REGULATION UNDER FINANCIAL CONSTRAINTS*

by

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ABSTRACT*: *This article studies a simple procurement problem (Laffont and Tirole,1993) where the regulator faces a cash-in-advance constraint. The introduction of such a constraint not only reduces the amount of public good provided but also limits the instruments available to the regulator. The wealth constraint could change the optimal regulatory contract from a two-part tariff, where the quantities produced depend on the firm's cost, to a less efficient fixed fee where the firm produces the same quantity whatever its cost.*

1 Introduction

A simple procurement problem[1] consists in the following: a public authority (regulator) faces a monopolistic producer. The firm provides an essential facility (bridge, road, infrastructure or sewer system) to a bunch of consumers and the authority regulates the term under which the producer delivers the service to them. A procurement contract specifies an amount of public good the firm should produce and an associated financing scheme.

When the regulator knows the technology of the firm, the optimal procurement contract requires that the firm produces the quantity of public good that maximizes the total surplus i.e. the consumer's surplus net of the firm's production cost and the authority fully reimburses the firm's cost.

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***   *Résumé en fin d'article; Zusammenfassung am Ende des Artikels; resumen al fin del artículo.*

1   Laffont and Tirole (1993).

This contract could not be implemented when the firm's cost is private information. In this setting, the procurement contract should provide the firm with incentives to reveal its true cost to the regulator. The optimal procurement contract then requires that (1) the low cost firm still produces the quantity that maximizes the total surplus but it receives a compensation larger than its cost and (2) the high cost firm produces less than under symmetric information but its cost is still fully reimbursed. Distortion in the quantity produced by the high cost firm aims to lower the compensation of the low cost firm (rent extraction efficiency trade-off). This second best contract could be implemented with a non-linear reimbursement scheme.

This paper adds financial constraints to the standard procurement problem. It considers an environment where the regulator is endowed with a limited amount of public funds. Consequently, the transfer from the regulator to the firm is bounded.

Financial constraints need to be considered when one studies regulatory problems in developing countries.[2] Governments in developing countries could not finance all their infrastructure projects because they lack the financial resources. Moreover, local governments often have no credible political commitment to long-term financial obligations and even if long-term private capital is available, local governments generally can borrow only at very high rate of interest, if at all.[3] Hence, despite clearly identified benefits, public authorities cannot finance all their investment projects because of capital market imperfections. But, even with a limited budget, some infrastructure building could not be delayed and will be financed even if funds are scarce.

A cash in advance constraint modifies the procurement contract in a twofold manner. The first and the most obvious consequence is the under-provision of public good by the firm (or equivalently a lower quality infrastructure) compared to the asymmetric information case. Second, the shape of the procurement contract could be modified by the financial constraint. The wealth constraint could change the procurement contract from a two-part tariff, where the quantities produced depend on the firm's cost, to a fixed fee where the firm produces the same quantity whatever its cost. Hence, a wealth constrained regulator may use a less efficient regulatory instrument.[4]

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2   Laffont (2000).
3   World Bank (2000).
4   Laffont (2000) documents that developing countries failed to implement high-powered procurement contracts.

In different context, Che and Gale (1998), Lewis and Sappington (2000) and Thomas (2002) study mechanism design problems with financial constraints. Like in this paper, the addition of financial constraints to the standard informational constraints modifies both the outcome and the nature of the optimal mechanism. For example, Lewis and Sappington (2000) consider privatizations to wealth-constrained operators where the most efficient operator is not necessarily the one who has the largest resources. Hence, privatization through an auction is not *ex post* efficient. The financial constraint modifies both the shape of the optimal mechanism and the amount (or the timing) of trade. It is optimal to limit the operator's stake in the privatized firm to achieve an *ex post* efficient privatization. Thomas (2002) considers the problem of selling a good to buyers with both unknown willingness to pay (informational constraint) and known ability to pay (financial constraint). In the financially constrained selling mechanism, price discrimination is no longer optimal for high valuation buyers. Like in this paper, bunching appears in the financially constrained mechanism. However in Thomas (2002), bunching always occurs while it is not necessarily the case in this paper.

