

A spherical metric for the field-oriented analysis of complex urban open spaces

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Abstract. The author deals with the analysis of urban open spaces, once conceived as part and parcel of our urban heritage. He introduces a mathematical modelling technique that is capable of mapping the variation of the sky visible from points distributed throughout space. The resulting maps overcome the limits of orthographic (plan, section, and elevation) and perspective methods of analysis by considering the dynamic qualities of the Gibsonian 'visual world' that takes account not only of bifocal vision but also of the relatively free movement of the head and shoulders, that is, vision as part of the human ecology. The maps show how a person might experience those volumes of a void that define a space, not from a fixed point but from moving about inside the entire urban open space.

1 Introduction

Urban open spaces can be understood in a variety of ways depending on the scientific approach to define and describe them. This notion actually appears to be closely related to political, social, or environmental concepts, such as the *public realm* (Joseph, 1992), *meeting places* (Sennet, 1990), or *green areas* (EGUE, 1996). From a morphological point of view, urban open spaces are usually defined as the empty space, or the void, separating the built volumes and surfaces. Defined as such, the shape of such space would obviously be characterised by a relation of strict duality with the 'filled elements' that surround it: buildings, vegetation, fences, and screens. Accordingly, an open space will be considered as 'dominant' when its geometrical form tends to determine the building layout. In some cases, this relation of dominancy implies a deformation of the walls of the buildings, as in the case of the Place des Victoires in Paris, where the façades are incurved to delineate better the circular open space. In the case of the Place des Vosges in Paris, the relation of dominancy explains the partial autonomy of the open space towards the urban pattern within which it was inserted. This type of spatial organisation always indicates some form of social organisation governing the construction of the space, either as a result of a *fait du Prince* (a royal edict to enforce arbitrary decisions, used by kings in ancient times) or as the result of the continued efforts of a highly integrated society (Lavedan, 1941). In contrast, open space is usually termed 'residual' when its geometry appears to be determined by the capricious layout of built structures. Its character would then rely upon the delicate coordination of all the individual buildings.

The dominant type of spatial organisation has long been mistaken for the mere notion of urban design itself. Most notably, followers of the modern movement have alleged that dominant spaces somehow testify to a more advanced stage of urban design (for instance, see Giedeon, 1941). Seeking to revive and further develop a rationalist tradition, some modernists have cited ideal patterns of the past, founded on 'rational principles'—for example, ideal Renaissance cities or Baroque compositions—based on a retrospective analysis of urban design. Accordingly, regular and geometrically shaped open spaces were to be considered superior to residual spaces, which were mostly denied any intrinsic quality. This 'evolutionist' view appeared also

to be supported by a number of urban morphology pioneers, such as Lavedan (1941), who somehow considered the square-shaped French royal places as a benchmarking standard for assessing any open space.

As brilliantly illustrated by Bacon (1967), this view appears to be contradicted by numerous outstanding examples of residual open spaces such as, for instance, the Athenian Agora of the Hellenistic period, or some medieval Italian places. Citations of remarkable historical examples of residual open spaces can be found amongst the work of a number of other authors, from Sitte (1945) to Kostoff (1991). It could be argued that these residual spaces are somehow exceptional and that they are either accidental or characteristic of the premodern period, once considered as a sort of lost paradise. Still, the same remark could be addressed to most dominant spaces, as many of them date from the Renaissance and Baroque periods. Furthermore, one could easily find residual spaces of great quality within contemporary urban settings, as, for instance, the esplanade of La Defense in Paris or the squares and parks of Barcelona (Favole, 1995; Rowe, 1997). Arguably, these examples would lead some authors to consider that the collapse of the enclosed perspective space is a key aspect of the new visual urban space (for instance, see Chelkoff and Thibaud, 1992).

It has to be acknowledged that there is not a strict dichotomy between residual and dominant open spaces. As suggested by Thiel (1961), these prototypical situations are better understood as two intermediate situations within a larger continuum, ranging from 'vagues'—ambiguous spaces that result from the use of very few establishing elements, as in some low-density modern environments—to 'volumes'—completely defined spaces resulting from the use of complete and continuous surfaces, as in the case of public courtyards. Hence it would be interesting to develop an analysis method that can be applied to the entire spectrum of urban open spaces to characterise their shape.

In this paper, a new metric for analysing complex open spaces is proposed. The paper begins with an examination of existing methods of open-space analysis. Two different approaches are distinguished: the traditional Euclidean analyses, and the emerging field-oriented approaches. Spherical indicators are then introduced and discussed, as they appear to combine the advantages of both sets of methods. These indicators are then applied to a real-world situation, the analysis of Place Saint-Lambert in Liège, as a 'live testing' of the methodology. Finally, the environmental assessment of projects and plans affecting urban cultural heritage is suggested as a possible application for this kind of metric.

2 Field-oriented open-space analyses

Urban open spaces are 'abstract' by nature. They are not a material element of the physical world such as a building, road, or row of trees. Furthermore, open space is a continuous medium. Usually, the two-dimensional enclosure of a place or street is not complete, even though there are well-known exceptions to this statement (such as the forum at Pompeii). The limits of urban open spaces can thus be quite fuzzy or contestable (Peponis et al, 1997).

Notwithstanding these properties, urban open spaces have often been treated as 'open-air rooms' since the Renaissance. Such an analogy basically postulates that, even though virtual and continuous, the shape of an urban open space can be reduced to a composition of elementary geometric volumes. The properties of these volumes, or so-called 'urban boxes', can then be characterised through simple Euclidean ratios as proposed by Albertian theories. The most widely used Euclidean ratio is the prospect of a place (the ratio of the height to the midlength of the square). According to Spreiregen (1965), this ratio should lie between 1:1 and 1:3. More sophisticated

Euclidean ratios have also been proposed in the literature, as, for instance, the three-dimensional compactness of urban open spaces (Dupagne and Teller, 1999).

Obviously, such a conceptual model is not neutral because it tends to favour the above-mentioned dominant spatial configurations, which are easy to assimilate to finite volumes. From a more theoretical stance, it can be argued that the 'urban box' model tends to disregard the contemporary conception of space, where the identity of objects is not given a priori—as in the case of 'urban boxes'—but constructed by experience. With reference to Couclelis (1992), one can indeed distinguish between two main spatial ontologies, namely, the *atomist* and *plenum* conceptions of space.

