

# THE MODIFIED REYMENT'S MULTIVARIATE COEFFICIENT OF VARIATION

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The univariate coefficient of variation  $CV = \sigma/|\mu|$  measures the relative variability of a random variable  $X$  to its mean ( $\mu \neq 0$ ). The  $CV$  is for example widely used in laboratory medicine for comparing the reproducibility of assay techniques/equipments: the lower the  $CV$ , the better the analytical precision. The multivariate coefficient of variation deals with the relative variability of a  $p$ -dimensional random vector  $\mathbf{X}^T = (X_1, \dots, X_p)$  with mean  $\boldsymbol{\mu} \neq \mathbf{0}$  and variance-covariance matrix  $\boldsymbol{\Sigma}$ . R. Reymont (1960) was the first to extend the univariate  $CV$  concept to the multivariate setting by defining  $CV_{RR} = \left( |\boldsymbol{\Sigma}|^{1/p} / \boldsymbol{\mu}^T \boldsymbol{\mu} \right)^{\frac{1}{2}}$ . He also derived a formula for the standard error of the estimate. There is supporting evidence however that Reymont's multivariate  $CV$  yields values that are too low compared to the methods which were introduced later on and that some flaws slipped into the standard error formula (Zhang & Albert, 2010). The talk starts with reviewing existing multivariate  $CV$  definitions. Then, we focus on correcting Reymont's definition by including a scaling factor  $\sqrt{p}$ . The estimated modified Reymont's  $CV$  and its standard error write  $\widehat{CV_{RR}^*} = \left( p |\mathbf{S}|^{1/p} / \bar{\mathbf{x}}^T \bar{\mathbf{x}} \right)^{\frac{1}{2}}$  and  $\text{s.e.}(\widehat{CV_{RR}^*}) = \widehat{CV_{RR}^*} \left[ \frac{(2n-p+1)}{4p(n-p)(n-p+1)} + \frac{\bar{\mathbf{x}}^T \mathbf{S} \bar{\mathbf{x}}}{n(\bar{\mathbf{x}}^T \bar{\mathbf{x}})^2} \right]^{\frac{1}{2}}$ . The various multivariate  $CV$ s are illustrated on electrophoretic data from the French and Belgian External Quality Assessment programmes (Zhang et al., 2010). The modified Reymont's approach provides results which are consistent with and comparable to those of the other multivariate coefficients of variation.

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## *References:*

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