2   The model

The model proposed in this paper is a standard procurement model. A public authority (regulator, principal) contracts with a monopolistic private firm (agent) for the provision of a public good. Production is financed by public transfers. The firm produces with a constant return to scale technology; its cost function is θq, where θ is a constant marginal cost and q the quantity produced.[5] The marginal cost is private information to the firm. The principal only knows that (1) $\theta \in \Theta \equiv \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$ and (2) $\text{Prob}(\theta = \theta_1) = v_1$ and $\text{Prob}(\theta = \theta_2) = v_2 = 1 - v_1$. We call $\Delta\theta = \theta_2 - \theta_1$.

The procurement contract specifies a quantity transfer pair: $<q, T>$. The firm's profit is the difference between the transfer T and the production cost: $U^A = T - \theta q$. The firm accepts the contract only if it realizes a non-negative profit (individual rationality (**IR**) constraint).

When the agent produces a quantity q, the principal collects a surplus S(q). We assume that $S' \geq 0$, $S'(0) = +\infty$, $S'' < 0$ and $S''' > 0$. The assumptions on S ensure that it is optimal to have the agent producing whatever its cost. The utility of the principal is the difference between the surplus and the transfer: $U^P = S(q) - T$.

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5   Equivalently, q could be interpreted as the quality of the good produced.

The cash in advance constraint limits the transfer: the regulator cannot transfer more than **T** to the firm.

3   Results

3.1   Second best equilibrium

Without the cash-in-advance constraint, the objective of the principal is to maximize its expected utility, subject to incentive compatible and participation constraints. The incentive constraints (**IC**) ensure that the agent of type θ_i selects the contract $<q_i, T_i>$ rather than the contract $<q_j, T_j>$, i,j = 1,2, i ≠ j. Without financial constraints, the regulator solves:

Program [P1]

$$\underset{q_1,q_2,T_1,T_2}{Max}\ v_1(S(q_1) - T_1) + v_2(S(q_2) - T_2)$$

subject to:

$$T_1 - \theta_1 q_1 \geq T_2 - \theta_1 q_2 \tag{IC_1}$$

$$T_2 - \theta_2 q_2 \geq T_1 - \theta_2 q_1 \tag{IC_2}$$

$$T_1 - \theta_1 q_1 \geq 0 \tag{IR_1}$$

$$T_2 - \theta_2 q_2 \geq 0 \tag{IR_2}$$

The two relevant constraints of this problem are IC_1 and IR_2.

Proposition 1 (*Baron and Myerson, 1982): The solution to the problem [P1] is given by***:

$$q_1^{SB} = S'^{-1}(\theta_1) \tag{1}$$

$$q_2^{SB} = S'^{-1}(\theta_2 + \frac{v_1}{v_2}\Delta\theta) \tag{2}$$

$$T_1^{SB} = \theta_1 q_1^{SB} + \Delta\theta q_2^{SB} \tag{3}$$

$$T_2^{SB} = \theta_2 q_2^{SB} \tag{4}$$

This solution is standard. The low cost firm θ_1 produces the quantity that maximizes the total surplus ($U^A + U^P$) and makes a positive profit (information rent). This information rent is necessary to fulfill the incentive constraint. The inefficient firm θ_2 makes a zero-profit and produces less than the quantity that maximizes the total surplus. The quantity q_2^{SB} is reduced compared to the first best in order to lower the profit made by the low cost firm (rent extraction-efficiency trade-off).

This second best solution could be implemented with a non linear financing scheme where the firm receives a payment T_2^{SB} for the production of q_2^{SB} and a bonus of $\Delta T = T_1^{SB} - T_2^{SB}$ for the additional quantities $\Delta q = q_1^{SB} - q_2^{SB}$. With this two parts tariff, the firm self-selects and only the low cost firm θ_1 produces the additional quantities.