In the atomist conception, space is assimilated to a simple referential where objects are identified and defined by clear and stable limits (a point, a line, a surface). The objects exist independently of their attributes (colour, texture, and so on) and can be manipulated in space (through translations, rotations, scaling, and so on) without altering their identity. This conception is pertinent mostly for quotidian objects of our world that we can handle directly, such as a chair, a piece of paper, and so on. In principle, urban open space does not belong to this type of object. Its limits are not entirely material and it can never be 'handled'. Its identity derives from the position and characteristics of the surrounding buildings. Accordingly, the notion of 'urban boxes' is nothing but an analogy. It is nevertheless quite structuring given the ubiquity of the atomist ontology. It assumes that, mentally, we tend to organise open space as defined by discrete entities similar to the objects we interact with in our daily lives.

It has already been noted that residual urban spaces somehow resist this kind of simplification, as their boundaries are often unclear or even unstable. Moreover, the identity of a place can be determined by a series of external factors that do not belong to it once reduced to a box—for instance, landmarks and glimpses (Lynch, 1960), tactile or olfactory sensations (Hall, 1966), or even cultural and behavioural references (Rapoport, 1975). Typically, the plenum conception postulates that the space does not exist a priori; rather, its definition results from the singular combination of a whole series of factors, such as the dominant axes, the layout of built masses, and so on. These different factors tend to affect configurations of attributes, which can be more or less homogeneous and stable. Accordingly, the plenum ontology tends to view a space as a continuous field, such as the image of a 'magnetic field'. In this conception of space it is easy to conceive that the boundaries of urban open spaces can be fuzzy and heavily reliant upon external conditions—on modification of building mass at one point of the system, for instance. This ontology also accommodates possible overlaps between different open spaces. As such, it is quite close to the notions of territory and territoriality formalised by the anthropological sciences. It also corresponds to the ecological model of perception proposed by Gibson (1950), as it tends to consider dynamic variations as the mainstay of our interaction with the environment.

The plenum conception of space underlies some recent spatial analysis methods: isovist fields (Batty, 2001; Benedikt, 1979; Turner et al, 2001), space syntax (Hillier and Hanson, 1984), and convex partitions (Peponis et al, 1997). All these methods share an interest in variations within space conceived as a continuous substrate. According to this approach, spatial configurations are proposed as a privileged way to characterise the identity of open spaces. When compared with traditional Euclidean ratios, these methods are nevertheless characterised by a major drawback; they do not fully consider the third dimension of urban open spaces.

The different field-oriented methods outlined above have all been developed for the analysis of plans, either of buildings or of urban patterns. Though some authors

claim that their methods can be applied to three-dimensional spatial configurations (Turner et al, 2001) such extensions are usually neither developed nor tested. Hillier and Hanson (1984, page 272) reject the utility of a three-dimensional analysis:

“Perhaps contrary to appearance, human spatial organisation is not three-dimensional in the same sense that it is two-dimensional—for the simple reason that human beings do not fly and buildings do not float in the air. Human space is in fact full of strategies—stairs, lifts, etc.—to reduce the three-dimensional structures to the two dimensions in which human beings move and order space.”

Even if there is some truth in this, does it still justify ignoring the height of the buildings surrounding our view? It probably depends on the scale of analysis and the type of urban pattern. In actual spaces, buildings exhibit large variations in height, especially in contemporary urban environments. These variations may have strong perceptual effects even if observed along a two-dimensional path.

The method of isovist fields could probably be generalised at the third dimension, especially as its computation is based on the graph theories proposed by Turner et al (2001). It would therefore be sufficient to consider the topological relations between points located in a plan and a three-dimensional grid filling the entire space. Still, this would apply only to spatial configurations of which the overall limits can be fixed beforehand, namely, bounded spatial configurations, such as spaces within a building. Urban open space is not like this, as it is never entirely enclosed: the sky is its only ceiling and it is usually assumed to be at an infinite distance from the observer. The application of isovist fields to the three-dimensional analysis of urban configurations would thus require an arbitrary height limit for unbounded open space. Such a limit would probably be hard to justify from a theoretical point of view.

Accordingly, a different mathematical approach has been adopted for the calculation of the field-mapped properties of urban open spaces. It is based on the use of non-Cartesian geometry, in order to enable a three-dimensional analysis of unbounded configurations, and it is assumed that the sky is, theoretically, located at an infinite distance from the observer.

3 Indicators based on spherical analyses

The open-space metric developed here is based on the use of spherical projections. It has to be stressed that the use of spherical views to analyse urban open spaces is not entirely new, but it has been applied mostly in the field of energy analysis. Markus and Morris (1980) describe a spherical construction method for calculating solar paths and masks. More recently, Bosselmann (1998) has used fish-eye photographs to analyse and compare solar availability within urban open spaces in San Francisco. In relation to these approaches, the main originality of the work in this paper lies in the development of morphological indicators derived from spherical views and their subsequent application to the field-oriented analysis of open spaces. Spherical views of urban scenes should hence be computed automatically, on numerical models of urban scenes, so as to allow such large-scale application.

The spherical projection of a three-dimensional point is computed in two stages: (1) projection of the point from the three-dimensional Cartesian space to a sphere and (2) projection from the sphere to a plane (figure 1). The computer algorithm to perform spherical projections of faces and volumes has been described elsewhere (Teller and Azar, 2001). Its input is a three-dimensional model of the urban environment in DXF format.

The second projection (see figure 1), from sphere to plane, establishes the properties of different spherical projections. It is well known, from the representation of the terrestrial globe, that a spherical surface cannot mathematically be ‘unrolled’ onto a

plane surface without some ‘deformations’. Snyder (1987) has provided a comprehensive review of the different spherical projection methods applied in cartography, discussing their respective mathematical construction methods and application domains. So-called equidistant projections respect angular distances. Isoarea projections respect relative solid angles as intercepted onto the sphere. Stereographical projections respect relative angles. Gnomonic projections respect the linearity of segments of the faces. There is no single projection that presents all these properties simultaneously, and each type of projection will support the analysis of some specific relations between the built elements and the open spaces but exclude others (Teller, 2001).

