

$$\begin{aligned}
0 = & -aN_n(x, \vartheta) - c \cos \vartheta \frac{dN_n(x, \vartheta)}{dx} \\
& + \alpha \int_0^{\pi/2} \int_0^{2\pi} N_{n-1}(n, \vartheta') f(\psi) \sin \vartheta' d\vartheta' d\chi \\
& + \alpha \int_0^{\pi/2} \int_0^{2\pi} N_{n-1}^+(x, \vartheta') f(\psi) \sin \vartheta' d\vartheta' d\chi,
\end{aligned}$$

where $\cos \psi$ is given by the same relation.

It may be observed that we have not introduced in our treatment the absorption of photons. This is, however, easily possible. We only have to replace then the coefficient of loss by the sum of $\alpha_s + \alpha_a$, α_s and α_a being the probabilities of loss by scattering and by real absorption, in the first term of the differential equation. In the term which represents the production of photons of the given kind α must then be replaced by α_s , the probability of scattering. The mathematical treatment does, however, not change in any essential point.

It may be remarked that we have given our deduction in the language of photons, as it was suggested by the similar treatment of scattering of neutrons given by the author in a series of papers in the *Proceeding of the Amsterdam Academy* (1936). This, however, is unessential; it would be just the same to treat the problem on the base of beams of light scattered zero, one, two, etc. times, introducing then, instead of the photon density, the density of radiation of these different classes.

The advantage of the treatment given is that it enables us to find, to any desired degree of approximation, the solution of the problem of scattering. The calculation, however, is somewhat longer than the approximate methods used up to this moment. A great advantage is that it clearly shows the physical meaning of each step in the calculations, and gives the proof of the existence of a solution.

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A NOTE ON MOLECULAR ABSORPTION IN INTERSTELLAR SPACE.

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(Communicated by Sir Arthur Eddington)

1. For a long time the H and K lines of Ca^+ and the D line of Na have been the only observed stationary lines. But, recently, Merrill* has discovered four interstellar lines in the yellow and red region of the spectrum; the wave-lengths of these lines are 5780.4, 5796.9, 6283.9 and 6613.9 Å. Instead of being narrow and sharp, they are somewhat widened and have rather diffuse edges.

The origin of these lines is thus far unknown. As the ultimate lines

* *P.A.S.P.*, 46, 206, 1934; *Ap. J.*, 83, 126, 1936.

of all the abundant atoms are now thoroughly known, and as none of them coincides with one of Merrill's lines, it has been suggested * that they must rather be molecular bands; the fuzzy character of the lines adds some weight to this hypothesis.

2. Actually, when trying to find a molecular identification of these lines, the types of molecular spectra to be considered are

- (a) ultimate electronic bands;
- (b) rotatory oscillation bands.

A systematic examination of the electronic bands likely to appear has not given us, so far, any real physical identification.

In the case of rotatory oscillation bands a striking coincidence appears. Two of Merrill's lines agree closely with the calculated wave-lengths of CO_2 bands.† The interstellar line λ 6613.9 ($\nu = 15115.5 \text{ cm.}^{-1}$) coincides with the origin of the band ($\nu_1 + 4\nu_2 + 5\nu_3$) at $\nu = 15115$; another stationary line λ 6283.9 ($\nu = 15909.3 \text{ cm.}^{-1}$) falls near the calculated CO_2 band at 15917 cm.^{-1} ($7\nu_3$). The second coincidence is not so good, but it is admissible owing to the fact that the wave-numbers of the CO_2 bands are only calculated (not yet observed in the laboratory), and also owing to the diffuse character of the stationary lines.

This is of course a purely tentative identification. It will be possible to find an observational criterion. As these bands correspond to rather high vibrational quantum numbers, they must be weaker than the "Venus bands" at $\lambda\lambda$ 7820 and 7883, and the latter bands ought to be strong in the spectra of all sufficiently distant stars. Dr. Merrill, to whom this coincidence has been announced, will be on the look-out for further evidence, and hopes to undertake a search for the "Venus bands" in the future.‡

That the other calculated bands of CO_2 have not been observed as interstellar lines may be due to lower transition probabilities, unfortunate blendings,§ or technical difficulties.

The numerical coincidences given here may be purely accidental; future observations will decide it. They are merely cited as an illustration of the actual interest of the consideration of the interstellar molecules.

3. An immediate objection to a molecular identification of the lines observed by Merrill is that these observed lines, though fuzzy, are not broad enough for molecular origin. For instance, the CO_2 bands when observed in the laboratory average about 100 cm.^{-1} in breadth throughout the spectrum.||

This objection disappears when we assume a very low temperature for the Boltzmann distribution of the molecules in the different rotational levels of the lowest electronic state. Owing to the selection rule concerning the rotational quantum number J ($J' - J'' = \pm 1$ or sometimes 0), only a few

* H. N. Russell, *M.N.*, **95**, 635, 1935.

† Adel and Dennison, *Physical Review*, **43**, 723, 1933.

‡ Private communication.

§ *I.e.* the band at $\nu = 15234$ (λ 6562.4) falls on *H α* .

|| Kindly communicated by Dr. Adel.

individual lines of the absorption bands will appear, and actually only those corresponding to small quantum numbers \mathcal{J} ; in other words, only a few lines near the band origins will appear.