3.2 Wealth-constrained equilibria

Consider now the financial constraint. The constraint implies that the principal cannot transfer to the agent more than $\mathbf{T}$. Obviously, the constraint is irrelevant if the highest possible transfer $\mathbf{T}$ is lower than the highest transfer paid in the second best equilibrium T_1^{SB}. In the remaining, we assume: $\mathbf{T} \leq \theta_1 q_1^{SB} + \Delta\theta q_2^{SB}$. When the principal is wealth-constrained, its optimization program becomes:

Program [P2]:

$$\underset{q_1, q_2, T_1, T_2}{Max} \quad v_1(S(q_1) - T_1) + v_2(S(q_2) - T_2)$$

subject to: (IC$_1$), (IC$_2$), (IR$_1$), (IR$_2$) and

$$T_1, T_2 \leq \mathbf{T} \tag{WC}$$

The solution of this optimization program is given proposition 2.

Proposition 2:

(i) *if* $\theta_1 \geq v_1\theta_2$ *and* $\mathbf{T} \leq \mathbf{T}^* = \theta_2 S'^{-1}(\frac{v_2\theta_1\theta_2}{\theta_1 - v_2\theta_2})$, *the equilibrium is a **pooling equilibrium**:*

$$q_1^{WC} = q_2^{WC} = \mathbf{T}/\theta_2 \tag{5}$$

$$T_1^{WC} = T_2^{WC} = \mathbf{T} \tag{6}$$

(ii) *otherwise, the equilibrium is a **separating equilibrium** characterized by the following first order conditions*:

$$q_1^{WC} = S'^{-1}(\frac{\mu_1}{v_1}\theta_1) \tag{7}$$

$$q_2^{WC} = S'^{-1}(\theta_2 + \frac{\mu_1}{v_2}\Delta\theta) \tag{8}$$

$$\mathbf{T} - \theta_1 q_1^{WC} - \Delta\theta q_2^{WC} = 0 \tag{9}$$

And the transfers are given by the constraints (WC) and (IR$_2$).

$$T_1^{WC} = \mathbf{T} \tag{10}$$

$$T_2^{WC} = \theta_2 q_2^{WC} \tag{11}$$

(iii) *In the separating equilibrium, the value of μ_1 is a decreasing and convex function of $\mathbf{T}$, with* $\lim_{\mathbf{T}\to 0}\mu_1 = 0$ *and* $\lim_{T\to T_1^{SB}}\mu_1 = v_1$.

The proof of proposition 2 is relegated to an appendix. Figure 1 illustrates the quantities produced when the wealth constraint is relevant.

Consider first the separating equilibrium. The transfer paid to the agent of type θ_1 could be decomposed into (i) a compensation for its production cost ($\theta_1 q_1$) and (ii) an informational rent ($\Delta\theta q_2$) which is necessary to satisfy the incentive constraint. If this transfer is fixed
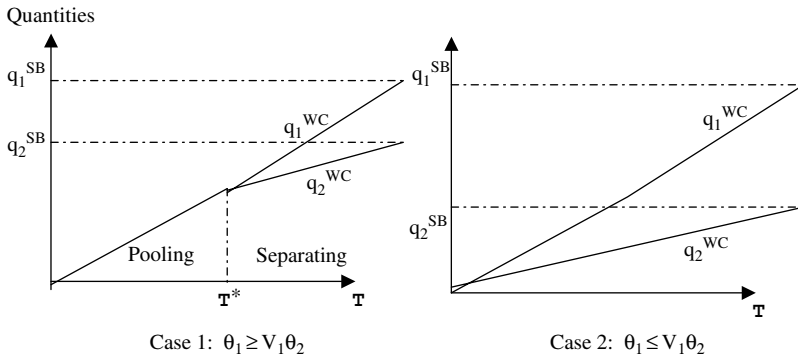

Figure 1 – Quantities produced when the principal is wealth constrained

to **T**, the principal has to reduce both the direct compensation and the informational rent i.e. reduces the quantity produced by both types of firm to fulfill the wealth constraint. On the top of the traditional second best trade-off between efficiency and rent extraction that leads to distortions in q_2, the wealth constraint implies a third best distortion in quantities q_1 and q_2.