Figure 2 shows a stereographical view of the Amalienborg Plads in Copenhagen. The outer circle of the figure corresponds to the reference circle of figure 1. The projection was computed from the centre of the space. The square is of the dominant type and is characterised by an octagonal shape. Four symmetrical buildings border it, each composed of a main building and two smaller wings. The place is located at the

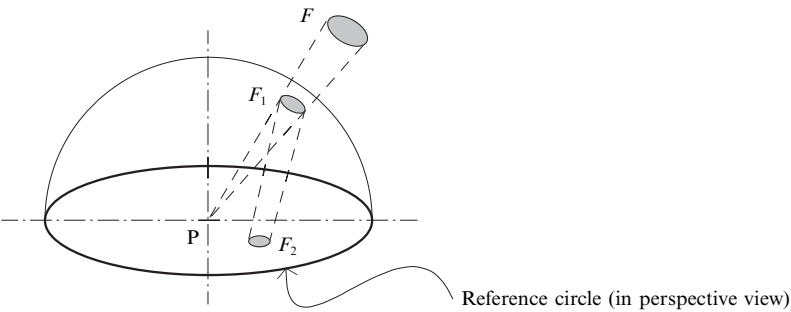


Figure 1. Spherical projection methods of a face, *F*. Note: P, projection point (centre of the sphere); projection 1, $F \rightarrow F_1$; projection 2, $F_1 \rightarrow F_2$.

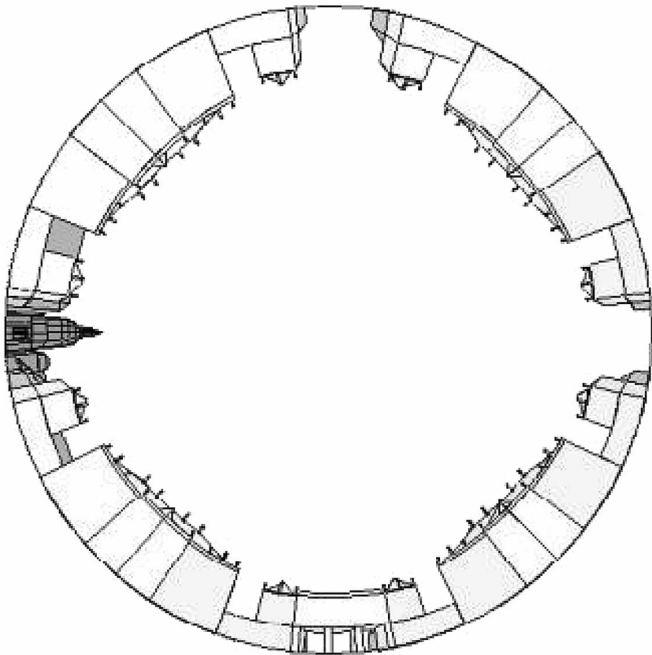


Figure 2. Example of a stereographical projection: Amalienborg Plads, Copenhagen.

intersection of two important axes. The Amaliengade street (the vertical axis in figure 2) crosses the historical city of Copenhagen. The other axis (the horizontal axis in figure 2) is largely opened towards the harbour on one side and frames a perspective towards the monumental Frederiks Kirke on the other side.

This view shows how spherical projections can represent three-dimensional relations between an open space and its surrounding buildings. The shape of the sky opening is significant in this view as it expresses the relations between the buildings and the open space viewed from a specific point. These relations can be formalised through parameters directly measured within spherical views, such as the sky opening or the regularity of the skyline, and so on. If repeated along a great number of points distributed on a matrix covering the entire model, these measures can be used to study the variation of a given parameter throughout the open space. The matrix of attributes thus obtained can then be represented as a field, emphasising the dynamic experience of an open space, a central concern of the ecology of vision developed by Gibson (1950).

Most importantly, spherical projections can transform an unbounded three-dimensional environment into a mathematically bounded two-dimensional view in a clear and controllable way. This has an important implication for the analysis of urban open spaces as a field of attributes. The analysis of an unbounded, continuous, space, such as an urban open space, becomes trivial because the points located at an infinite distance, the sky for instance, will be projected within the reference circle (figure 1). As a result, it is not necessary to fix an arbitrary vertical dimension, as would be the case in other field-oriented approaches. Addressing the three-dimensional nature of the built environment does not entail any extra calculation as it is inherent in the spherical projection. Furthermore, the dimension of the projection area, namely the radius of the reference circle, is one of the parameters of the computation. It is not influenced by the characteristics of the space. This reference dimension can be used to 'normalise' the different computations, which greatly facilitates comparison between different open spaces and the use of external references.

It is possible to retrieve many parameters from spherical projections, and each of these will have to be computed for a given type of spherical projection. In this paper four such parameters or 'indicators' are presented: sky opening, view lengths, skyline regularity, and moments of inertia. The construction of each indicator and its application to the urban open space analysis is introduced and discussed below.

3.1 Sky opening

The sky opening is defined as the amount of sky visible from a point. In an isoarea projection, it is given by the proportion of sky area (the central white area in figure 2), S_{sky} , relative to the surface area of the reference circle (the outer circle in figure 2), S_{ref} :

$$\text{sky opening} = \frac{S_{\text{sky}}}{S_{\text{ref}}} \times 100, \quad \text{sky opening} \in [0, 100] . \quad (1)$$

This indicator varies according to a 'massing factor' characterising the relations between the buildings and the open space, observed from a given point. It will be equal to 0% in a totally enclosed volume and 100% in a clear field: in the first case, the reference circle will be entirely filled by surrounding faces whereas in the second case it will be entirely empty. As such, it should correlate to the feeling of confinement perceived at a point, even if one knows that this feeling depends on a number of other factors such as the colours of façades, the presence of glimpses, the ground declivity, and so on. The spatial variation of the sky opening is characteristic of the modification of the massing factor within a space.

