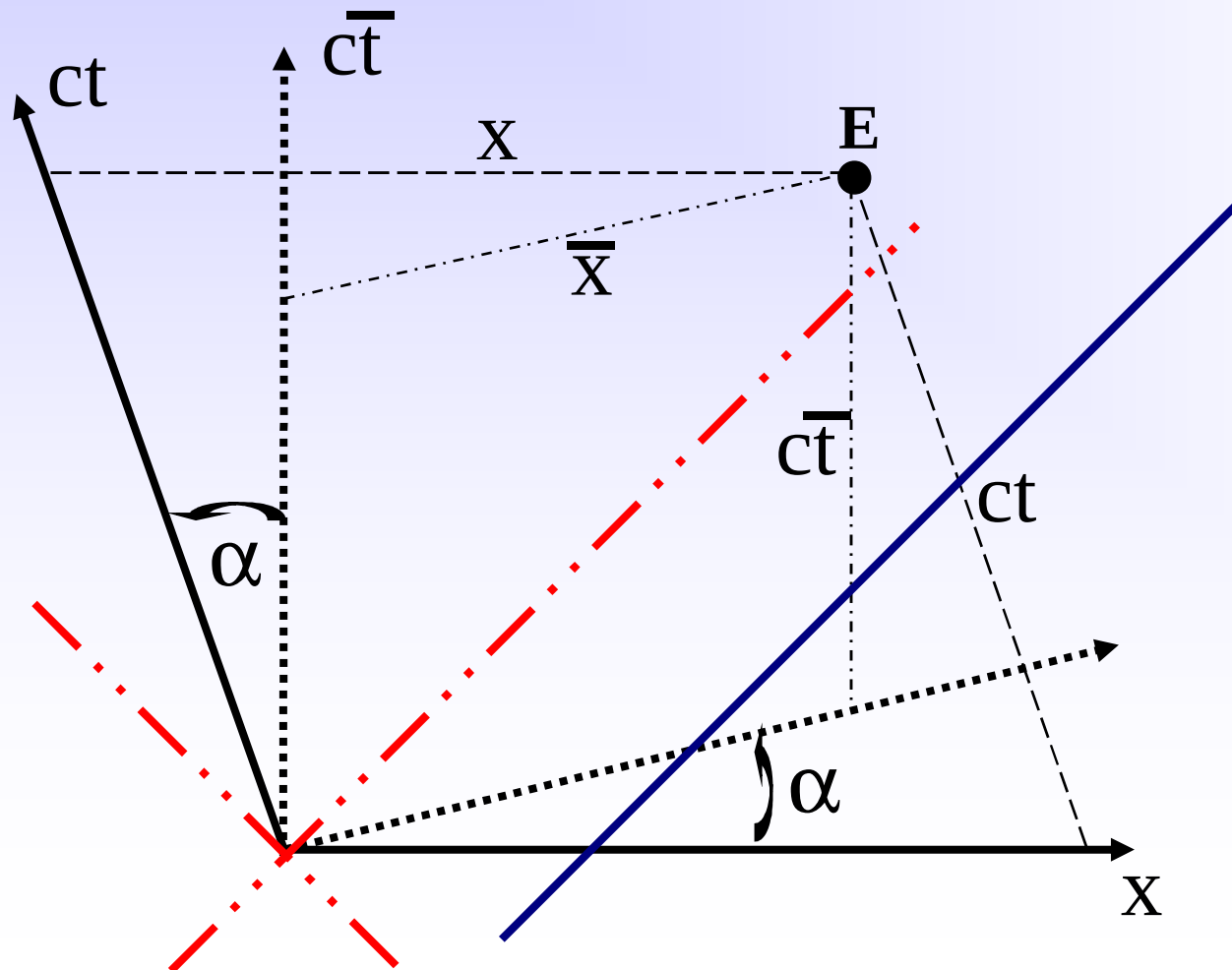
- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 2 :