If we call $\mu_1 = \mu_1 - v_1$, we can rewrite (7) and (8) in order to isolate the second and third best distortions:

$$q_1^{WC} = S'^{-1}(\theta_1 + \frac{\mu_1'}{v_1}\theta_1) \qquad (12)$$

$$q_2^{WC} = S'^{-1}(\theta_2 + \frac{v_1}{v_2}\Delta\theta + \frac{\mu_1'}{v_1}\Delta\theta) \qquad (13)$$

In these two expressions, the last terms on the right hand member measure the distortions imposed to keep the transfer T_1 equals to **T**. These distortions depend on the highest possible transfer **T**. As part (iii) of proposition 2 shows, μ_1' increases when the wealth constraint becomes more severe. The lower **T** is, the larger are the distortions in quantities. But these distortions are unequally distributed among q_1 and q_2. The wealth constrained regulator selects the most efficient way to reduce the transfer paid. For that, it compares the efficiency cost of reducing the information rent (reducing q_2) and the efficiency cost of reducing the production cost (reducing q_1). The efficiency cost is measured by the reduction in the expected surplus. If $v_1\Delta\theta$ is larger (resp. lower) than θ_1, a given reduction in q_2 reduces more (resp. less) the transfer than a similar reduction in q_1. Consequently, q_2 will be relatively more (resp. less) reduced than q_1.

The addition of a third best distortion in q_1 and q_2 may lead to the collapse of the incentive system. It will be the case if the quantities q_1^{WC} and q_2^{WC} do not satisfy the necessary condition for implementation, namely keeping q_1 greater than q_2.[6] If μ_1 is such that $q_1^{WC} < q_2^{WC}$, the only feasible mechanism is a pooling mechanism where the firm produces the same quantity whatever its cost. If $\theta_1 > v_1\theta_2$, there is a level of μ_1' and a corresponding level of **T** (called **T***) such that the value of q_1 given by (7) is smaller than the value of q_2 given by (8).

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6   If $q_2 < q_1$, it is impossible to satisfy the two incentive constraints at the same time. This is not an issue in the second best problem, because only the action of the inefficient firm θ_2 is distorted.

Therefore, for $\mathbf{T} \leq \mathbf{T}^*$, the only feasible mechanism is a pooling mechanism.[7]

Given that the equilibrium is not always separating, the implications in terms of regulatory policy are drawn in the following corollary:

Corollary 1: *The optimal regulation policy for a wealth-constrained principal is a fixed fee* $\mathbf{T}$ *for quantity* $\mathbf{q} = \mathbf{T}/\theta_2$ *when the separating equilibrium does not exist and, otherwise, a two part tariff where the firm receives* $\mathrm{T}_2 = \theta_2 \mathrm{q}_2^{\mathrm{WC}}$ *for the quantity* $\mathrm{q}_2^{\mathrm{WC}}$ *and a bonus* $\Delta \mathrm{T} = \mathbf{T} - \mathrm{T}_2$ *if the firm produces the additional quantities* $\Delta \mathrm{q} = \mathrm{q}_1^{\mathrm{WC}} - \mathrm{q}_2^{\mathrm{WC}}$.

The wealth constraint has two consequences on the regulatory scheme designed by the principal. First, there is under provision of the public good compared to the second best equilibrium. Second, in the case where the separating equilibrium does not exist, the regulation policy is less efficient. In this case, the principal uses a low-powered regulation scheme rather than a (high powered) two-parts tariff. Regulation with fixed fee implies additional welfare losses due to the use of a less efficient regulatory instrument. The wealth constraint not only reduces the quantities of the good but also the number of instruments available to the regulator.

4   Concluding remarks

The development of high powered incentive regulatory scheme is a challenge for developing countries. Cook (1999) and Laffont (2000) document that governments often lack the power to enforce sophisticated regulatory structures. This paper argues that a possible explanation for not using those kind of financing schemes could come from financial constraints that regulators face. Governments might (optimally) prefer low powered incentive scheme like fixed fee, not only because they cannot manage more sophisticated ones but also because it is efficient to use those schemes when they are financially constrained. Using incentive mechanisms in procurement contracts is optimal only if the benefits exceed the costs. This is not always true

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7   In the approach of Thomas (2002), there is bunching whenever $\mathbf{T} \leq \theta_2 \mathrm{q}_2^{\mathrm{SB}}$. The difference comes from the fact that Thomas considers that all types of agent for which the second best transfer exceeds the financial constraint receives the same contract while it is not necessarily the case in this paper.

when public funds are scarce. Financially constrained governments may prefer to spend all their money for infrastructure spending rather than using part of it to finance incentive procurement contract.