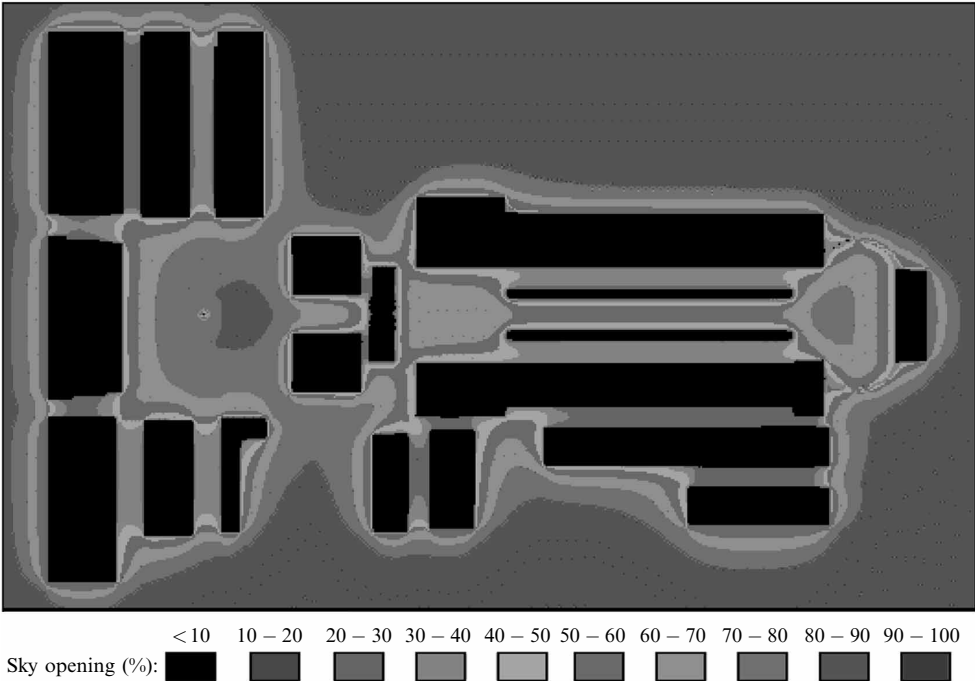


Figure 3. Sky opening map of the royal Places of Nancy: Place Stanislas (left), Place de la Carrière (centre), and Place Royal (right).

Sky opening maps can be used to singularise points and/or areas that appear to be central as indicated by this parameter. This operation is far from trivial when the building heights vary, even though the ground surface is regular. This is particularly well illustrated by the case of the system of Baroque Places in Nancy (figure 3). The sky opening map confirms the existence of an overall spatial structure and of different types of connections between the three main open spaces: the Place Stanislas (left), the Place de la Carrière (centre), and the Place Royale (right). In the Place Stanislas it can be observed that the ‘centre of mass’ does not coincide with the geometrical centre of the base rectangle (the centre is materialised by a statue). A significant shift is a result of the relative height of the pavilions framing the triumphal arch (5 m in height) compared with the other buildings surrounding the space (of some 20 m in height). The whole space is thus ‘oriented’ towards the Place de la Carrière.

This characteristic of the Place explains the overall triangular shape of the sky opening curves in the centre of the Place. A symmetrical phenomenon can be observed on the Place Royale, because of the shape of the colonnade framing the open space. Accordingly, the overall spatial structure of the urban setting is one of two poles closely intertwined, as a result of subtle variations in the building heights. It is clearly shown in the sky opening graph. The dynamic path between the two extreme Places is punctuated by different interruptions and diaphragms that are more or less transparent: the two pavilions, the triumphal arch, and the double row of trees.

3.2 Mean and characteristic view lengths

The notion of sky opening is intended to convey the idea that a set of large and distant volumes may have a visual mass quite similar to a set of smaller volumes that is closer to the observer. The rationale of this indicator, therefore, lies in the fact that it is a relative measure that can be used to compare open spaces with different absolute dimensions.

Such a relative indicator also has its own limits, as it totally neglects the importance of the urban scale. The calculation of scale factor must consider the view length distribution of faces surrounding an observer. When considering the distances L_i, \dots, L_n of the set of all visible faces F_i, \dots, F_n surrounding a point, and the solid angles they project on the spheres (giving areas S_i, \dots, S_n), it is possible to build two indicators: the mean view length (L_m) and the characteristic view length, L_{char} :

$$L_m = \sum_{i=1}^n S_i L_i / \sum_{i=1}^n S_i, \quad L_m \in [0, \infty] ; \quad (2)$$

$$L_{\text{char}} = L_m S_{\text{ref}} / \sum_{i=1}^n S_i, \quad L_{\text{char}} \in [0, \infty] . \quad (3)$$

The mean view length is quite similar to the measure of the radial length of an isovist in that it relies heavily upon the global dimensions of the open space. In highly regular configurations, characterised by a simple geometrical figure and almost constant building heights, the value of the mean view length in the centre of the space will be roughly equal to half the dimension of the space. When applied to residual configurations, this indicator gives an idea of the overall dimension of the open space, but the mean view length does not really support comparison of different configurations. In fact, the mean view length of a set of faces located 50m from an observer and occupying 80% of the sky will be equal to one for a set of faces at a similar distance but occupying 20% of the sky. The characteristic view length is intended to differentiate between such situations by weighting the mean view lengths according to the importance of the mask (given by the area of the reference circle, S_{ref} , divided by the area of the whole set of visible faces, $S_1 + S_2 + \dots + S_n$). This last value is, of course, theoretical. However, it can be used to compare very different spaces, with quite distinct morphologies.

As for the sky opening indicator, these two indicators can be applied to the analysis of spatial variations within an urban open space (figure 4). Hence, for a very irregular space such as the Place Cathédrale in Liège, the a priori estimation of the space dimension is quite complex. It can be seen from figure 4 that the mean view length of this square is 47 m, and that the dimensions of the base rectangle are 63 m \times 83 m. The fact that the mean view length is larger than the maximal half dimension of the base rectangle (41.5 m) can be explained by the presence of a large and deep opening on the axis of the rectangle. This opening widens the space considerably, as perceived from the central area. It can be further stressed that despite this large opening the central area remains largely contained by the base rectangle, given the presence of an important statue that breaks the otherwise continuous scale decrease between the two open spaces.