$$x = \pm ct + C^{te}$$

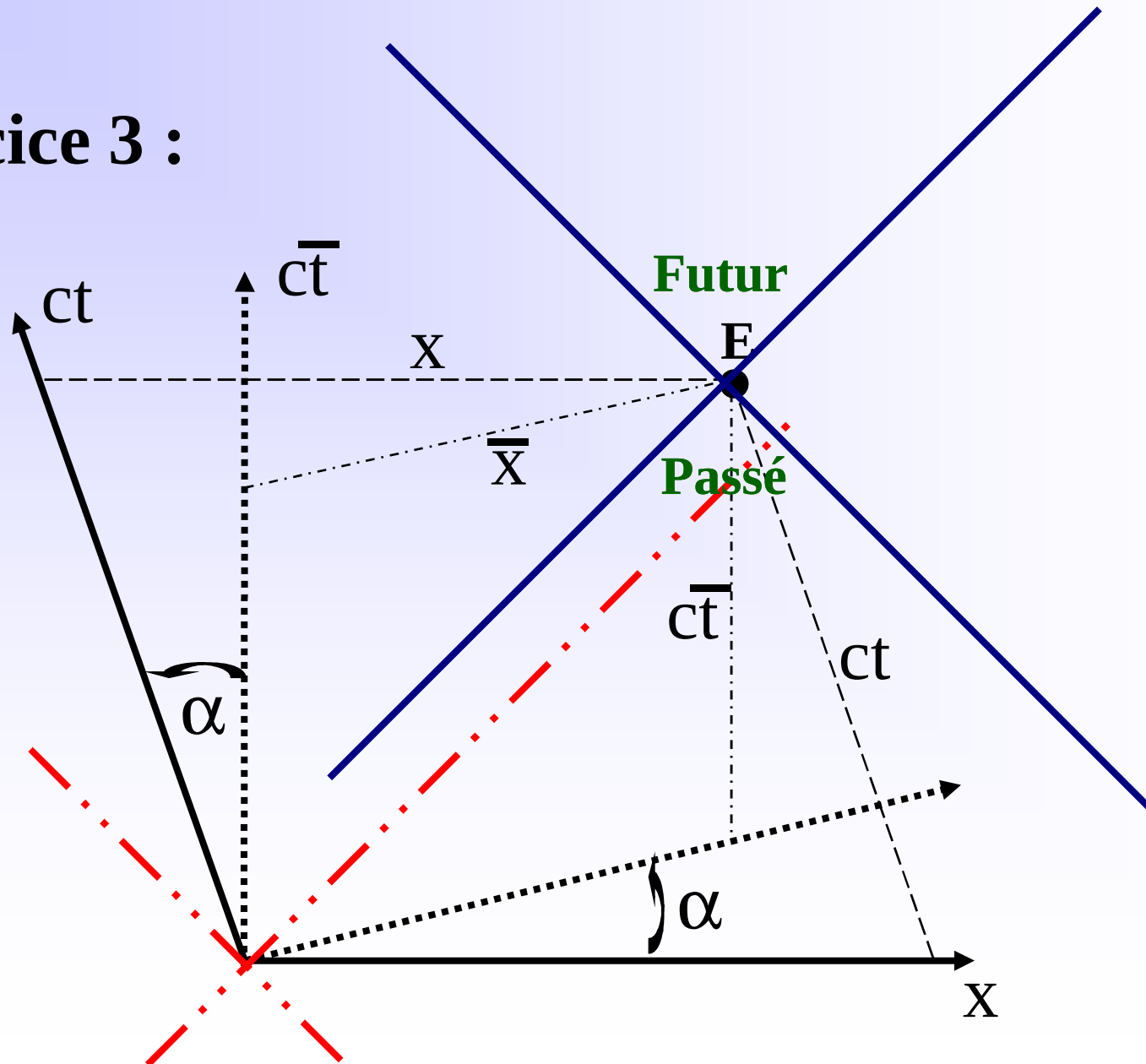
$\Leftrightarrow$

$$\bar{x} = \pm c\bar{t} + C^{te'}$$



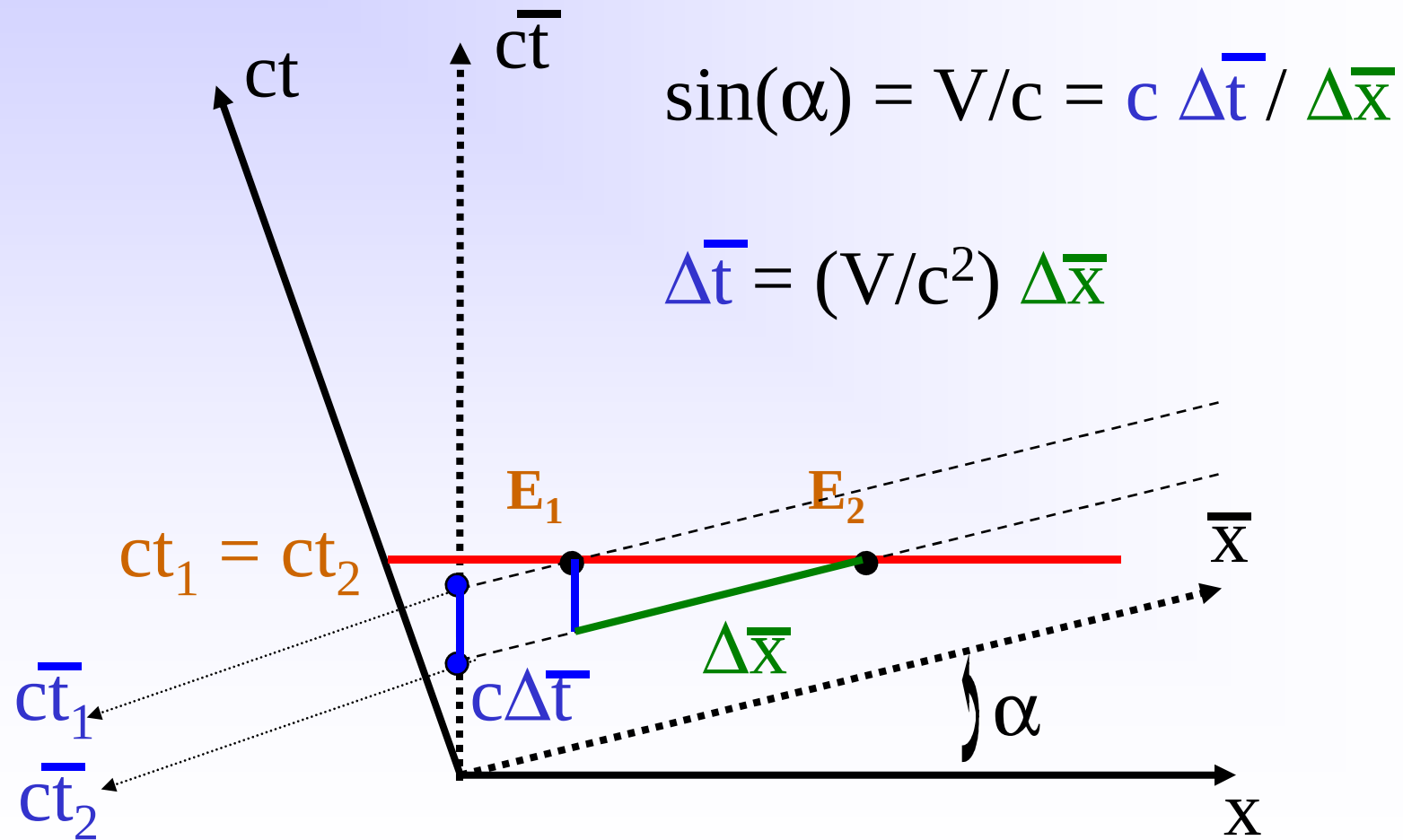
- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 3 :



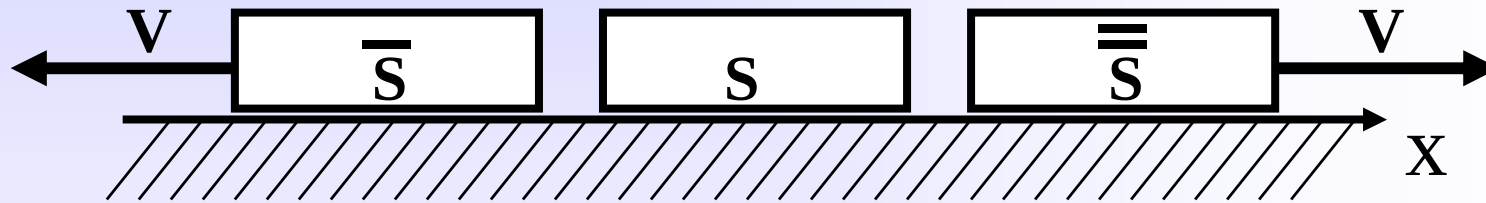
- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 4 :