4. Let us consider first the case of a molecule which has a pure rotational spectrum, *i.e.* which has an electric dipole in the normal electronic state. We know that the speeds of the electrons in interstellar space correspond to a temperature of the order $10,000^\circ$, and the translatory velocities of the molecules will be brought up to equilibrium with the electrons. But in this case, owing to the strong selective absorption between the different rotational levels, we must consider for each rotational absorption line λ an equivalent temperature T_λ , exactly as in the treatment of atomic excitations *; this equivalent temperature T_λ is such that the actual energy-density for wave-length λ is equal to that of equilibrium radiation at temperature T_λ .

We shall use the following formula for the molecular energy of rotation :

$$E = \frac{h^2}{8\pi^2 I} \cdot \mathcal{J}(\mathcal{J} + 1) \quad (\mathcal{J} = 0, 1, 2, \dots), \quad (1)$$

I being the moment of inertia and \mathcal{J} the rotational quantum number.

We know from the experimental investigation of the infra red spectra of pure rotation that these appear in the region of wave-lengths greater than 10μ . For small moments of inertia (or great energies E , following formula (1)), the spectra lie between 10 and 100μ ; for higher values of I —*e.g.* of the same order as that of CO_2 —the first rotational lines have still greater wave-lengths, which may be calculated by the usual formula,

$$\nu = \frac{h}{4\pi^2 c I} (\mathcal{J} + 1) \quad (\mathcal{J} = 0, 1, 2, \dots). \quad (2)$$

For I of the order of $70 \cdot 10^{-40}$ g. cm.² (value for CO_2), this would give for the first ten lines values extending from about $\lambda = 10,000 \mu$ to $\lambda = 1000 \mu$. It seems therefore useful to estimate the equivalent temperatures T_λ for $\lambda = 10, 100$ and 1000μ .

For the determinations of T_λ we have followed Eddington's procedure. For $\lambda = 1000 \mu$, T_λ is already smaller than $T = 3^\circ.2$ K.† For 10 and 100μ we obtain

$$T_{\lambda=10\mu} = 43^\circ K;$$

$$T_{\lambda=100\mu} = 4^\circ.8 K.$$

Thus for the lowest value of \mathcal{J} the extreme value of T_λ to be applied—even for the smallest moments of inertia I —is about $5^\circ K$. For the very high values of \mathcal{J} , T_λ could be as high as $43^\circ K$, at least for very small moments of inertia.

Table I gives the population for $T_\lambda = 10, 5$ and $3^\circ.2$ K, corresponding to $I = 70 \cdot 10^{-40}$ g. cm.² and computed according to Boltzmann's formula,

$$N_J = (2\mathcal{J} + 1) e^{-\frac{E_J}{kT}}, \quad (3)$$

* A. S. Eddington, *Internal Constitution of Stars*, 2nd ed., p. 371.

† This is the value of T obtained by A. S. Eddington by applying the formula $E = aT^4$ to the total radiation of the stars ($7.67 \cdot 10^{-13}$ ergs/cm.³).

TABLE I

Populations of the rotational levels of a dipolar molecule, for $T_\lambda = 10^\circ$, 5° and $3^\circ.2$ K, assuming $I = 70 \cdot 10^{-40}$ g. cm.².

J	Population for $T_\lambda = 10^\circ$ K	Population for $T_\lambda = 5^\circ$	Population for $T_\lambda = 3^\circ.2$
0	1	1	1
1	2.67	2.37	2.07
2	3.55	2.55	1.75
3	3.57	1.82	0.84
4	2.97	0.99	0.27
5	2.01	0.36	0.055
6	1.17	0.12	0.009
7	0.61	0.027	...
8	0.31	0.005	...

Evidently these populations do not exactly correspond to the intensities of the individual lines of the absorption bands; for these intensities we should also have to consider the transition probabilities (Hönl-London's formulæ). But this would not modify substantially our conclusions.

Table I shows that in our assumptions only the four or five first lines of the P, Q and R branches will be really observable; this means that the breadth of the bands will be only 6 or 7 cm.⁻¹, or approximately 2 Å. near 6000 Å.; the edges must be diffuse. This seems to agree perfectly with Merrill's observations.

5. Let us now consider a molecule—such as CO₂—which has no electric dipole in the lowest electronic state. It seems at first that the collisions will bring the rotations up to a distribution corresponding to a temperature of the order 10,000°. But this is not necessarily true, as the molecule may still have a “quadrupolar transition probability” for the pure rotational spectrum, which may be appreciable; thus, owing to the very low density of interstellar matter, the molecule may still be able to absorb and emit these low-frequency radiations faster than the exchanges by collision processes. There are still other reasons which indicate that the consideration of a low temperature for the distribution of the rotational quantum numbers is appropriate.

Consequently results similar to those of Table I would also be expected here.

6. When once the identifications have been definitely made the determination of the profile of the interstellar bands will give a possibility of direct measure of the temperatures corresponding to the actual Boltzmann distribution of the rotational levels.

It is quite clear that our estimations of T_λ are rather rough; * on the

* For instance, in our calculations of T_λ we have not considered any star of temperature lower than 3000°.