3.3 Skyline regularity

Cartesian indicators focus on the absolute height of buildings or even street prospects. They treat the regularity of building heights in simple, monodimensional terms. In contrast, it can be interesting to consider the statistical distribution of angular heights as they are observable from a given point. Here, one evaluates the regularity of the skyline as it is actually perceived from the urban open space, taking into account the position of the observer as well as the urban layout. This type of analysis should typically be realised as an equidistant projection, a spherical projection that respects angular heights. This is not the case for the isoarea projections that were used for sky opening and view length distribution analyses.

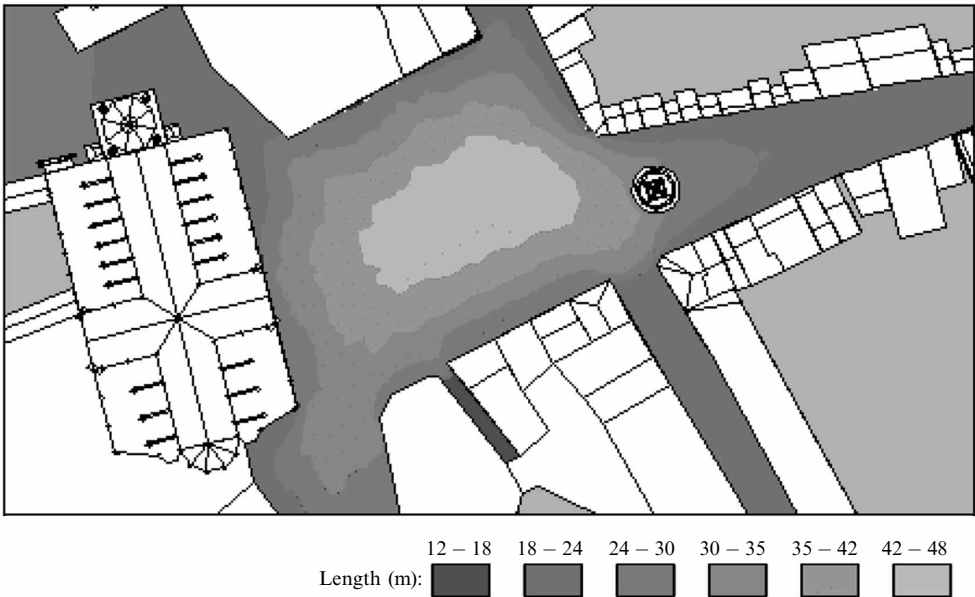


Figure 4. Map of the mean view length of the Place Cathédrale in Liège.

As such a measure is representative of the relative dispersion of a statistical distribution, standard deviation has been proposed as the main indicator of skyline regularity. It is calculated from the azimuthal distribution of building angular heights, measured in an equidistant projection. It can be supplemented by other statistical indicators such as the mean, maximal, and minimal values, but these are mostly accompanying figures that help to interpret the standard deviation (Teller, 2001).

This technique has been applied to the Place des Vosges in Paris (figure 5, over). The trees have deliberately not been included in the three-dimensional model in order to focus on the structure of the royal Place, in its initial 17th-century configuration, before its transformation into a square in 1797 (Joffet, 1957). It can be seen on the map that the standard deviation observable in the Place des Vosges is quite small. It is equal to 1.9° in the centre and remains at less than 10° in most of the space. In some spatial configurations, a standard deviation of 10° will be the minimum observable. The observed regularity of angular heights can be explained through the uniformity of the façades and roofs of this Place. The fact that the Place base adopts the shape of a perfect square also contributes to the regularity.

It should be remembered that the standard deviation is quite limited in scope as, basically, it tends to compare the shape of the skyline with a circle, centred on the origin and with a radius equal to the mean angular height. Accordingly, the standard deviation observable in a street will always be quite important, given that its overall shape in spherical projection tends towards an oblong ellipse, regardless of the constancy of building heights. This definition of regularity cannot be mistaken for the notion of ‘morphological deviation’ or ‘continuity’ which are measured in two dimensions (Maïzia, 1999).

3.4 Eccentricity and spread

A statistical indicator such as the standard deviation is intended to operate on an ordered distribution of values—the angular heights observable from one point, in this case. It neglects what truly makes a shape; namely, the relations between these values.

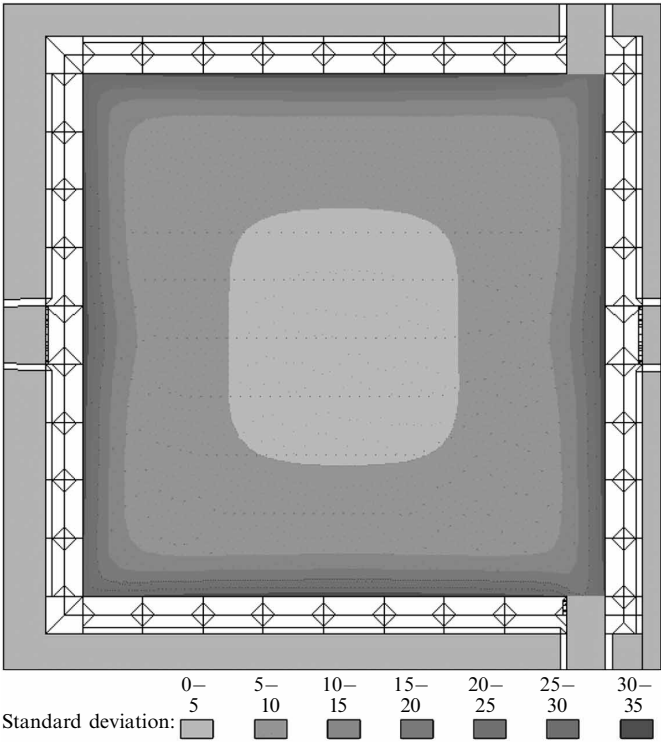


Figure 5. Variation of the angular height standard deviation (in degrees): Place des Vosges.

An alternative way to analyse a skyline is to consider the sky as a true figure, in gestaltist terms, as suggested in the seminal work of Arnheim (1977). This figure can then be characterised by geometrical properties, such as compactness, directionality, symmetry, and so on. Here, two main shape factors are proposed, namely, eccentricity and spread, based on the moments of inertia of a shape. The sky shape is therefore treated as an homogeneous surface. It is computed in stereographical view, as this spherical projection is mathematically ‘conform’ and thus better respects the appearance of shapes.