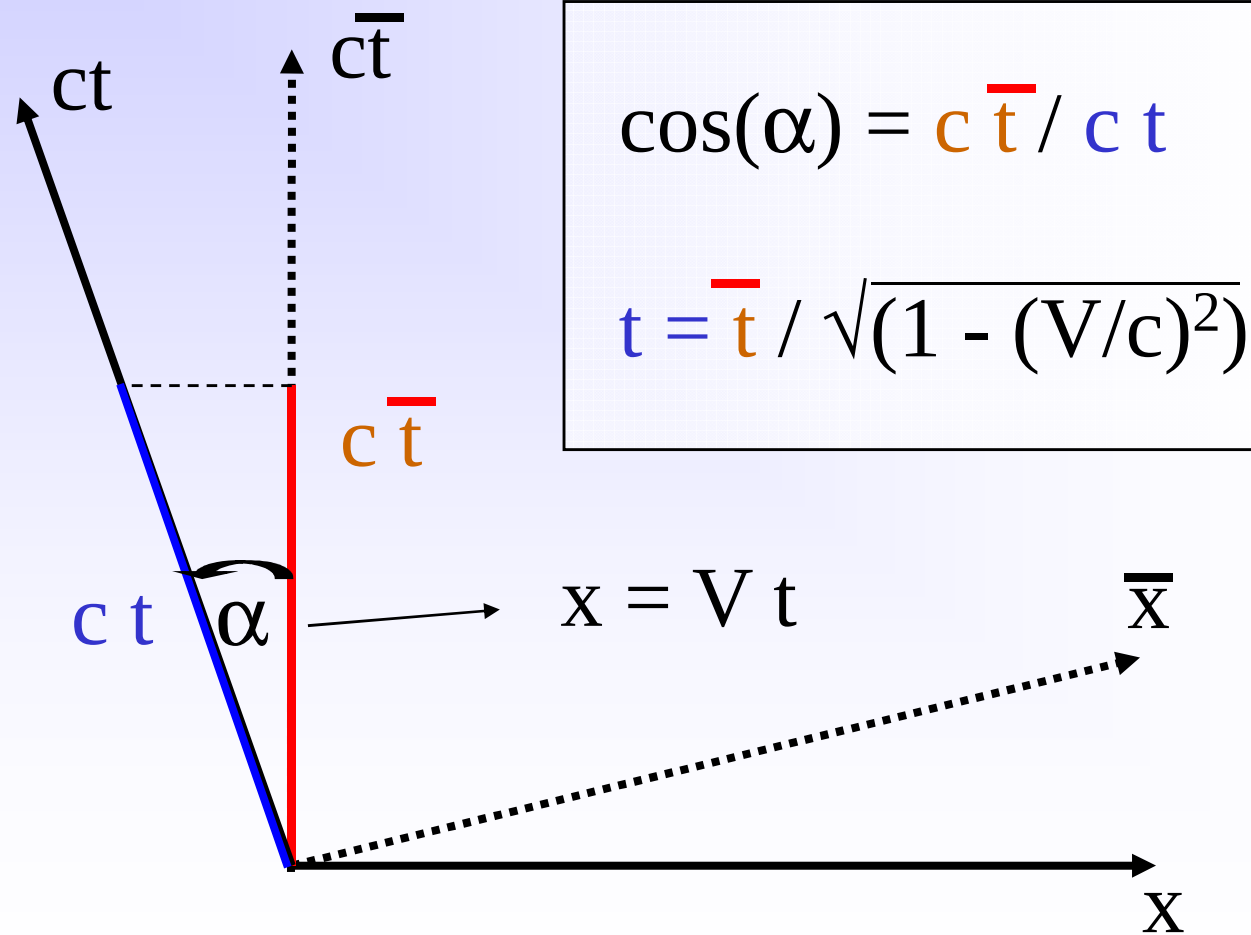
- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 5 :