Competition for the procurement contract would be another way to overcome the adverse effect of the wealth constraint. Competition in procurement results in lower information rent for the elected firm.[8] Hence, governments can finance larger infrastructure with the same funds. However, competition does not guarantee the use of the more efficient regulatory instrument[9] and the results of this paper continue to hold when several firms compete for the procurement contract.[10]

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8   Laffont and Tirole (1993).

9   Competition for the procurement contract changes the incentive constraints but the logic behind the proof of proposition 2 remains the same.

10   Cook (1999) and Laffont (2000) show that achieving a sufficient degree of competition in procurement is not easy to settle in developing countries.

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Régulation en présence d'une contrainte financière

Cet article étudie un problème standard de régulation (Laffont et Tirole, 1993) où le régulateur fait face à une contrainte de liquidité. L'introduction d'une telle contrainte réduit non seulement le montant de bien public produit mais également les instruments disponibles pour le régulateur. La contrainte financière peut modifier le contrat de régulation optimal d'un tarif en deux parties, où les quantités produites par la firme dépendent de son coût, à un payement forfaitaire, moins efficace, et où la firme produit la même quantité indépendamment de son coût.

Regulierung unter finanziellen Zwängen

Der Beitrag untersucht ein simples Vergabeproblem (Laffont und Tirole, 1993), bei dem der Regulator einem Vorauszahlungszwang („cash-in-advance constraint') ausgesetzt ist. Ein solcher Zwang reduziert nicht nur die Menge des bereitgestellten öffentlichen Gutes, sondern schränkt auch die Instrumente ein, die dem Regulator zur Verfügung stehen. Der „wealth constraint' könnte zu einem Wechsel des optimalen regulatorischen Kontrakts führen: von einem Two-part-Tarif, bei dem die produzierten Mengen von den Kosten des Unternehmens abhängen, zu einem weniger effizienten Festbetrag („fixed fee'), bei dem das Unternehmen dieselbe Menge, unabhängig von seinen Kosten, produziert.

Regulación en presencia de un apremio financiero

Este artículo estudia un problema estándar de regulación (Laffont y Tirole, 1993), en el que el regulador hace frente a un apremio de liquidez. La introducción de tal apremio reduce no solamente el valor del bien público producido sino también los instrumentos disponibles por el regulador. El apremio financiero puede modificar el contrato de regulación óptimo de una tarifa en dos aspectos, o bien las cantidades producidas por la firma dependen de su coste, con un pago a tanto alzado, menos eficiente, o la firma produce la misma cantidad inde-pendientemente de su coste.

Appendix: Proof of proposition 2

We use the following result to simplify [P2]:

Lemma 1: When $\mathbf{T} \leq \theta_1 q_1^{SB} + \Delta\theta q_2^{SB}$, *the transfer paid to the low cost firm equals* $\mathbf{T}$.

Proof: If $\mathbf{T} < T_1^{SB}$, the solution of [P1] cannot be replicated in [P2]. Then, at least one of the transfers in [P2] is given by the constraint (WC). A necessary condition for implementation is: $q_1 \geq q_2$ and $T_1 \geq T_2$. Then the constraint (WC) binds (at least) for T_1.

Using the relevant constraints (IC_1), (IR_2) and the result of lemma 1, the program [P2] can be rewritten as:

Program[P3]:

$$\underset{q_1, q_2}{Max} \quad v_1(S(q_1) - \mathbf{T}) + v_2(S(q_2) - \theta_2 q_2)$$

subject to:

$$\mathbf{T} = \theta_1 q_1 + \Delta\theta q_2 \qquad (\mu_1)$$

$$T_2 = q_2 \theta_2 \leq \mathbf{T} \qquad (\mu_2)$$

Where we indicate the Lagrange multipliers in brackets. The second constraint is equivalent to $q_2 \leq q_1$.