The moment i, j of an homogeneous shape is given by the following equation:

$$M_{i,j} = \iint x^i y^j \, dx \, dy \quad (4)$$

The order of a moment is given by the sum of i and j in equation (4). The area of a shape is given by its moment of order 0 (M_{00}). This value is totally independent of the position and orientation of the reference axes used to calculate the integral. The position of the barycentre is given by the ratio between first-order moments (M_{10} and M_{01}) and M_{00} . The second-order moment is also characterised by significant properties. In opposition to the zero-order moment, the values of M_{02} , M_{20} , and M_{11} will vary according to the position and orientation of the reference axes. The preferential orientation of the axes will then be given by the eigenvalues of the inertia matrix, namely:

$$\text{inertia matrix} = \begin{pmatrix} M_{20} & M_{11} \\ M_{11} & M_{02} \end{pmatrix}, \quad (5)$$

and

$$I_1 = \frac{M_{20} + M_{02} - [(M_{20} - M_{02})^2 + 4M_{11}]^{1/2}}{2} ,$$

(6)

$$I_2 = \frac{M_{20} + M_{02} + [(M_{20} - M_{02})^2 + 4M_{11}]^{1/2}}{2} .$$

(7)

The value I_1 corresponds to the smaller M_{02} moment of the shape. The value I_2 is the maximal value of M_{02} when the reference axes are located on the barycentre. Interestingly, these two values characterise the mass distribution within a shape. As such, they are heavily used in mechanics as well as in image analysis. In this case, it is the ratio between these two inertia values that will be of most use. Actually, these values can be used to build two adimensional parameters, independent of the area of the analysed shape, namely, the eccentricity E and the spread Sp :

$$E = \frac{I_2}{I_1} , \quad E \in [1, \infty] ,$$

(8)

$$Sp = \frac{I_1 + I_2}{(\text{area})^2} , \quad Sp \in [0.159, \infty] .$$

(9)

The values of eccentricity and spread for three elementary shapes—a circle, a rectangle, and a cross—is given in table 1, each characterised by two parameters, d_1 and d_2 . In the case of a circle, the value of d_1 is equal to the radius of the circle. In the case of a rectangle, the values of d_1 and d_2 are equal to the width and length of the rectangle, respectively. In the case of a cross, the value of d_1 is equal to the width of the cross branches, and d_2 is equal to their length.

It can be seen from table 1 that the eccentricity of a circle, a square, and a cross are equal to 1. This is because these are perfectly symmetrical shapes, considered from their gravity centre. For each point (x, y) of the shape, there exists a point $(-x, -y)$ also belonging to the shape. The two inertia axes will thus have the same value. It can also be observed that the eccentricity of a rectangle increases very rapidly with its elongation (ratio of height to width). This is because the inertia axes vary as the fourth power of the base dimensions of the shape. The eccentricity can thus be considered as representative of the ‘anisotropy’ of the shape.

It is possible to calculate these coefficients describing the shape of the sky within a spherical projection, to measure the anisotropy of the open space, as perceived from specific points. When this calculation is repeated for a great number of points it highlights the variation of anisotropy throughout the space.

Table 1. Spread and eccentricity of elementary shapes.

Shape	d_1 (unit length)	d_2	Spread ^a	Eccentricity
Circle	20	na	0.159	1.000
Rectangle	20	20	0.167	1.000
	20	40	0.208	4.000
	20	100	0.433	25.000
Cross	20	20	0.187	1.000
	20	40	0.263	1.000
	20	100	0.506	1.000

na: not applicable. Note: for a circle d_1 = radius; for a rectangle, d_1 = width, d_2 = length; for a cross, d_1 = width of cross bar, d_2 = length of cross vertical bar.

^a Spread is a-dimensional.

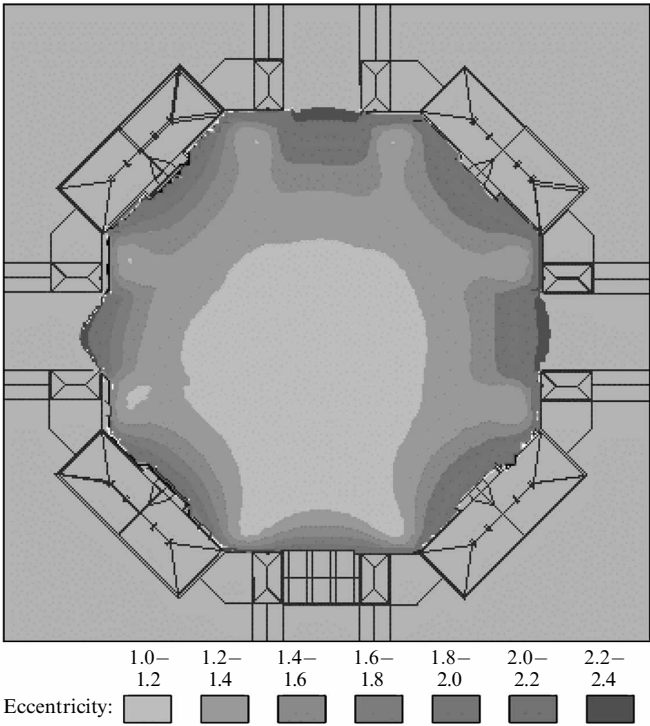
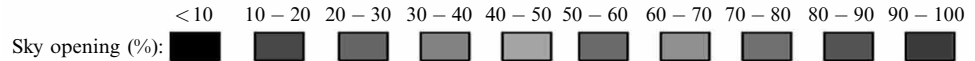
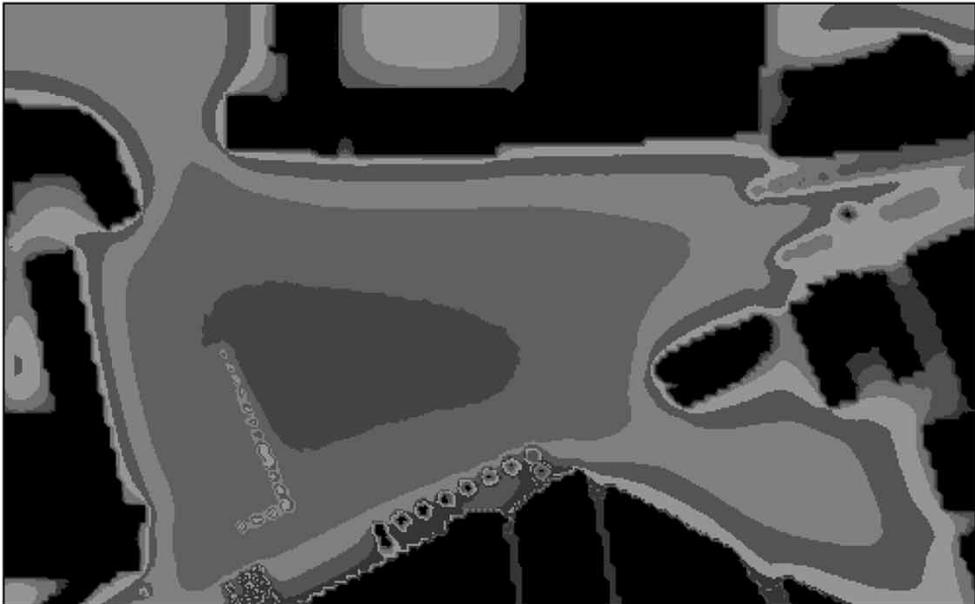