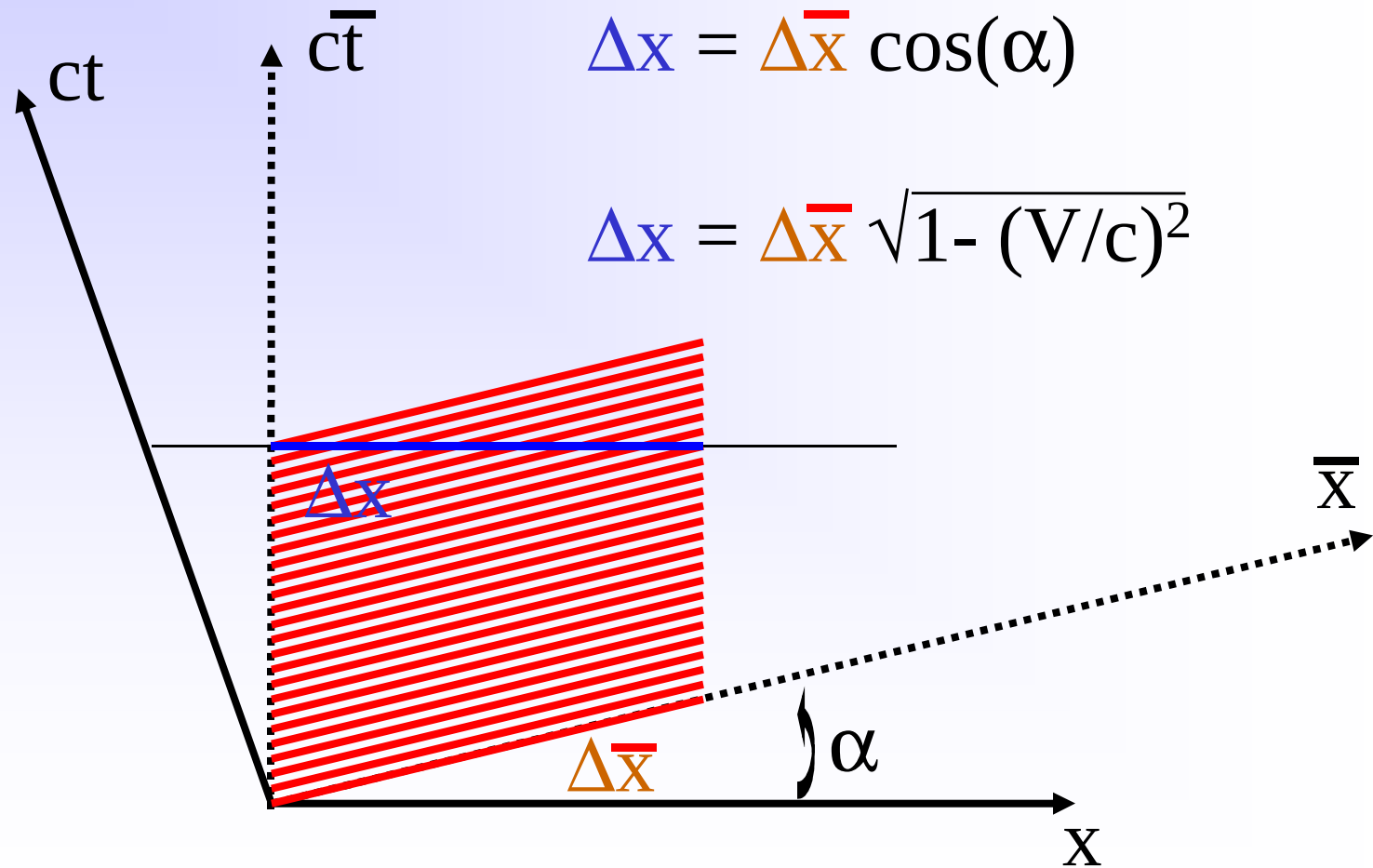
- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 6 :