The first order conditions of [P3] are:

$$S'(q_1) = \frac{\mu_1}{v_1}\theta_1 \qquad (14)$$

$$S'(q_2) = \theta_2 + \frac{\mu_1}{v_2}\Delta\theta + \frac{\mu_2}{v_2}\theta_2 \qquad (15)$$

$$\mathbf{T} - \theta_1 q_1 - \Delta\theta q_2 = 0 \qquad (16)$$

$$\mu_2(\mathbf{T} - \theta_2 q_2) = 0 \qquad (17)$$

We know by lemma 1 that $\mu_1 > 0$ if the wealth constraint is relevant. There are two possible solutions to this system of equation: a separating solution when $\mu_2 = 0$ and a pooling solution when μ_2 is positive.

If $\mu_2 > 0$, (17) becomes $\mathbf{T} = \theta_2 q_2$, then $q_2 = \mathbf{T}/\theta_2$. Replacing this value in (16), we have $q_1 = q_2 = \mathbf{T}/\theta_2$.

If $\mu_2 = 0$, the separating solution is given by:

$$S'(q_1) = \frac{\mu_1}{v_1}\theta_1 \tag{18}$$

$$S'(q_2) = \theta_2 + \frac{\mu_1}{v_2}\Delta\theta \tag{19}$$

$$\mathbf{T} - \theta_1 q_1 - \Delta\theta q_2 = 0 \tag{20}$$

To know which solution applies, we check when μ_2 is positive. As long as q_2 is smaller than q_1, the transfer T_2 is smaller than $\mathbf{T}$. Therefore, the second wealth constraint is slack when $q_2 \leq q_1$. This corresponds to the following condition:

$$\frac{\mu_1}{v_1}\theta_1 \geq \theta_2 + \frac{\mu_1}{v_2}\Delta\theta \tag{21}$$

where the value of μ_1 is given by (20). Take the limit case where (21) is satisfied with equality, and solve for μ_1 we have:

$$\mu_1 = \frac{v_1 v_2 \theta_2}{\theta_1 - v_1 \theta_2}.$$

As long as the actual μ_1 is smaller than this value (call it $\mu_1{}^*$), q_2 is smaller than q_1. $\mu_1{}^*$ is negative if $\theta_1 \leq v_1\theta_2$. In this case, whatever $\mathbf{T}$, q_2 is smaller than q_1, except in the limit case where $\mathbf{T}$ is null where both quantities are equal to zero.

If $\theta_1 > v_1\theta_2$, we have to find the value of $\mathbf{T}$ that generates value of μ_1 equals to $\mu_1{}^*$. To do this, we solve (18), (19) and (20) for $\mathbf{T}$ when $\mu_1 = \mu_1{}^*$. This gives a value $\mathbf{T}^* = \theta_2 S'^{-1}(\frac{\theta_1\theta_2 v_2}{\theta_1 - v_1\theta_2})$.

Hence, when $\mathbf{T} \leq \mathbf{T}^*$ and $\theta_1 > v_1\theta_2$, μ_2 is positive and the solution is the pooling equilibrium. When $\mathbf{T} \geq \mathbf{T}^*$, μ_2 is null and the solution is the separating equilibrium.

Now we can show that μ_1 increases when $\mathbf{T}$ decreases: (i) From the first order conditions we have: $\mathbf{T} = \theta_1 S'^{-1}(\frac{\mu_1}{v_1}\theta_1) + \Delta\theta S'^{-1}(\theta_2 + \frac{\mu_1}{v_2}\Delta\theta)$. Call the right hand side $G(\mu_1)$. Then $\mu_1 = G^{-1}(\mathbf{T})$. Given our assumptions on $S(.)$, $G(.)$ is increasing and concave, because S'^{-1} is increasing and concave. Then G^{-1} is decreasing and convex. (ii) At the limit when $\mathbf{T}$ goes to T_1^{SB}, the problem is identical to [P1] and therefore the solution is identical. i.e. $\mu_1 = v_1$. When $\mathbf{T}$ goes to zero, the right hand side of equation (9) must go to zero. Given that $S'(0) = +\infty$, we have that $G^{-1}(0) = +\infty$.

The second order conditions of [P3] are always satisfied thanks to the concavity of the problem.