Figure 6. Eccentricity variation within the Amalienborg Plads, Copenhagen.

This method has been applied to Amalienborg Plads in Copenhagen (figure 6). This figure is very different from those obtained up to now. In fact, the central area of the field, characterised by an eccentricity ranging between 1.0 and 1.4, is no longer convex. It adopts a kind of octopus shape. Its ‘arms’ tend to spread towards the corners of the square. Two types of elements can induce an anisotropy of sky, namely, the buildings and the street openings. Typically, the buildings tend to orient the skyline along a direction parallel to their façade. Hence they repel the least eccentric curves away from the centre of their façade where this effect is maximal. The streets produce an increase of anisotropy along their axes, which tends to affect points located within the square, along the axis of the street. Also, the Triumphal Arch, located to the south of the square, tends to attract the whole central area, as it has lower anisotropy when compared with street openings.

4 Application to a case study

In the previous discussion, different spherical indicators were introduced progressively and each of them was illustrated through an application to existing European open spaces. These open spaces can thus be considered as ‘references’ to constitute common culture, to be shared by different stakeholders of the morphological regulation: public authorities, developers, designers, and so on (Teller, 2002). We can now consider a specific case, as a ‘live testing’ of the different aspects mentioned above. The development of a building block at the eastern corner of the Place Saint-Lambert in Liège was chosen [see figures 7(a) and 7(b) for the existing situation and projected scheme]. The proposed development is part of a more general urban project, consisting of the rehabilitation of the entire Place Saint-Lambert. A new building block has already been constructed at the west side of the space. The intention is to balance this with another construction on the east side.



(a)



(b)

Figure 7. Sky opening variation in the Place Saint-Lambert: (a) existing situation and (b) projected scheme.

It has to be stressed that the proposed construction area is characterised by many constraints. Its historical value is exceptional because of the presence of underground archaeological vestiges. Additionally, it is surrounded by many historical buildings. Also, its future occupancy may create strong perturbation within the Place Saint-Lambert, which is an important circulation node at the city scale (for cars, public transport, pedestrians, bicycles, and so on). Furthermore, the site connects or separates two residual places with very different qualities. This obviously constituted a unique opportunity to test the methodology described in this paper. The tools were applied to measure the sensibility of the open space configuration with respect to the characteristics of a building located here.

4.1 Sky opening maps

The objective of the development of the block is to close one side of the Place Saint-Lambert to improve its legibility. It was decided to analyse the space by using sky opening maps, to delineate the central area of the open space. Accordingly, the area to be analysed should not be restricted to the 'expected' open space, because its limits are hard to determine beforehand and could possibly be affected by the construction of a new building. The analysis has to address a much larger fragment of the urban pattern so as to include the context of the space. The difference introduced by the development of the eastern block is highlighted in figures 7(a) and 7(b). It can be seen from these that the dimension and geometry of the central area would be strongly affected by the construction of an eastern block.

The sky opening map of the Place Saint-Lambert without this construction [figure 7(a)] highlights the importance of developing this block. The central area takes on a triangular shape, which tends to extend it towards surrounding open spaces. The legibility of the place is quite weak. Figure 7(b) highlights three important properties of the projected open space:

1. A very homogeneous central area appears in the middle of the figure. The sky opening percentage (80%) is representative of the whole open space.
2. Lateral effects of the façades surrounding the open space are restricted to a very limited area when compared with the whole area of the square. The contour lines are very dense at the perimeter of the open space. Façades, therefore, would produce a localised sharp gradient in sky opening percentage. This property is very typical of wide open spaces.
3. Perturbation effects, caused by the surrounding street connections, are quite limited in scope and extent. This is an important point when one takes into account that these openings are rather wide. Isoline containment has to be related to the positive effect of trees and plantations.

4.2 Comparison with other open spaces

Another possible use of the spherical instrumentation consists of comparisons between different urban open spaces. It may be interesting to compare the results obtained in the centre of the Place Saint-Lambert with the ones obtained in the four open spaces analysed above: Place Stanislas (section 3.1), Place Cathédrale (section 3.2), Place des Vosges (section 3.3), and Amalienborg Plads (section 3.4). Some of these spaces are characterised by a complex shape (Place Cathédrale and the existing Place Saint-Lambert). Practically, it means that it is not always possible to define a priori a representative 'centre point'. The computations were thus systematically achieved in the point where the sky opening is maximal. This point was identified with sky opening maps computed for the entire open space.

It can be seen from table 2 that the Place Saint-Lambert belongs to the family of largely opened spaces: the maximal value of sky opening observed within the space is

Table 2. Spherical indicators for the Place Saint-Lambert and four reference spaces.