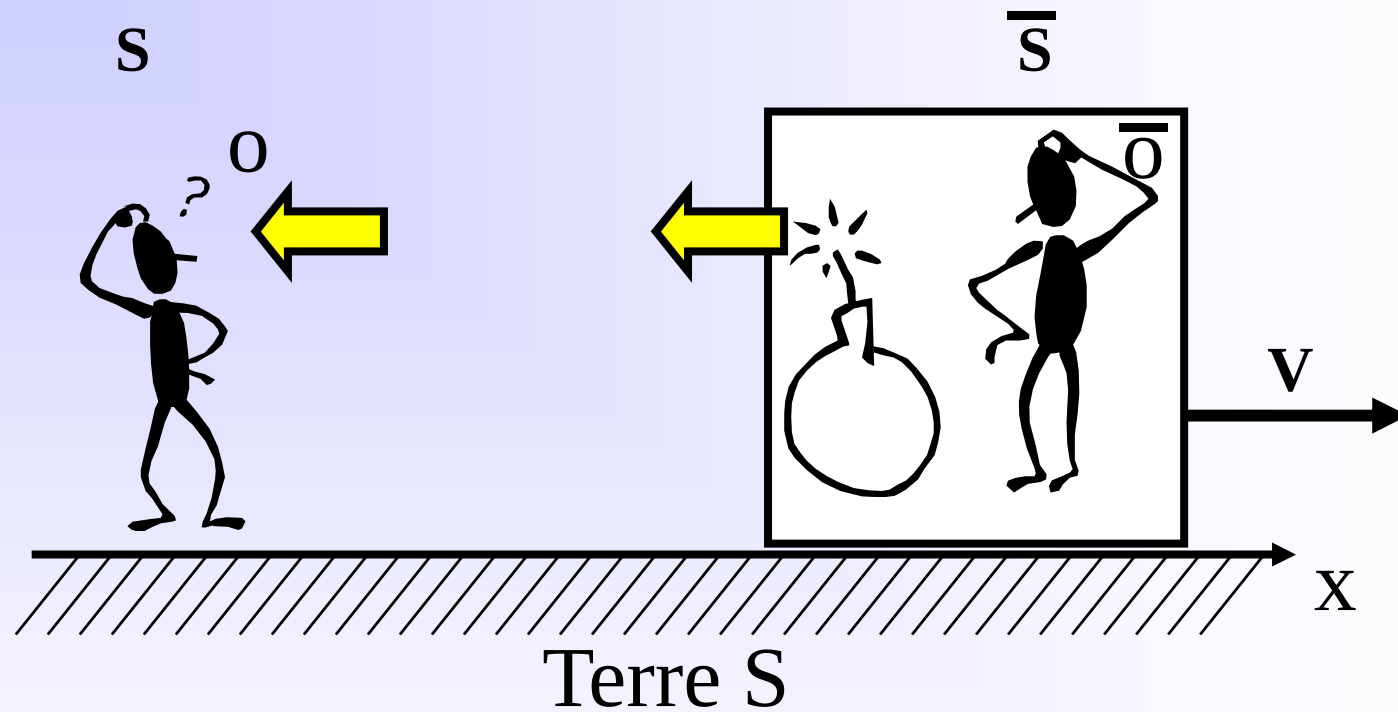


- 3.5 L'espace de Minkowski et la distance sp.-temp.

Exercice 7 :



- 3.6 L'effet Doppler longitudinal



	$\bar{S}$	S
E_1	$\bar{x}_1 = 0, \quad \bar{t}_1 = \bar{t}$	
E_2		
		$x_2 = 0, \quad t_2 = t$

- 3.6 L'effet Doppler longitudinal

	$\bar{S}$	S
E_1	$\bar{x}_1 = 0, \quad \bar{t}_1 = \bar{t}$	
E_2		$x_2 = 0, \quad t_2 = t$

	$\bar{S}$	S
E_1	$\bar{x}_1 = 0, \quad \bar{t}_1 = \bar{t}$	$x_1 = \frac{V\bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad t_1 = \frac{\bar{t}}{\sqrt{1 - \frac{V^2}{c^2}}}$
E_2	$\bar{x}_2 = \frac{-Vt}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad \bar{t}_2 = \frac{t}{\sqrt{1 - \frac{V^2}{c^2}}}$	$x_2 = 0, \quad t_2 = t$

- 3.6 L'effet Doppler longitudinal

$$(\bar{x}_2 - \bar{x}_1)^2 = c^2(\bar{t}_2 - \bar{t}_1)^2 \quad (3.35)$$

$$c(\bar{t}_2 - \bar{t}_1) = -\bar{x}_2$$

$$c \left(\frac{t}{\sqrt{1 - \frac{V^2}{c^2}}} - \bar{t} \right) = \frac{Vt}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$t = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} \bar{t} \quad (3.36)$$

$$\Delta t = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} \Delta \bar{t}$$

- 3.6 L'effet Doppler longitudinal

$$\Delta t = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} \Delta \bar{t}$$

$$\tau = \Delta t$$

$$\bar{\tau} = \Delta \bar{t}$$

$$\tau = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} \bar{\tau}$$

$$\lambda = c\tau \quad \bar{\lambda} = c\bar{\tau}$$

$\Rightarrow$

$$\boxed{\lambda = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} \bar{\lambda}}$$

(3.37)

- 3.6 L'effet Doppler longitudinal

$$\lambda = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} \bar{\lambda}$$

$$\lambda > \bar{\lambda} \text{ (red-shift)}$$

$$z = \frac{\Delta\lambda}{\bar{\lambda}} = \frac{\lambda - \bar{\lambda}}{\bar{\lambda}}$$

$$\Delta\lambda = \lambda - \bar{\lambda}$$

$$z = \frac{\sqrt{1 + \frac{V}{c}}}{\sqrt{1 - \frac{V}{c}}} - 1$$

$$z = 6.28$$

$$V/c = 0,963$$