Space	Sky opening (%) ^a	Mean view length (m) ^b	Characteristic view length (m) ^c	Standard deviation (°) ^d	Eccentricity ^e
Place Saint-Lambert:					
existing situation	79.91	83.75	391.95	4.52	1.38
projected scheme	78.95	78.72	352.72	4.21	1.30
Place Stanislas	80.41	67.07	342.57	3.70	1.04
Place Cathédrale	60.98	46.77	119.80	7.32	1.39
Place des Vosges	78.76	80.99	380.67	1.94	1.00
Amalienborg Plads	78.68	69.16	322.93	5.03	1.01

^a See text, equation (1). ^b See text, equation (2). ^c See text, equation (3). ^d See text, section 3.3.
^e See text, equation (8).

79.91%, which is slightly lower than the maximal value observed in the Place Stanislas but higher than the one observed in all the other open spaces studied. The characteristic view length in the existing situation (391.95 m) is larger than the one observed in the other cases studied here. This is partly because of the deep opening in the axis of the open space. Actually, in the projected scheme, which closes this opening, it would decrease to 352.72 m. This last value is still larger than the one measured in Place Stanislas, but it is smaller than that of Place des Vosges.

Given the scale of the open space, development of the east side of the Place would not greatly affect the sky opening, regularity of the skyline, and the eccentricity of the sky shape. The sky opening would decrease from 79.91% to 78.95%, the standard deviation from 4.52° to 4.21°, and the eccentricity from 1.38 to 1.30. These differences are small when compared with the variation of these three indicators observed across the different cases studied. This can be explained by the fact that measures were taken as perceived from the point of maximal sky opening. If they had been taken from points located close to the project, the effect would have been much more important [see figures 7(a) and 7(b)]. This underlines the importance of field-oriented representations in assessing the effects of any modification upon the open space as a whole.

Finally, it can be observed that the Place Saint-Lambert is rather anisotropic: the eccentricity of the space is much larger than that measured in the other places, with the exception of Place Cathédrale, which is oriented towards the large opening governing the space. This result can be explained by the fact that the sky shape of the Place Saint-Lambert, as perceived from the point of maximal sky opening, is largely oriented by the façades of the main northern buildings.

5 Conclusions

A new field-oriented metric of open space has been introduced and discussed. It is based on spherical projections and, more specifically, on measures applied to spherical views. Several indicators based on this technique have been proposed: the sky opening, the mean view length and the characteristic view length, the skyline regularity, the eccentricity, and the spread of the sky shape. These characterise different properties of urban form, perceived from specific viewpoints. When repeated for a large number of points, distributed on a grid covering a significant area of the urban pattern, they provide a field-oriented representation of urban open spaces. In this way, the limits of a space do not have to be determined a priori; they can be analysed from observed variations within the field of attributes. As a consequence, this method

suggests a way to overcome the unproductive dichotomy between dominant and residual open spaces. The Place Saint-Lambert provides a good example of a space typically lying between these two extremes. Although its spatial limits were not easy to identify in advance, sky opening maps clearly distinguish between central and lateral areas. They also aid assessment of the effect of a new construction upon their configuration.

This method can provide reliable arguments as well as practical references to assess the effects of development projects on urban open spaces. Environmental impact assessment (EIA) and strategic environmental assessment (SEA) appear to be promising application domains for such a metric. Interestingly, these two procedures signalled a shift from the traditional prescription-based regulation of the built environment to a performance-based approach.

Compared with existing field-oriented approaches (isovists, e-partitions, and so on), the main advantage of the method proposed in this paper lies in the fact that it is truly three-dimensional. This point is particularly important because the height of buildings is quite variable in many urban environments. The potential effect of this factor has been clearly highlighted through the case of Place Stanislas, where a shift of the centre of mass from the two-dimensional barycentre of the place can be observed in sky opening maps. The computational cost of introducing the third dimension is offset by the use of spherical projections. By definition, spherical views transform an unbounded three-dimensional scene to a bounded two-dimensional view, respecting given relations between objects. It has been observed that, even for large urban models, the maps can be calculated with a step of 1m without difficulty. Such a resolution ensures reliable results in small open spaces, such as narrow streets.

The metric presented here has been inspired largely by Gibson's (1950) ecology of vision in an attempt to make salient perceived variations within the space. The main hypothesis of Gibson's work was indeed that:

"the basis of the so-called perception of space is the projection of its objects and elements as an image, and the consequent gradual change of size and density in the image as the objects and elements recede from the observer" (page 78).

Accordingly, the motion-perspective was defined by Gibson as the perceived variation of the size, texture and relative speed of objects that can be observed when we are in motion. It was regarded by Gibson as fundamental to the perception. Interestingly, spherical views tend to give the whole environment surrounding an observer, and variations between points appear clearly through the reading of contour maps. Further research should now be directed towards the analysis of differences between views. At this moment, spatial variations are analysed through a sole comparison of indicator values at different points. It is not yet possible to trace the displacement or the possible deformation of an object—whether it be a point or a polygon—from one spherical view to another. Analysis of 'spherical optic flow', in Gibson's terms, suggests a promising field of further investigation for the instrumentation proposed.

It would also be tempting to develop a standard, or benchmark, to assess the value of places before and after intervention, or even to make cross-comparisons of the overall value of different spaces. Such a benchmarking method would obviously facilitate the environmental assessment of plans and projects and, especially, environmental impact reporting. However, apart from the technical and theoretical difficulties of such an approach, in my opinion it is irrelevant to urban open spaces. In fact, open spaces can be considered as part and parcel of our common cultural heritage. Accordingly, their conservation and enhancement should basically obey the criterion of diversity, a situation that is not necessarily the case for other environmental factors (the quality of air or water, for instance). The definition of some 'open

space standard' would certainly contradict this basic objective. In contrast, the notion of contextual references, possibly mundane ones as in the case of Place Cathédrale in Liège, appears much more flexible. References are intention driven and must always be clearly justified. If greater regularity is desired, one could consider references such as Arras or Place des Vosges. If, the objective is to enhance the axial character of a space, Place Stanislas probably provides a better reference. Hence the use of references provides local decisionmakers and stakeholders with greater freedom to create opinions and innovations.

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